Pr: $\int \frac{7x^5+14x^4+41x^3+42x^2+20x-6}{x^4+2x^3+6x^2+10x+5}\,dx = \dots = \int \left(7x - \frac{3}{(x+1)} + \frac{2}{(x+1)^2} + \frac{2x-1}{x^2+5}\right)dx =$

$= 7\int x\,dx - 3\int \frac{1}{x+1}dx + 2\int \frac{1}{(x+1)^2}dx + \int \frac{2x-1}{x^2+5}dx =$

$\int \frac{f'}{f}dx = \ln|f| + c$; $\left| \begin{matrix} x+1 = t \\ dx = dt \end{matrix} \right| \to \int \frac{1}{t}dt$

$= 7\frac{x^2}{2} - 3\ln|x+1| - 2\cdot\frac{1}{x+1} + \ln|x^2+5| - \frac{1}{\sqrt{5}}\operatorname{arctg}\frac{x}{\sqrt{5}} + c =$

$= \frac{7}{2}x^2 - \frac{2}{x+1} + \ln\left|\frac{x^2+5}{(x+1)^3}\right| - \frac{1}{\sqrt{5}}\operatorname{arctg}\frac{x}{\sqrt{5}} + c$

$\int \frac{1}{(x+1)^2}dx = \left| \begin{matrix} x+1 = t \\ dx = dt \end{matrix} \right| = \int \frac{1}{t^2}dt = \int t^{-2}dt = \frac{t^{-1}}{-1} + c = -\frac{1}{t} + c = -\frac{1}{x+1} + c$

$\left\{ \int x^n dx = \frac{x^{n+1}}{n+1} + c \right.$

$\int \frac{2x-1}{x^2+5}dx = \int \frac{2x}{x^2+5}dx - \int \frac{1}{x^2+5}dx = \ln|x^2+5| - \frac{1}{\sqrt{5}}\operatorname{arctg}\frac{x}{\sqrt{5}} + c$

$\left\{ \begin{matrix} \int \frac{f'}{f}dx = \ln|x^2+5| + c \\ \int \frac{1}{x^2+a^2}dx \\ = \frac{1}{a}\operatorname{arctg}\frac{x}{a} + c \end{matrix} \right.$

$\boxed{\begin{matrix} a\ln x = \ln x^a \\ \ln r - \ln s = \ln\frac{r}{s} \end{matrix}}$

Pr: $\int \frac{x-1}{\sqrt[3]{x+1}}dx = \left| \begin{matrix} x+1 = t^3 \\ dx = 3t^2dt \end{matrix} \right| = \int \frac{(t^3-1)-1}{\sqrt[3]{t^3}} \cdot 3t^2dt =$

$x = t^3 - 1$; $t = \sqrt[3]{x+1}$

$\left\{ \sqrt[r]{a^s} = a^{\frac{s}{r}} \right.$

$= \int \frac{t^3-2}{\cancel{t}} \cdot 3t^{\cancel{2}}dt = 3\int (t^4 - 2t)dt = 3\left(\frac{t^5}{5} - \cancel{2}\frac{t^2}{\cancel{2}}\right) + c =$

$= \frac{3}{5}\sqrt[3]{(x+1)^5} - 3\sqrt[3]{(x+1)^2} + c$

Pr: $\int \frac{\sqrt[3]{x}}{x(\sqrt{x} + \sqrt[3]{x})}dx = \left| \begin{matrix} x = t^6 \\ dx = 6t^5dt \end{matrix} \right| = \int \frac{\sqrt[3]{t^6}}{t^6(\sqrt{t^6} + \sqrt[3]{t^6})} \cdot 6t^5dt =$

$t = \sqrt[6]{x}$

$= \int \frac{t^{\frac{6}{3}}}{t^6(t^{\frac{6}{2}} + t^{\frac{6}{3}})} 6t^5dt = 6\int \frac{t^2}{t^6(t^3+t^2)} \cdot t^5dt = 6\int \frac{1\cancel{t^2}}{\cancel{t^2}t(t+1)}dt =$

$= 6\int \frac{1}{t(t+1)}dt = 6\int \left(\frac{1}{t} - \frac{1}{t+1}\right)dt = 6(\ln|t| - \ln|t+1|) + c =$

$= 6\ln\left|\frac{\sqrt[6]{x}}{\sqrt[6]{x}+1}\right| + c$

Pr: $\int \frac{\sqrt[3]{7x-2}}{(7x-2) + \sqrt[6]{(7x-2)^5}}dx = \left| \begin{matrix} 7x-2 = t^6 \\ 7dx = 6t^5dt \end{matrix} \right| = \int \frac{t^2}{t^6+t^5} \cdot \frac{6}{7}t^5dt =$

$t^{\frac{6\cdot5}{6}}$

$dx = \frac{6}{7}t^5dt$; $t = \sqrt[6]{7x-2}$

$= \frac{6}{7}\int \frac{t^2}{\cancel{t^5}(t+1)} \cdot \cancel{t^5}dt = \frac{6}{7}\int \frac{t^2-1+1}{t+1}dt =$

$\left\{ \begin{matrix} a^2 - b^2 = (a+b)(a-b) \\ t^2 - 1\ (t+1) \end{matrix} \right.$

$(t+1)(t-1)$

$= \frac{6}{7}\int\left(\frac{t^2-1}{\cancel{t+1}} + \frac{1}{t+1}\right)dt = \frac{6}{7}\int\left(t - 1 + \frac{1}{t+1}\right)dt = \frac{6}{7}\left(\frac{t^2}{2} - t + \ln|t+1|\right) + c$

$\int \frac{f'}{f}dx$

$= \frac{3}{7}\sqrt[3]{7x-2} - \frac{6}{7}\left(\sqrt[6]{7x-2} - \ln\left|\sqrt[6]{7x-2} + 1\right|\right) + c$

Pr: $\int \frac{\sqrt[6]{4x+5} - 1}{\sqrt[3]{(4x+5)^4} + \sqrt[4]{(4x+5)^5}}dx = \left| \begin{matrix} 4x+5 = t^{12} \\ t = \sqrt[12]{4x+5} \\ 4dx = 12t^{11}dt \\ dx = 3t^{11}dt \end{matrix} \right| =$

$= \int \frac{t^2-1}{t^{16}+t^{15}} \cdot 3t^{11}dt = 3\int \frac{(\cancel{t+1})(t-1)}{t^{\cancel{15}4}(\cancel{t+1})} \cdot \cancel{t^{11}}dt =$

$\left\{ \int x^n dx = \frac{x^{n+1}}{n+1} + c \right.$

$3\int \frac{t-1}{t^4}dt = 3\int\left(\frac{\cancel{t}1}{t^{\cancel{4}3}} - \frac{1}{t^4}\right)dt = 3\int (t^{-3} - t^{-4})dt$

$= 3\left(\frac{t^{-2}}{-2} - \frac{t^{-3}}{-3}\right) + c = -\frac{3}{2} \cdot \frac{1}{t^2} + \frac{1}{t^3} + c = -\frac{3}{2}\frac{1}{\sqrt[6]{4x+5}} + \frac{1}{\sqrt[4]{4x+5}} + c$

$t^2 = \sqrt[12]{(4x+5)^2} = (4x+5)^{\frac{2}{12}} = (4x+5)^{\frac{1}{6}}$

$t^3 = \sqrt[12]{(4x+5)^3} = (4x+5)^{\frac{3}{12}} = (4x+5)^{\frac{1}{4}}$

Pr: $\int \frac{2x+3}{2x+3+\sqrt{2x+3}}dx = \left| \begin{matrix} 2x+3 = t^2 \\ t = \sqrt{2x+3} \\ 2dx = 2t\,dt \\ dx = t\,dt \end{matrix} \right| = \int \frac{t^2}{t^2+t} \cdot t\,dt =$

$= \int \frac{t^2-1+1}{\cancel{t}(t+1)} \cdot \cancel{t}\,dt = \int\left(\frac{(t-1)(\cancel{t+1})}{\cancel{t+1}} + \frac{1}{t+1}\right)dt = \int\left(t - 1 + \frac{1}{t+1}\right)dt =$

$= \frac{t^2}{2} - t + \ln|t+1| + c = \frac{1}{2}(2x+3) - \sqrt{2x+3} + \ln\left|\sqrt{2x+3} + 1\right| + c$

Pr: $\int \frac{x^2}{(4x+5)\sqrt{4x+5}}dx = \left| \begin{matrix} 4x+5 = t^2 \\ x = \frac{t^2-5}{4} \\ 4dx = 2t\,dt \\ dt = \frac{t}{2}dt \end{matrix} \right| = \int \frac{\left(\frac{t^2-5}{4}\right)^2}{t^2\cdot\cancel{t}} \cdot \frac{\cancel{t}}{2}dt =$

$t = \sqrt{4x+5}$

$= \frac{1}{2}\cdot\frac{1}{16}\int \frac{t^4 - 10t^2 + 25}{t^2}dt = \frac{1}{32}\int (t^2 - 10 + 25\cdot t^{-2})dt =$

$= \frac{1}{32}\left(\frac{t^3}{3} - 10t + 25\cdot\frac{t^{-1}}{-1}\right) + C = \frac{1}{32}\left(\frac{1}{3}\sqrt{(4x+5)^3} - 10\sqrt{4x+5} - \frac{25}{\sqrt{4x+5}}\right) + c$

ALEBO: $\frac{1}{32}\sqrt{4x+5}\left(\frac{4x-25}{3} - \frac{25}{4x+5}\right) + C$