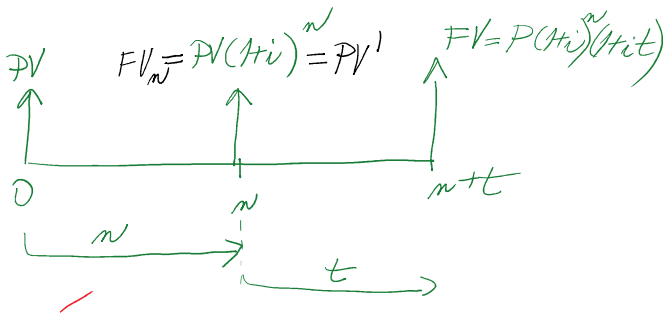
Zmiešané úrokovanie, spojité úrokovanie

streda 26. februára 2025 10:00

ZMIEŠANÉ ÚROKOVANIE

$$FV = PV(1+i)^n(1+it)$$

PV'



ÚROČENIE

PR1 $PV = 1500$

$i = 0{,}05$

$t = \frac{212}{360}$

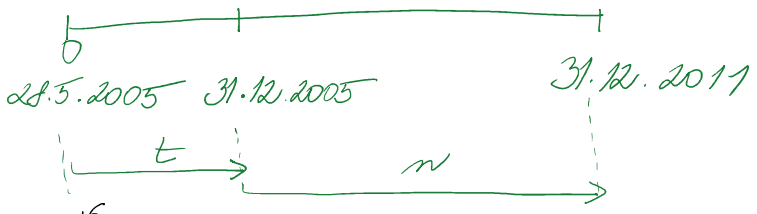
$n = 6$

$FV = ?$

$FV = PV(1+it)(1+i)^n$

$FV = 1500\left(1+0{,}05\cdot\frac{212}{360}\right)(1+0{,}05)^6$

$FV = 2069{,}331$



$$t = \frac{30-28+7\cdot 30}{360} = \frac{2+210}{360} = \frac{212}{360}$$

$n = 6\ (= 2011-2005)$

PR2 $PV = 1500$

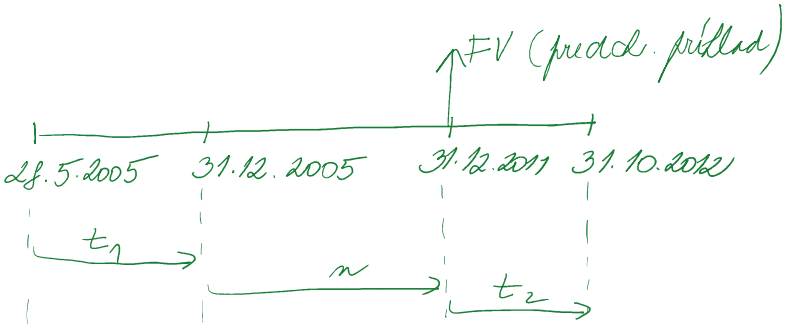
$i = 0{,}05$

$t_1 = \frac{212}{360}$

$n = 6$

$t_2 = \frac{10}{12}$

$FV = ?$



$FV = PV(1+it_1)(1+i)^n(1+it_2)$

$FV = 1500\left(1+0{,}05\cdot\frac{212}{360}\right)(1+0{,}05)^6\left(1+0{,}05\cdot\frac{10}{12}\right)$

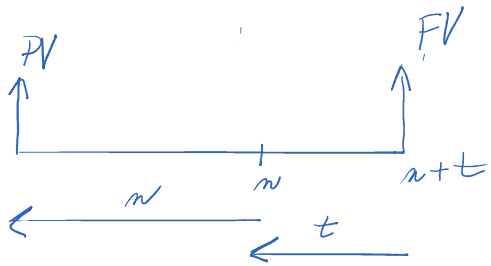
2069,331 (viď predošlý príklad)

$FV = 2155{,}553$

$FV = PV(1+it)(1+i)^n \;/\cdot \frac{1}{(1+it)(1+i)^n}$

$PV = \frac{FV}{(1+it)(1+i)^n}$

ODÚROČENIE



PR 3 $FV = 3200$

$i = 0{,}04$

$n = 3$

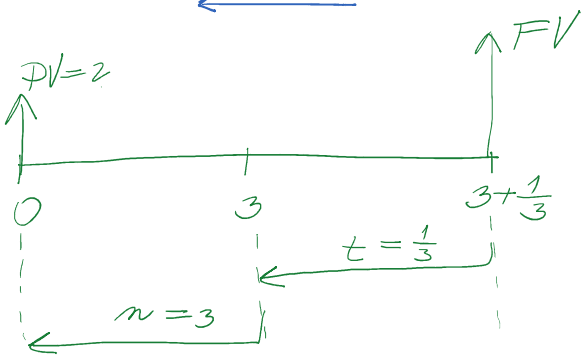
$t = \frac{4}{12} = \frac{1}{3}$

$PV = ?$

$PV = \frac{FV}{(1+i)^n(1+it)}$

$PV = \frac{3200}{(1+0{,}04)^3\,(1+0{,}04\cdot\frac{1}{3})}$

$PV = 2807{,}35$ €



SPOJITÉ ÚROKOVANIE

! $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e$

• $\lim_{n\to\infty}\left(1+\frac{k}{n}\right)^n = e^k$ resp. $\lim_{x\to\infty}\left(1+\frac{k}{x}\right)^x = e^k$ •

$FV_n = PV\cdot\left(1+\frac{j}{m}\right)^{m\cdot n}$! $(x^a)^b = x^{a\cdot b}$!

$\lim_{m\to\infty}\left(1+\frac{j}{m}\right)^{m\cdot n} = \lim_{m\to\infty}\left[\left(1+\frac{j}{m}\right)^m\right]^n =$

$= \left[\lim_{m\to\infty}\left(1+\frac{j}{m}\right)^m\right]^n = \left[e^j\right]^n = e^{j\cdot n}$

e^j $\quad t$

$FV_t = PV\,e^{jt}$

PR 1 1. $j = 9\%$ $\quad FV_n = 2PV$ $\quad m = 12$ $\quad n = ?$

2. $j = 9\%$ $\quad FV_t = 2PV$ $\quad m \to \infty$ $\quad t = ?$

1. $FV_n = PV\left(1+\frac{j}{m}\right)^{m\cdot n}$ $/\cdot\frac{1}{PV}$ 2 $FV_t = PV\,e^{jt}$ $/\cdot\frac{1}{PV}$

1. $FV_n = PV\left(1+\frac{j}{m}\right)^{m\cdot n}$ $/\cdot\frac{1}{PV}$

$2PV = PV\left(1+\frac{0{,}08}{12}\right)^{12\cdot n}$

$\left(1+\frac{0{,}08}{12}\right)^{12n} = 2 \quad /\ln(\,)$

$\ln\left(1+\frac{0{,}08}{12}\right)^{12n} = \ln 2$

$12n\cdot\ln\left(1+\frac{0{,}08}{12}\right) = \ln 2 \quad /\cdot\frac{1}{12\ln\left(1+\frac{0{,}08}{12}\right)}$

$n = \frac{\ln 2}{12\ln\left(1+\frac{0{,}08}{12}\right)}$

$n = 8{,}69$ rokov

2. $FV_t = PV e^{jt}$ $/\cdot\frac{1}{PV}$

$2PV = PV e^{0{,}08t}$

$e^{0{,}08t} = 2 \quad /\ln(\,)$

$\ln e^{0{,}08t} = \ln 2$

$0{,}08t\cdot\ln e = \ln 2 \quad /\cdot\frac{1}{0{,}08}$

$\ln e = 1$

$\log_a a = 1$

$0{,}08t = \ln 2$

$t = \frac{\ln 2}{0{,}08}$

$t = 8{,}66$ rokov

PR2 $PV = 16\,000$

$FV_4 = 21\,597{,}74$

$t = 4$

$m \to \infty$

$j = ?$

$FV_t = PV\cdot e^{jt}$

$21\,597{,}74 = 16\,000\cdot e^{j\cdot 4} \quad /:16\,000$

$e^{4j} = \frac{21\,597{,}74}{16\,000} \quad /\ln(\,)$

$\ln e^{4j} = \ln\frac{21\,597{,}74}{16\,000}$

$4j\cdot\ln e = \ln\frac{21\,597{,}74}{16\,000} \quad /:4$

$j = \frac{\ln\frac{21\,597{,}74}{16\,000}}{4}$

$j = 0{,}075$

$j\cdot 100\% = 7{,}5\%$

PR3 $FV_{10} = 50\,000$

$j = 0{,}05$

TKJ $FV_{10} = 50\,000$

$j = 0{,}05$

$t = 10$

$m \to \infty$

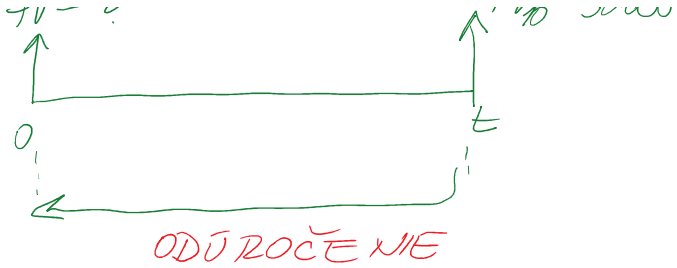
$PV = ?$

$FV_t = PV\, e^{jt}$

$PV = \frac{FV_t}{e^{jt}}$

$PV = \frac{50\,000}{e^{0{,}05 \cdot 10}}$

$PV = 30\,326{,}53$



OPTIMÁLNA DOBA VLASTNENIA

$\frac{V'(t)}{V(t)}$ ← zmena

… relatívna miera (zmeny)

← pôvodná hodnota

PR $V(t) = 10\,000\, e^{7t}$ ← g

$j = 0{,}05$ ← f

$m \to \infty$

$t = ?$

$(f(g(x)))' = f'(g(x)) \cdot g'(x)$

$(t^{\frac{1}{2}})' = \frac{1}{2} \cdot t^{-\frac{1}{2}}$

$\frac{V'(t)}{V(t)} = \frac{10\,000 \cdot e^{7t} \cdot (7t)'}{10\,000\, e^{7t}} = \frac{1}{2} t^{-\frac{1}{2}} = \frac{1}{2\sqrt{t}}$ (klesajúca ↓)

$\frac{1}{2\sqrt{t}} = 0{,}05$

$\frac{1}{2\sqrt{t}} = \frac{5}{100}$ / $\frac{1}{(\,)}$

$2\sqrt{t} = \frac{100}{5}$ / 1

$2\sqrt{t} = 20$ /:2

$\sqrt{t} = 10$ /()²

$t = 100$

OPLATÍ SA PREDAŤ PO 100 ROKOCH