NMPaMŠ – 3.cvičenie

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Riešenie sústav nelineárnych rovníc

Newtonova iteračná metóda

sústava rovníc $f_1(x, y) = 0$
$f_2(x, y) = 0$

po úpravách

$$\frac{\partial f_1(x^{(k)}, y^{(k)})}{\partial x}(x^{(k+1)} - x^{(k)}) + \frac{\partial f_1(x^{(k)}, y^{(k)})}{\partial y}(y^{(k+1)} - y^{(k)}) = -f_1(x^{(k)}, y^{(k)}),$$

$$\frac{\partial f_2(x^{(k)}, y^{(k)})}{\partial x}(x^{(k+1)} - x^{(k)}) + \frac{\partial f_2(x^{(k)}, y^{(k)})}{\partial y}(y^{(k+1)} - y^{(k)}) = -f_2(x^{(k)}, y^{(k)}).$$

Neznáme $(x^{(k+1)} - x^{(k)})$, $(y^{(k+1)} - y^{(k)})$, k – iterácia (k = 0, 1, 2, ...)

Cramerovo pravidlo pre riešenie sústavy

$$W_1^{(k)} = \begin{vmatrix} -f_1(x^{(k)}, y^{(k)}) & \frac{\partial f_1(x^{(k)}, y^{(k)})}{\partial y} \\ -f_2(x^{(k)}, y^{(k)}) & \frac{\partial f_2(x^{(k)}, y^{(k)})}{\partial y} \end{vmatrix}, \quad W_2^{(k)} = \begin{vmatrix} \frac{\partial f_1(x^{(k)}, y^{(k)})}{\partial x} & -f_1(x^{(k)}, y^{(k)}) \\ \frac{\partial f_2(x^{(k)}, y^{(k)})}{\partial x} & -f_2(x^{(k)}, y^{(k)}) \end{vmatrix},$$

Cramerovo pravidlo pre riešenie sústavy

$$\frac{\partial f_1(x^{(k)},y^{(k)})}{\partial x}(x^{(k+1)}-x^{(k)})+\frac{\partial f_1(x^{(k)},y^{(k)})}{\partial y}(y^{(k+1)}-y^{(k)})=-f_1(x^{(k)},y^{(k)}),$$

$$\frac{\partial f_2(x^{(k)},y^{(k)})}{\partial x}(x^{(k+1)}-x^{(k)})+\frac{\partial f_2(x^{(k)},y^{(k)})}{\partial y}(y^{(k+1)}-y^{(k)})=-f_2(x^{(k)},y^{(k)}).$$

$$W^{(k)}=\begin{vmatrix}\frac{\partial f_1(x^{(k)},y^{(k)})}{\partial x} & \frac{\partial f_1(x^{(k)},y^{(k)})}{\partial y}\\ \frac{\partial f_2(x^{(k)},y^{(k)})}{\partial x} & \frac{\partial f_2(x^{(k)},y^{(k)})}{\partial y}\end{vmatrix}$$

$$x^{(k+1)}-x^{(k)}=\frac{W_1{}^{(k)}}{W^{(k)}}$$

$$y^{(k+1)}-y^{(k)}=\frac{W_2{}^{(k)}}{W^{(k)}}$$

Pr. 1: Newtonovou iteračnou metódou s presnosťou $\varepsilon = 0{,}05$ riešte sústavu nelineárnych rovníc $xy - x + 1 = 0,\ \ 2x^2 + y - 3 = 0$ z počiatočného bodu $x^{(0)} = 1, y^{(0)} = 0$.

1. **Zapíšeme sústavu rovníc $f_1(x, y)$, $f_2(x, y)$, určíme $-f_1(x, y)$, $-f_2(x, y)$, urobíme ich parciálne derivácie podľa *x* a *y***

2. **Pomocou Cramerovho pravidla určíme determinanty $W^{(k)}$, $W_1^{(k)}$, $W_2^{(k)}$**

$$W_1^{(k)} = \begin{vmatrix} -f_1(x^{(k)}, y^{(k)}) & \dfrac{\partial f_1(x^{(k)}, y^{(k)})}{\partial y} \\ -f_2(x^{(k)}, y^{(k)}) & \dfrac{\partial f_2(x^{(k)}, y^{(k)})}{\partial y} \end{vmatrix}$$

$$W_2^{(k)} = \begin{vmatrix} \dfrac{\partial f_1(x^{(k)}, y^{(k)})}{\partial x} & -f_1(x^{(k)}, y^{(k)}) \\ \dfrac{\partial f_2(x^{(k)}, y^{(k)})}{\partial x} & -f_2(x^{(k)}, y^{(k)}) \end{vmatrix}$$

$$W^{(k)} = \begin{vmatrix} \dfrac{\partial f_1(x^{(k)}, y^{(k)})}{\partial x} & \dfrac{\partial f_1(x^{(k)}, y^{(k)})}{\partial y} \\ \dfrac{\partial f_2(x^{(k)}, y^{(k)})}{\partial x} & \dfrac{\partial f_2(x^{(k)}, y^{(k)})}{\partial y} \end{vmatrix}$$

3. Hodnoty neznámych $x^{(k+1)}$, $y^{(k+1)}$ sú určené interačným procesom

$$x^{(k+1)} = x^{(k)} + \frac{W_1^{(k)}}{W^{(k)}}$$

$$y^{(k+1)} = y^{(k)} + \frac{W_2^{(k)}}{W^{(k)}}$$

4. Hodnoty zapíšeme do tabuľky

k	$x^{(k)}$	$y^{(k)}$	$W_1^{(k)}$	$W_2^{(k)}$	$W^{(k)}$	$\|\overline{x}^{(k+1)} - \overline{(k)}\| < \varepsilon$

5. Iteračný proces ukončíme

a) ak je daná presnosť ε

$$\|\overline{x}^{(k+1)} - \overline{(k)}\| = \max\{|x^{(k+1)} - x^{(k)}|, |y^{(k+1)} - y^{(k)}|\} < \varepsilon$$

b) ak je daný počet iterácií *k*, proces ukončíme pre danej iterácií

potom chyba výpočtu $\max\{|x^{(k+1)} - x^{(k)}|, |y^{(k+1)} - y^{(k)}|\}$

6. Aproximujeme neznáme hodnotami $x^{(K)}$, $y^{(K)}$

Pr. 2

Newtonovou iteračnou metódou riešte danú sústavu rovníc:

$$9x^2 + y^2 - 10 = 0$$
$$2xy + 4y - 5 = 0$$

z počiatočného bodu $x^{(0)} = 1, y^{(0)} = 0{,}5$. Urobte jednu iteráciu a určte chybu výpočtu.

$$f_1(x, y) = 9x^2 + y^2 - 10 \qquad -f_1(x, y) = -9x^2 - y^2 + 10$$

$$f_2(x, y) = 2xy + 4y - 5 \qquad -f_2(x, y) = -2xy - 4y + 5$$

$$\frac{\partial f_1(x, y)}{\partial x} = 18x \qquad \frac{\partial f_1(x, y)}{\partial y} = 2y$$

$$\frac{\partial f_2(x, y)}{\partial x} = 2y \qquad \frac{\partial f_2(x, y)}{\partial y} = 2x + 4$$

$$\boldsymbol{x^{(0)} = 1, y^{(0)} = 0,5}$$

$$W_1{}^{(k)} = \begin{vmatrix} -9x^2 - y^2 + 10 & 2y \\ -2xy - 4y + 5 & 2x + 4 \end{vmatrix}$$

$$W_1{}^{(0)} = \begin{vmatrix} 0,75 & 1 \\ 2 & 6 \end{vmatrix} = 2,5$$

$$W_2{}^{(k)} = \begin{vmatrix} 18x & -9x^2 - y^2 + 10 \\ 2y & -2xy - 4y + 5 \end{vmatrix}$$

$$W_2{}^{(0)} = \begin{vmatrix} 18 & 0,75 \\ 1 & 2 \end{vmatrix} = 35,25$$

$$W^{(k)} = \begin{vmatrix} 18x & 2y \\ 2y & 2x + 4 \end{vmatrix}$$

$$W^{(0)} = \begin{vmatrix} 18 & 1 \\ 1 & 6 \end{vmatrix} = 107$$

$$x^{(1)} = x^{(0)} + \frac{W_1{}^{(0)}}{W^{(0)}} = 1 + \frac{2,5}{107} = 1,023$$

$$y^{(1)} = y^{(0)} + \frac{W_2{}^{(0)}}{W^{(0)}} = 0,5 + \frac{35,25}{107} = 0,829$$

k	$x^{(k)}$	$y^{(k)}$	$W_1^{(k)}$	$W_2^{(k)}$	$W^{(k)}$
0	1	0,5	2,5	35,25	107
1	1,023	0,829			

$$\max\{|x^{(k+1)} - x^{(k)}|, |y^{(k+1)} - y^{(k)}|\}$$

$$max\{|1{,}023 - 1|, |0{,}829 - 0{,}5|\} = max\{|-0{,}023|, |0{,}329|\} = 0{,}329$$

$x \approx 1{,}023, y \approx 0{,}829$ s chybou 0,329

Pr. 3

Newtonovou iteračnou metódou riešte danú sústavu rovníc: $\sin y - x - 1{,}2 = 0$

$$2y + \cos x - 1{,}7 = 0$$

z počiatočného bodu $x^{(0)} = -1, y^{(0)} = 0{,}2$ (hodnoty sú dané v radiánoch). Urobte jednu iteráciu a určte chybu výpočtu.

$$f_1(x,y) = \sin y - x - 1{,}2 \qquad -f_1(x,y) = -\sin y + x + 1{,}2$$

$$f_2(x,y) = 2y + \cos x - 1{,}7 \qquad -f_2(x,y) = -2y - \cos x + 1{,}7$$

$$\frac{\partial f_1(x,y)}{\partial x} = -1 \qquad \frac{\partial f_1(x,y)}{\partial y} = \cos y$$

$$\frac{\partial f_2(x,y)}{\partial x} = -\sin x \qquad \frac{\partial f_2(x,y)}{\partial y} = 2$$

$$W^{(k)} = \begin{vmatrix} -1 & \cos y \\ -\sin x & 2 \end{vmatrix}$$

$$\boldsymbol{x^{(0)} = -1, y^{(0)} = 0,2}$$

$$W^{(0)} = \begin{vmatrix} -1 & \cos 0{,}2 \\ -\sin(-1) & 2 \end{vmatrix} = -2 - 0{,}8247 = -2{,}8247$$

$$W_1{}^{(k)} = \begin{vmatrix} -\sin y + x + 1{,}2 & \cos y \\ -2y - \cos x + 1{,}7 & 2 \end{vmatrix}$$

$$W_1{}^{(k)} = \begin{vmatrix} 0{,}00133 & 0{,}98006 \\ 0{,}75969 & 2 \end{vmatrix} = -0{,}7419$$

$$W_2{}^{(k)} = \begin{vmatrix} -1 & -\sin y + x + 1{,}2 \\ -\sin x & -2y - \cos x + 1{,}7 \end{vmatrix}$$

$$W_2{}^{(0)} = \begin{vmatrix} -1 & 0{,}00133 \\ 0{,}8415 & 0{,}75969 \end{vmatrix} = -0{,}7609$$

$$x^{(1)} = x^{(0)} + \frac{{W_1}^{(0)}}{W^{(0)}} = -1 + \frac{-0,7419}{-2,8247} = -0,7374$$

$$y^{(1)} = y^{(0)} + \frac{{W_2}^{(0)}}{W^{(0)}} = 0,2 + \frac{-0,7609}{-2,8247} = 0,4693$$

k	$x^{(k)}$	$y^{(k)}$	${W_1}^{(k)}$	${W_2}^{(k)}$	$W^{(k)}$
0	- 1	0,2	- 0,7419	- 0,7608	- 2,8247
1	- 0,7374	0,4693			

$$\max\{|x^{(k+1)} - x^{(k)}|, |y^{(k+1)} - y^{(k)}|\}$$

$$max\{|-0,7374 + 1|, |0,4693 - 0,2|\} = max\{|-0,2626|, |0,2693|\} = 0,2693$$

$x \approx -0,7374, y \approx 0,4693$ s chybou 0,2693

Dú: Príklady na riešenie 1: B / 1 - 5