

$\int dx = x + c$	$\int \sin x dx = -\cos x + c$
$\int k dx = kx + c \quad k \in \mathbb{R}$	$\int \cos x dx = \sin x + c$
$\int x^n dx = \frac{x^{n+1}}{n+1} + c \quad \begin{matrix} n \in \mathbb{R} \\ n \neq -1 \end{matrix}$	$\int \frac{1}{\cos^2 x} dx = \tan x + c$
$\int \frac{1}{x} dx = \ln x  + c$	$\int \frac{1}{\sin^2 x} dx = -\cot x + c$
$\int e^x dx = e^x + c$	$\int \frac{1}{a^2+x^2} dx = \frac{1}{a} \arctan \frac{x}{a} + c \quad \begin{matrix} a \neq 0 \\ a \in \mathbb{R} \end{matrix}$
$\int a^x dx = \frac{a^x}{\ln a} + c \quad \begin{matrix} a > 0 \\ a \neq 1 \\ a \in \mathbb{R} \end{matrix}$	$\int \frac{1}{a^2-x^2} dx = \frac{1}{2a} \ln \left  \frac{a+x}{a-x} \right  + c \quad \begin{matrix} a \neq 0 \\ a \in \mathbb{R} \end{matrix}$
$\int \frac{f'(x)}{f(x)} dx = \ln f(x)  + c$	$\int \frac{1}{\sqrt{a^2-x^2}} dx = \arcsin \frac{x}{a} + c \quad \begin{matrix} a \neq 0 \\ a \in \mathbb{R} \end{matrix}$
$c \in \mathbb{R}$	$\int \frac{1}{\sqrt{x^2+a^2}} dx = \ln x + \sqrt{x^2+a^2}  + c$

$(a \pm b)^2 = a^2 \pm 2ab + b^2$	$\sin^2 x + \cos^2 x = 1$
$a^2 - b^2 = (a+b)(a-b)$	$\sin^2 x = \frac{1 - \cos 2x}{2}$
$\sin 2x = 2 \sin x \cos x$	$\cos^2 x = \frac{1 + \cos 2x}{2}$
$\cos 2x = \cos^2 x - \sin^2 x$	

P1:  $\int \left( 4x^3 - 6x^2 + 2x - 3 + \frac{1}{4x^4} - \frac{3}{5x} + 4\sqrt{x^5} - \frac{3}{\sqrt{x^2}} \right) dx =$

$= \int 4x^3 dx - \int 6x^2 dx + \int 2x dx - \int 3 dx + \int \frac{1}{4x^4} dx - \int \frac{3}{5x} dx + \int 4\sqrt{x^5} dx - \int \frac{3}{\sqrt{x^2}} dx =$

$= 4 \int x^3 dx - 6 \int x^2 dx + 2 \int x dx - 3 \int 1 dx + \frac{1}{4} \int x^{-4} dx - \frac{3}{5} \int \frac{1}{x} dx + 4 \int x^{\frac{5}{2}} dx - 3 \int x^{-\frac{1}{2}} dx =$

$= 4 \cdot \frac{x^4}{4} - 6 \cdot \frac{x^3}{3} + 2 \cdot \frac{x^2}{2} - 3x + \frac{1}{4} \cdot \frac{x^{-3}}{-3} - \frac{3}{5} \ln|x| + 4 \cdot \left( \frac{x^{\frac{5}{2}+1}}{\frac{5}{2}+1} \right) - 3 \cdot \frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + c =$

$= x^4 - 2x^3 + x^2 - 3x - \frac{1}{12x^3} - \frac{3}{5} \ln|x| + 2\sqrt{x^7} - 9\sqrt{x} + c$

Formulas:  $\int x^n dx = \frac{x^{n+1}}{n+1} + c \quad \begin{matrix} c \in \mathbb{R} \\ n \in \mathbb{R} \\ n \neq -1 \end{matrix}$        $\int \frac{1}{x} dx = \ln|x| + c$        $\int \frac{1}{\sqrt{x}} dx = 2\sqrt{x} + c$

P2:  $\int \frac{(4-x)^2}{4x^2} dx = \int \frac{16-8x+x^2}{4x^2} dx = \int \left( \frac{16}{4x^2} - \frac{8x}{4x^2} + \frac{x^2}{4x^2} \right) dx =$

$\int \left( \frac{4}{x^2} - \frac{2}{x} + \frac{1}{4} \right) dx = 4 \int x^{-2} dx - 2 \int \frac{1}{x} dx + \frac{1}{4} \int 1 dx = -\frac{4}{x} - 2 \ln|x| + \frac{x}{4} + c = -\frac{4}{x} - 2 \ln|x| + \frac{x}{4} + c$

$\int \frac{1}{x^2} dx = \int x^{-2} dx = \frac{x^{-2+1}}{-2+1} = -\frac{1}{x} + c$

$(a-b)^2 = a^2 - 2ab + b^2$

Formulas:  $\int x^n dx = \frac{x^{n+1}}{n+1} + c$        $\int \frac{1}{x} dx = \ln|x| + c$        $\int k dx = k \int dx = kx + c$

P3:  $\int \frac{(3^x+4^x)^2}{12^x} dx = \int \frac{3^{2x} + 2 \cdot 3^x \cdot 4^x + 4^{2x}}{12^x} dx =$

$= \int \left( \frac{3^x \cdot 3^x}{3^x \cdot 4^x} + \frac{2 \cdot 3^x \cdot 4^x}{3^x \cdot 4^x} + \frac{4^x \cdot 4^x}{3^x \cdot 4^x} \right) dx =$

$= \int \left( \left(\frac{3}{4}\right)^x + 2 + \left(\frac{4}{3}\right)^x \right) dx = \frac{\left(\frac{3}{4}\right)^x}{\ln \frac{3}{4}} + 2x + \frac{\left(\frac{4}{3}\right)^x}{\ln \frac{4}{3}} + c$

Formulas:  $\int a^x dx = \frac{a^x}{\ln a} + c \quad \begin{matrix} a > 0 \\ a \neq 1 \\ a \in \mathbb{R} \end{matrix}$

$5^{2x} = 5^x \cdot 5^x$   
 $4^{2x} = 4^x \cdot 4^x$   
 $12^x = (3 \cdot 4)^x = 3^x \cdot 4^x$   
 $(a \cdot b)^x = a^x \cdot b^x$   
 $\frac{a^x}{b^x} = \left(\frac{a}{b}\right)^x$

P4:  $\int \frac{\cos 2x}{\sin^2 2x} dx = \int \frac{\cos^2 x - \sin^2 x}{4 \sin^2 x \cos^2 x} dx =$

$\int \left( \frac{\cos^2 x}{4 \sin^2 x \cos^2 x} - \frac{\sin^2 x}{4 \sin^2 x \cos^2 x} \right) dx = \int \left( \frac{1}{4 \sin^2 x} - \frac{1}{4 \cos^2 x} \right) dx =$

$= -\frac{1}{4} \cot x - \frac{1}{4} \tan x + c$

Formulas:  $\int \frac{1}{\cos^2 x} dx = \tan x + c$   
 $\int \frac{1}{\sin^2 x} dx = -\cot x + c$   
 $\cos 2x = \cos^2 x - \sin^2 x$   
 $\sin 2x = 2 \sin x \cos x$

P5:  $\int \cos^2 x dx = \int \frac{1 + \cos 2x}{2} dx =$

$= \int \left( \frac{1}{2} + \frac{\cos 2x}{2} \right) dx = \frac{1}{2}x + \frac{1}{2} \frac{\sin 2x}{2} + c = \frac{2x + \sin 2x}{4} + c$

Formulas:  $\int \cos x dx = \sin x + c$   
 $\int \cos kx dx = \frac{\sin kx}{k} + c$   
 $\cos^2 x = \frac{1 + \cos 2x}{2}$

P6:  $\int \sin^2 x dx = \int \frac{1 - \cos 2x}{2} dx = \int \left( \frac{1}{2} - \frac{\cos 2x}{2} \right) dx =$

$= \frac{1}{2}x - \frac{1}{2} \frac{\sin 2x}{2} + c = \frac{2x - \sin 2x}{4} + c$

P7:  $\int \frac{1}{1-x^2} \cdot \arcsin x dx = \int \frac{1}{\sqrt{1-x^2}} \cdot \frac{1}{\arcsin x} dx =$

$= \ln|\arcsin x| + c$

Formulas:  $\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + c$   
 $\int \frac{1}{x} dx = \ln|x| + c$

WAAR:  $\frac{d}{dx} \arcsin x = \frac{1}{\sqrt{1-x^2}}$

$\int \frac{dx}{\sqrt{1-x^2}} = \arcsin x = t$   
 $\frac{1}{\sqrt{1-x^2}} \cdot dx = 1 \cdot dt$   
 $\int \frac{dt}{t} = \int \frac{1}{t} dt = \ln|t| + c$

P8:  $\int \frac{x^2+7}{7+x^2} dx = \int \left( \frac{x^2+7}{7+x^2} - \frac{7}{7+x^2} \right) dx = \int \left( 1 - \frac{7}{7+x^2} \right) dx =$

$= x - 7 \cdot \frac{1}{\sqrt{7}} \arctan \frac{x}{\sqrt{7}} + c$

Formulas:  $\int \frac{1}{a^2+x^2} dx = \frac{1}{a} \arctan \frac{x}{a} + c \quad \begin{matrix} a \neq 0 \\ a \in \mathbb{R} \end{matrix}$

P9:  $\int \frac{x-3+3-3}{\sqrt{x+3}} dx = \int \frac{x+3-6}{\sqrt{x+3}} dx = \int \left( \frac{x+3}{\sqrt{x+3}} - \frac{6}{\sqrt{x+3}} \right) dx =$

$= \int (x+3)^{\frac{1}{2}} dx - 6 \int (x+3)^{-\frac{1}{2}} dx = \left. \frac{(x+3)^{\frac{3}{2}}}{\frac{3}{2}} - 6 \frac{(x+3)^{\frac{1}{2}}}{\frac{1}{2}} \right|_{x=0}^{x=5} = \frac{2}{3} (x+3)^{\frac{3}{2}} - 12 (x+3)^{\frac{1}{2}} + c$

$= \frac{2}{3} \cdot \frac{8\sqrt{3}}{3} - 12 \cdot \frac{2\sqrt{3}}{2} + c = \frac{16\sqrt{3}}{9} - 12\sqrt{3} + c = \frac{16\sqrt{3} - 108\sqrt{3}}{9} + c = -\frac{92\sqrt{3}}{9} + c$

Formulas:  $\int x^n dx = \frac{x^{n+1}}{n+1} + c$