PRÍKLAD 6: SIMPSONOVOU METÓDOU S PRESNOSŤOU $\varepsilon = 10^{-4}$ VYPOČÍTAJME $\int_1^2 x \cdot \ln^2 x \, dx$.

1) ZO VZORCA PRE ODHAD CHYBY URČÍME $\underline{\underline{n}}$ A POMOCOU NEHO VYPOČÍTAME DĹŽKU KROKU $\underline{\underline{h}}$

$$\frac{(b-a)^5}{180 n^4} \cdot M_4 < \varepsilon \Rightarrow 180 n^4 \cdot \varepsilon > (b-a)^5 \cdot M_4 \Rightarrow n > \sqrt[4]{\frac{(b-a)^5 \cdot M_4}{180 \cdot \varepsilon}}$$

$$f(x) = x \cdot \ln^2 x$$

$$f'(x) = 1 \cdot \ln^2 x + \cancel{x} \cdot \frac{2\ln x}{\cancel{x}} = \ln^2 x + 2\ln x$$

$$f''(x) = \frac{2\ln x}{x} + 2 \cdot \frac{1}{x} = \frac{2\ln x + 2}{x}$$

$$f'''(x) = \frac{2\frac{1}{\cancel{x}} \cdot \cancel{x} - (2\ln x + 2) \cdot 1}{x^2} = \frac{\cancel{2} - 2\ln x - \cancel{2}}{x^2} = \frac{-2\ln x}{x^2}$$

$$f^{(IV)}(x) = \frac{-2\frac{1}{\cancel{x}} \cdot \cancel{x^2} + (2\ln x) \cdot 2x}{x^4} = \frac{-2x + 4x\ln x}{x^4} = \frac{2\cancel{x}(2\ln x - 1)}{x^{\cancel{4}3}} = \frac{2(2\ln x - 1)}{x^3}$$

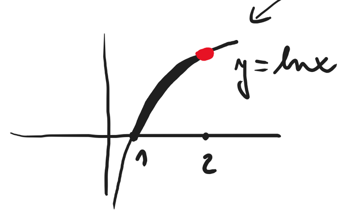
$$|f^{(IV)}(x)| = \left| \frac{2(2\ln x - 1)}{x^3} \right| = \frac{|2| \cdot |2\ln x - 1|}{|x^3|} = \frac{2|2\ln x - 1|}{|x^3|} \leq \frac{2(|2\ln x| + |-1|)}{|x^3|} =$$

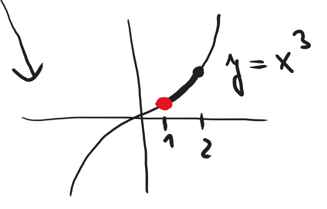
$$= \frac{2}{|x^3|}(|2\ln x| + 1) = \frac{2(|2\ln x| + 1)}{|x^3|}$$

HĽADÁM ČO NAJVÄČŠIU HODNOTU ČITATEĽA

$$M_4 = \max_{\langle 1,2 \rangle} |f^{(IV)}(x)| = \max_{\langle 1,2 \rangle} \frac{2(|2\ln x| + 1)}{|x^3|} \leq \frac{2(2\ln 2 + 1)}{1^3} \doteq 4{,}7725887$$

HĽADÁM ČO NAJMENŠIU HODNOTU MENOVATEĽA





$$n > \sqrt[4]{\frac{(2-1)^5 \cdot 4{,}7725887}{180 \cdot 10^{-4}}} \doteq 4{,}03525 \Rightarrow \boxed{n = 6} \Rightarrow h = \frac{b-a}{n} = \frac{2-1}{6} = \frac{1}{6}$$

2) VYTVORÍME TABUĽKU

i	x_i	$y_i = x_i \cdot \ln^2 x_i$
0	1	0
1	7/6	0,0277228
2	8/6 = 4/3	0,1103479
3	9/6 = 3/2	0,2466029
4	10/6 = 5/3	0,4349047
5	11/6	0,6735678
6	12/6 = 2	0,9609060

0,9478935

0,5452526

3) VÝPOČET INTEGR. DOSADENÍM DO VZORCA

$$\int_1^2 x \cdot \ln^2 x \, dx \approx \frac{\frac{1}{6}}{3}\left[0 + 4 \cdot 0{,}9478935 + 2 \cdot 0{,}5452526 + 0{,}960906\right] = 0{,}3246102 \pm 10^{-4}$$

DOMÁCA ÚLOHA

LICHOBEŽN. METÓDOU VYPOČÍTAJTE

1) $\int_2^6 \ln(x^3+1)\,dx \; ; \; n = 8$ [16,183535]

2) $\int_0^{10} \frac{1}{1+x^2}\,dx \; ; \; n = 10$ [1,476842]

3) LICHOBEŽN. MET. S PRESNOSŤOU $\varepsilon = 0{,}05$ VYPOČÍTAJTE
$\int_0^1 e^{x^2}\,dx$ [$n = 6$; 1,4752]

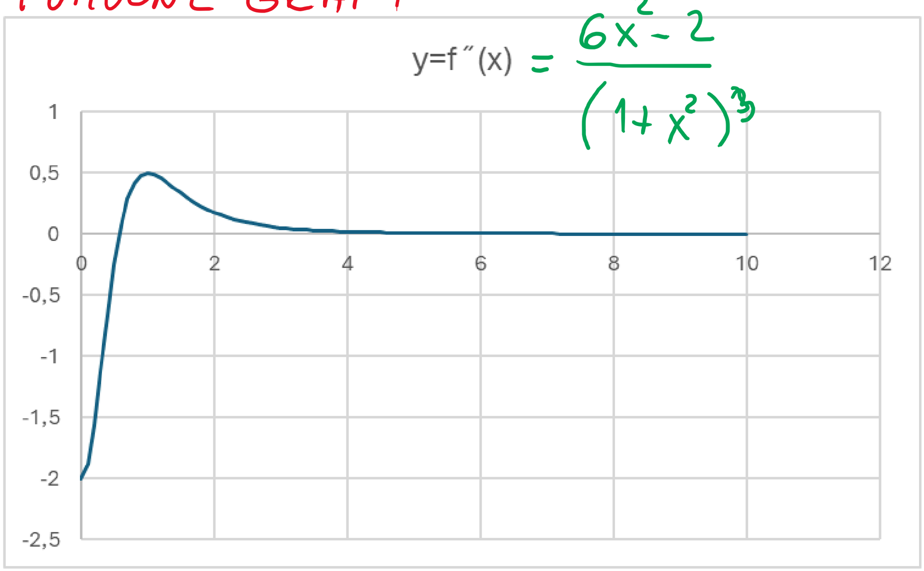
4) SIMPSON. MET. VYPOČÍTAJTE
$\int_0^5 \frac{1}{1+x}\,dx \; ; \; n = 10$ [1,793170]

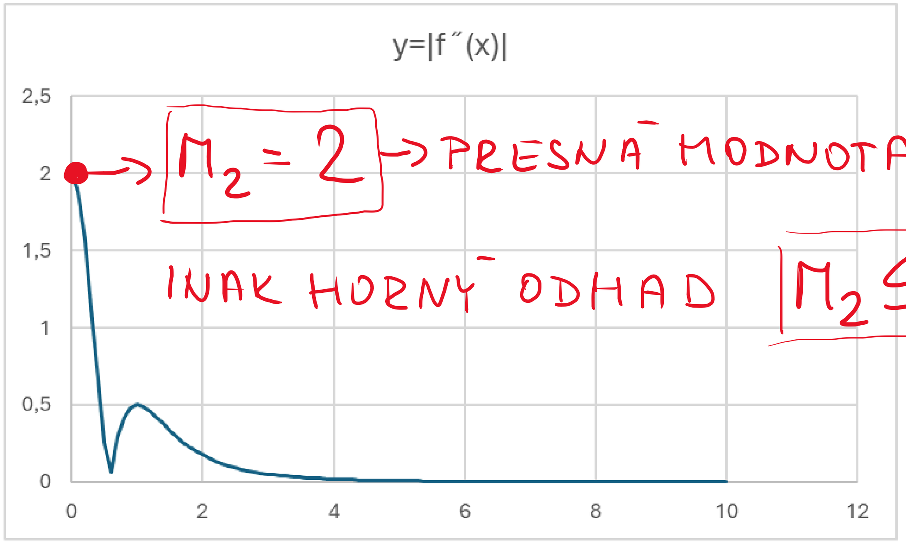
5) SIMPS. MET. S PRESNOSŤOU $\varepsilon = 0{,}001$ VYPOČÍTAJTE
$\int_0^1 \frac{1}{1+x}\,dx$ [$n = 4$; 0,693170]

6) ODHADNITE CHYBY, KTORÝCH STE SA DOPUSTILI V ÚLOHÁCH 2 a 4.

[1,66] [$4{,}17 \cdot 10^{-2}$]

POMOCNÉ GRAFY





$\boxed{M_2 = 2}$ → PRESNÁ HODNOTA,

INAK HORNÝ ODHAD $\boxed{M_2 \leq 602}$!