PER PARTES: $\boxed{\int u(x)\, v'(x)\, dx = u(x)\, v(x) - \int u'(x)\, v(x)\, dx}$

$\overset{u(x)}{P_n(x)}\ \overset{v'(x)}{e^{kx}}$	$\overset{v'(x)}{P_n(x)}\ \overset{u(x)}{\ln x}$	$\int e^{kx}dx = \frac{e^{kx}}{k} + c$
$P_n(x)\, a^x$	$P_n(x) \arcsin x$	$\int \cos kx\, dx = \frac{\sin kx}{k} + c$
$P_n(x) \sin kx$	$P_n(x) \arccos x$	$\int \sin kx\, dx = -\frac{\cos kx}{k} + c$
$P_n(x) \cos kx$	$P_n(x) \operatorname{arctg} x$	
	$P_n(x) \operatorname{arccotg} x$	

Pr: $\int (3x^2+2x+1)\cos(4x)\,dx = \left| \begin{matrix} u = 3x^2+2x+1 & v' = \cos 4x \\ u' = 6x+2 & v = \frac{\sin 4x}{4} \end{matrix} \right| =$

$= (3x^2+2x+1)\cdot\frac{\sin 4x}{4} - \frac{1}{2}\int (3x+1)\cdot\sin(4x)\,dx = \left| \begin{matrix} u = 3x+1 & v' = \sin 4x \\ u' = 3 & v = -\frac{\cos 4x}{4} \end{matrix} \right|$

$= (3x^2+2x+1)\cdot\frac{\sin 4x}{4} - \frac{1}{2}\left[(3x+1)\cdot\left(-\frac{\cos 4x}{4}\right) + \frac{3}{4}\int \cos 4x\,dx\right] =$

$= (3x^2+2x+1)\cdot\frac{\sin 4x}{4} + \frac{1}{8}(3x+1)\cos(4x) - \frac{3}{8}\cdot\frac{\sin 4x}{4} + c =$

$= \frac{1}{4}\left(3x^2-2x-\frac{5}{8}\right)\sin(4x) + \frac{1}{8}(3x+1)\cos(4x) + C$

Pr: $\int (4x^3+3x^2+2x+1)\ln x\,dx = \left| \begin{matrix} u = \ln x & v' = 4x^3+3x^2+2x+1 \\ u' = \frac{1}{x} & v = x^4+x^3+x^2+x \end{matrix} \right| =$

$= (x^4+x^3+x^2+x)\ln x - \int (x^3+x^2+x+1)\,dx =$

$= (x^4+x^3+x^2+x)\ln x - \frac{x^4}{4} + \frac{x^3}{3} + \frac{x^2}{2} + x + c$

Pr: $\int (4x+3)\cdot 7^x\,dx = \left| \begin{matrix} u = 4x+3 & v' = 7^x \\ u' = 4 & v = \frac{7^x}{\ln 7} \end{matrix} \right| =$

$= (4x+3)\cdot\frac{7^x}{\ln 7} - \frac{4}{\ln 7}\int 7^x\,dx = (4x+3)\frac{7^x}{\ln 7} - \frac{4}{\ln 7}\cdot\frac{7^x}{\ln 7} + c =$

$= \frac{7^x}{\ln 7}\left(4x+3-\frac{4}{\ln 7}\right) + c$

Pr: $\int 1\cdot\arcsin x\,dx = \left| \begin{matrix} u = \arcsin x & v' = 1 \\ u' = \frac{1}{\sqrt{1-x^2}} & v = x \end{matrix} \right| =$

$= \overset{v\cdot u}{x \arcsin x} - \int \frac{\overset{\to u'\cdot v}{x}}{\sqrt{1-x^2}}\,dx = x\arcsin x + \sqrt{1-x^2} + c$

$\int \frac{x}{\sqrt{1-x^2}}\,dx = \left| \begin{matrix} 1-x^2 = t \\ -2x\,dx = dt \\ x\,dx = \frac{dt}{-2} \end{matrix} \right| = -\frac{1}{2}\int \frac{1}{\sqrt{t}}\,dt = -\frac{1}{2}\int t^{-\frac{1}{2}}\,dt = -\frac{1}{2}\,\frac{t^{\frac{1}{2}}}{\frac{1}{2}} + c =$

$= -\sqrt{t} + c = -\sqrt{1-x^2} + C$

Pr: $\int \frac{\ln x}{\sqrt{x}}\,dx = \int \frac{1}{\sqrt{x}}\cdot\ln x\,dx = \left| \begin{matrix} u = \ln x & v' = \frac{1}{\sqrt{x}} = x^{-\frac{1}{2}} \\ u' = \frac{1}{x} & v = 2\sqrt{x} \end{matrix} \right| =$

$= 2\sqrt{x}\ln x - 2\int \frac{\sqrt{x}}{x}\,dx = 2\sqrt{x}\ln x - 2\int \frac{1}{\sqrt{x}}\,dx =$

$= 2\sqrt{x}\ln x - 4\sqrt{x} + c = 2\sqrt{x}(\ln x - 2) + c$

Pr: $\int 1\cdot\ln^2 x\,dx = \left| \begin{matrix} u = \ln^2 x & v' = 1 \\ u' = \frac{2\ln x}{x} & v = x \end{matrix} \right| = x\ln^2 x - 2\int 1\cdot\ln x\,dx = \left| \begin{matrix} u = \ln x & v' = 1 \\ u' = \frac{1}{x} & v = x \end{matrix} \right|$

$= x\ln^2 x - 2\left[x\ln x - \int dx\right] = x\ln^2 x - 2x\ln x + 2x + c =$

$= x(\ln^2 x - 2\ln x + 2) + c$

INTEGROVANIE RACIONÁLNEJ FUNKCIE:

$\int \frac{2x^4+5x^3-x^2+12x+14}{x^4+2x^3-x^2+4x+12}\,dx \overset{\text{ROZKLAD FCIE NA PARCIÁLNE ZLOMKY}}{=} \cdots = \int\left(2 - \frac{2}{(x+2)^2} + \frac{x-1}{x^2-2x+3}\right)dx$

$= \int 2\,dx - 2\int\frac{1}{(x+2)^2}\,dx - \int\frac{x-1}{x^2-2x+3}\,dx =$

$= 2x + \frac{2}{x+2} - \frac{1}{2}\ln|x^2-2x+3| + c$

$\int \frac{1}{(x+2)^2}\,dx = \int (x+2)^{-2}\,dx = \left| \begin{matrix} x+2 = t \\ dx = dt \end{matrix} \right| = \int t^{-2}\,dt = -\frac{1}{t} + c = -\frac{1}{x+2} + c$

$\int \frac{2}{2}\frac{(x-1)}{(x^2-2x+3)}\,dx = \frac{1}{2}\int \frac{\overset{f'}{2x-2}}{\underset{f}{x^2-2x+3}}\,dx =$

$f(x) = x^2-2x+3$

$f'(x) = 2x-2$

$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c$

$= \frac{1}{2}\ln|x^2-2x+3| + c$

Pr: $\int \frac{x-3}{x^4-3x^3+5x^2-5x+2}\,dx \overset{\text{ROZKLAD NA PARCIÁLNE ZLOMKY}}{=} \cdots = \int\left(\frac{1}{x-1} - \frac{1}{(x-1)^2} + \frac{-x+1}{x^2-x+2}\right)dx$

$= \int \frac{\overset{f'}{1}}{\underset{f}{x-1}}\,dx - \int\frac{1}{(x-1)^2}\,dx + \int\frac{-x+1}{x^2-x+2}\,dx =$

$\int \frac{f'}{f}\,dx = \ln|f| + c$

subs. $\left| \begin{matrix} x-1 = t \\ dx = dt \end{matrix} \right| = \int t^{-2}\,dt = -\frac{1}{t} + c$

$= \ln|x-1| + \frac{1}{x-1} - \frac{1}{2}\ln|x^2-x+2| + \frac{1}{\sqrt{7}}\operatorname{arctg}\frac{2x-1}{\sqrt{7}} + c =$

$\boxed{= \frac{1}{x-1} + \ln\left|\frac{x-1}{\sqrt{x^2-x+2}}\right| + \frac{1}{\sqrt{7}}\operatorname{arctg}\frac{2x-1}{\sqrt{7}} + C}$ → VÝSLEDOK !

$\int \frac{-x+1}{x^2-x+2}\,dx = -\int\frac{2(x-1)}{2(x^2-x+2)}\,dx$

$f(x) = x^2-x+2$

$f'(x) = 2x-1$

$-\frac{1}{2}\int\frac{2x-1-1}{x^2-x+2}\,dx = -\frac{1}{2}\left[\int\frac{2x-1}{x^2-x+2}\,dx - \int\frac{1}{x^2-x+2}\,dx\right] =$

$= -\frac{1}{2}\left[\ln|x^2-x+2| - \frac{2}{\sqrt{7}}\operatorname{arctg}\frac{2x-1}{\sqrt{7}}\right] + c$

$\int \frac{1}{x^2-x+2}\,dx = \int\frac{1}{(x-\frac{1}{2})^2+\frac{7}{4}}\,dx = \frac{2}{\sqrt{7}}\operatorname{arctg}\frac{2x-1}{\sqrt{7}} + c$

ÚPRAVA NA ŠTVOREC