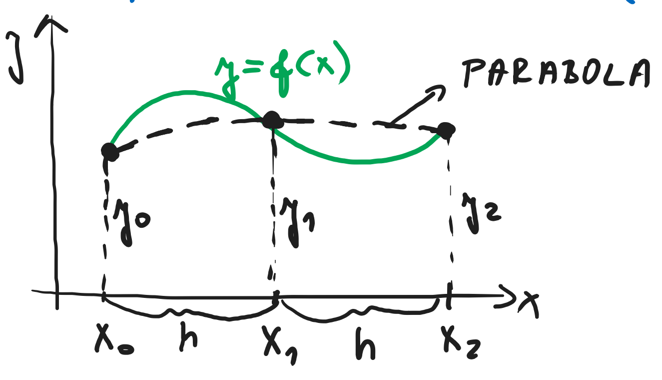
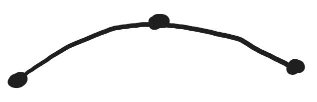
VÝPOČET INTEGRÁLU (SIMPSONOVA METÓDA)



ÚSEČKA

LICH. MET.



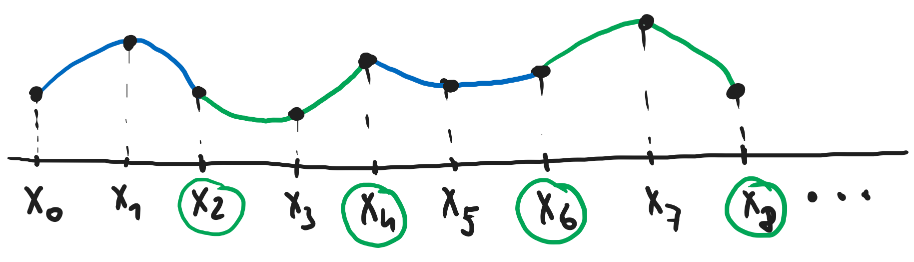
PARABOLA

SIMPSON. MET

NA INTERVALE $\langle x_0; x_2 \rangle$ APPROXIMUJEME ZELENÚ F-CIU POLYNÓMOM 2. STUPŇA (PARABOLA - POMOCOU LAGRANGEOVHO INTERPOLAČNÉHO POLYNÓMU).

PLATÍ (BEZ ODVODENIA):

$$\int_{x_0}^{x_2} f(x)\,dx \approx \frac{h}{3}\left[f(x_0)+4f(x_1)+f(x_2)\right] = \frac{h}{3}\left[y_0+4y_1+y_2\right]$$



K VYTVÁRANIU TROJÍC POTREBUJEME MAŤ K DISPOZÍCII PÁRNY POČET INTERVALOV $(n = 2k) \Rightarrow x_{2k}$

POSLEDNÝ UZLOVÝ BOD

PRE VÝPOČET INTEGRÁLU NA INTERVALE $\langle a, b \rangle$ PLATÍ:

$$\int_a^b f(x)\,dx = \int_{x_0}^{x_{2k}} f(x)\,dx = \int_{x_0}^{x_2} f(x)\,dx + \int_{x_2}^{x_4} f(x)\,dx + \int_{x_4}^{x_6} f(x)\,dx + \dots + \int_{x_{2k-2}}^{x_{2k}} f(x)\,dx \approx$$

$$\approx \frac{h}{3}\left[y_0+4y_1+y_2\right] + \frac{h}{3}\left[y_2+4y_3+y_4\right] + \frac{h}{3}\left[y_4+4y_5+y_6\right] + \dots + \frac{h}{3}\left[y_{2k-2}+4y_{2k-1}+y_{2k}\right] + R_S$$

R_S — CHYBA PRI SIMPS. METÓDE (PREDPOKLADÁ SA, ŽE JE MINIMÁLNA)

$$\approx \boxed{\frac{h}{3}\left[y_0+4(y_1+y_3+y_5+\dots+y_{2k-1})+2(y_2+y_4+\dots+y_{2k-2})+y_{2k}\right]}$$

ODHAD ABSOLÚTNEJ CHYBY JE:

$$\boxed{|R_S| \le \frac{(b-a)^5}{180 \cdot n^4} \cdot M_4 < \varepsilon} \; ; \; M_4 = \max_{\langle a, b\rangle} |f^{(IV)}(x)|$$

PRÍKLAD 5: SIMPSONOVOU METÓDOU VYPOČÍTAJME $\int_0^{\pi/2} \sin(x^2)\,dx$ PRE $n = 4$ (KALKULAČKA - RADIÁNY) A UROBME ODHAD CHYBY

1) VYPOČÍTAME DĹŽKU KROKU $h = \frac{b-a}{n} = \frac{\frac{\pi}{2}-0}{4} = \frac{\pi}{8}$

2) VYTVORÍME TABUĽKU

i	x_i	$y_i = \sin(x_i)^2$
0	0	0
1	$\pi/8$	0,153602
2	$\pi/4$	0,578469
3	$3\pi/8$	0,983323
4	$\pi/2$	0,624266

3) HODNOTY Z TABUĽKY DOSADÍME DO VZORCA PRE VÝPOČET INTEGRÁLU

$$\int_0^{\pi/2} \sin(x^2)\,dx \approx \frac{\frac{\pi}{8}}{3}\left[0+4(0{,}153602+0{,}983323)+2(0{,}578469)+0{,}624266\right] =$$

$$= \underline{\underline{0{,}828452}}$$

4) UROBÍME ODHAD CHYBY, POTREBUJEME URČIŤ (ODHADNÚŤ) M_4

$f(x) = \sin x^2$

$f'(x) = 2x \cdot \cos x^2$

$f''(x) = 2\cos x^2 + 2x \cdot 2x(-\sin x^2) = 2\cos x^2 - [4x^2 \sin x^2]$

$f'''(x) = 2 \cdot 2x(-\sin x^2) - [8x \cdot \sin x^2 + 4x^2 \cdot 2x \cos x^2] = -4x \cdot \sin x^2 - 8x \cdot \sin x^2 - 8x^3 \cdot \cos x^2 =$

$= -12x \cdot \sin x^2 - 8x^3 \cdot \cos x^2$

$f^{(IV)}(x) = -12\sin x^2 - \underset{\sim}{12x \cdot 2x\cos x^2} - \underset{\sim}{24x^2\cos x^2} - 8x^3 \cdot 2x(-\sin x^2) =$

$= -12\sin x^2 - 48x^2\cos x^2 + 16x^4 \sin x^2$

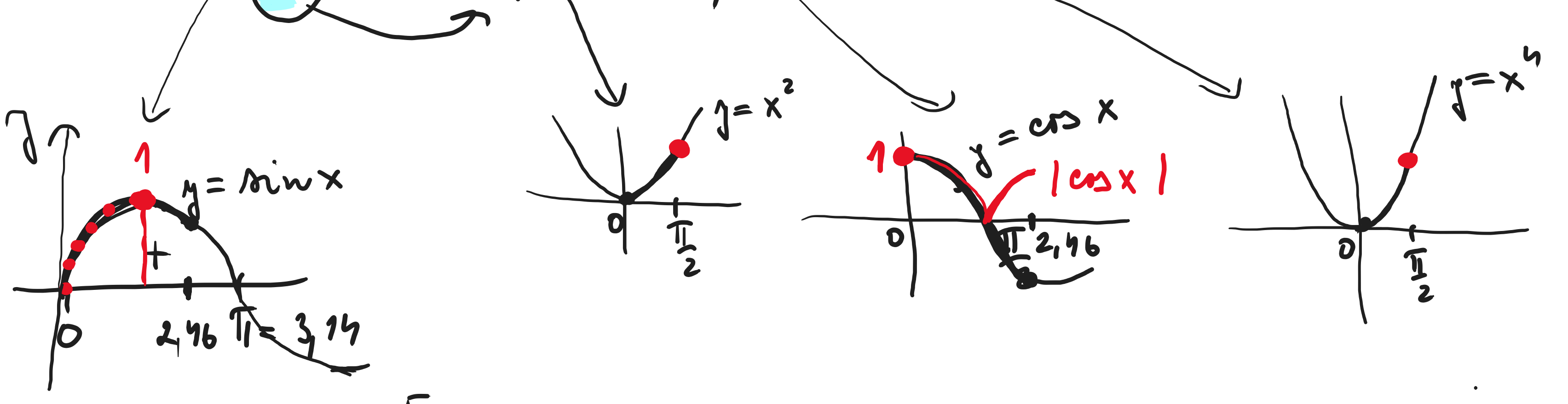
$|f^{(IV)}(x)| = |-12\sin x^2 - 48x^2\cos x^2 + 16x^4\sin x^2| \le 12|\sin x^2| + 48x^2|\cos x^2| + 16x^4|\sin x^2|$

$|a \pm b| \le |a| + |b|$

$|a \cdot b| = |a| \cdot |b|$

$$M_4 = \max_{x \in \langle 0; \frac{\pi}{2}\rangle} \left[12|\underbrace{\sin x^2}_{1}| + 48\underbrace{x^2}_{\frac{\pi^2}{4}}|\underbrace{\cos x^2}_{1}| + 16\underbrace{x^4}_{\frac{\pi^4}{16}}|\underbrace{\sin x^2}_{1}|\right] = 12 \cdot 1 + \cancel{48}^{12} \cdot \frac{\pi^2}{\cancel{4}} \cdot 1 + \cancel{16} \cdot \frac{\pi^4}{\cancel{16}} \cdot 1 = \underline{\underline{227{,}84}}$$

$x^2 \rightarrow \langle 0; \frac{\pi^2}{4} \rangle \doteq \langle 0; 2{,}46 \rangle$



$$|R_S| \le \frac{(\frac{\pi}{2}-0)^5}{180 \cdot (4)^4} \cdot 227{,}84 = \frac{\pi^5}{32 \cdot 180 \cdot 256} \cdot 227{,}84 = 0{,}04728 \doteq \underline{\underline{4{,}73 \cdot 10^{-2}}}$$

$$\int_0^{\pi/2} \sin x^2\,dx \approx 0{,}828452 \pm 4{,}73 \cdot 10^{-2}$$