PRÍKLAD 2: POMOCOU LAGRANGEOVHO INTERPOLAČNÉHO POLYNÓMU URČME HODNOTU V BODE $x=1$ PRE F-CIU DANÚ TABUĽKOU

	x_0	x_1	x_2	x_3
x_i	-1	0	2	4
y_i	-2	$0,5$	$5,4$	35
	y_0	y_1	y_2	y_3

NEPOTREBUJEME NÁJSŤ VŠEOBECNÝ TVAR POLYNÓMU, STAČÍ HĽADAŤ JEHO F-KČNÚ HODNOTU V BODE $x=1$

$$L_3(x)=\frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}\cdot y_0+\frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}\cdot y_1+\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)}\cdot y_2+$$

$$+\frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}\cdot y_3$$

$$L_3(1)=\frac{(1-0)(1-2)(1-4)}{(-1-0)(-1-2)(-1-4)}(-2)+\frac{(1+1)(1-2)(1-4)}{(0+1)(0-2)(0-4)}\cdot 0,5+\frac{(1+1)(1-0)(1-4)}{(2+1)(2-0)(2-4)}\cdot 5,4+\frac{(1+1)(1-0)(1-2)}{(4+1)(4-0)(4-2)}\cdot 35=$$

$$=\frac{1\cdot\cancel{(-1)}\cancel{(-3)}}{\cancel{(-1)}\cancel{(-3)}(-5)}(-2)+\frac{\cancel{2}(-1)(-3)}{1\cdot\cancel{(-2)}(-4)}\cdot 0,5+\frac{2\cdot 1\cdot(-3)}{3\cdot 2\cdot(-2)}\cdot 5,4+\frac{2\cdot 1\cdot(-1)}{5\cdot 4\cdot 2}\cdot 35=$$

$$=\frac{2}{5}+\dots=\underline{\underline{1,725}}$$

PRÍKLAD 3: VYPOČÍTAJME PRIBLIŽNÚ HODNOTU x^*, PRE KTORÚ F-CIA DANÁ TABUĽKOU NADOBÚDA HODNOTU $y=-3$ (POUŽIJEME INVERZNÝ LAGRANGEOV POLYNÓM)

	x_0	x_1	x_2	x_3
x_i	-3	-2	0	1
y_i	4	3	-1	-5
	y_0	y_1	y_2	y_3

$$x^*=L_3^{-1}(-3)=?$$

$$L_3^{-1}(y)=\frac{(y-y_1)(y-y_2)(y-y_3)}{(y_0-y_1)(y_0-y_2)(y_0-y_3)}\cdot x_0+\frac{(y-y_0)(y-y_2)(y-y_3)}{(y_1-y_0)(y_1-y_2)(y_1-y_3)}\cdot x_1+\frac{(y-y_0)(y-y_1)(y-y_3)}{(y_2-y_0)(y_2-y_1)(y_2-y_3)}\cdot x_2+$$

$$+\frac{(y-y_0)(y-y_1)(y-y_2)}{(y_3-y_0)(y_3-y_1)(y_3-y_2)}\cdot x_3$$

$$x^*=L_3^{-1}(-3)=\frac{(-3-3)(-3+1)(-3+5)}{(4-3)(4+1)(4+5)}(-3)+\frac{(-3-4)(-3+1)(-3+5)}{(3-4)(3+1)(3+5)}(-2)+0+\frac{(-3-4)(-3-3)(-3+1)}{(-5-4)(-5-3)(-5+1)}\cdot 1=$$

$$=\frac{(-\cancel{6})(-2)\cdot 2\,\cancel{(-3)}\cdot 2}{5\cdot\cancel{9}\cdot\cancel{3}}+\frac{(-7)\cancel{(-2)}\cancel{2}\cancel{(-2)}}{(-1)\cdot 4\cdot\cancel{8}}+\frac{(-7)\cancel{(-6)}\cancel{(-2)}\cancel{3}}{3\cancel{(-9)}(-8)\cancel{(-4)}\cancel{2}}=-\frac{8}{5}+\frac{7}{4}+\frac{7}{24}\doteq\underline{\underline{0,441\overline{6}}}$$

PRÍKLAD 4: POZNÁME HODNOTY F-CIE $y=\cos x$ V BODOCH $x=0;\ \frac{\pi}{6};\ \frac{\pi}{4}$.

a) URČME POMOCOU LAGR. INTERP. POLYNÓMU HODNOTU $\cos\frac{\pi}{5}$; x

b) ODHADNIME CHYBU, KTOREJ SME SA DOPUSTILI

c) VYPOČÍTAJME (PRESNÚ) CHYBU —II—

	x_0	x_1	x_2
x_i	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$
y_i	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$
	y_0	y_1	y_2

a) $$L_2\left(\frac{\pi}{5}\right)=\frac{\left(\frac{\pi}{5}-\frac{\pi}{6}\right)\left(\frac{\pi}{5}-\frac{\pi}{4}\right)}{\left(0-\frac{\pi}{6}\right)\left(0-\frac{\pi}{4}\right)}\cdot 1+\frac{\left(\frac{\pi}{5}-0\right)\left(\frac{\pi}{5}-\frac{\pi}{4}\right)}{\left(\frac{\pi}{6}-0\right)\left(\frac{\pi}{6}-\frac{\pi}{4}\right)}\cdot\frac{\sqrt{3}}{2}+\frac{\left(\frac{\pi}{5}-0\right)\left(\frac{\pi}{5}-\frac{\pi}{6}\right)}{\left(\frac{\pi}{4}-0\right)\left(\frac{\pi}{4}-\frac{\pi}{6}\right)}\cdot\frac{\sqrt{2}}{2}=$$

$$=\frac{\frac{\cancel{\pi}}{30}\cdot\left(-\frac{\cancel{\pi}}{20}\right)}{\frac{\cancel{\pi}}{6}\cdot\frac{\cancel{\pi}}{4}}+\frac{\frac{\cancel{\pi}}{5}\cdot\left(-\frac{\cancel{\pi}}{20}\right)}{\frac{\cancel{\pi}}{6}\cdot\left(-\frac{\cancel{\pi}}{12}\right)}\cdot\frac{\sqrt{3}}{2}+\frac{\frac{\cancel{\pi}}{5}\cdot\frac{\cancel{\pi}}{30}}{\frac{\cancel{\pi}}{4}\cdot\frac{\cancel{\pi}}{12}}\cdot\frac{\sqrt{2}}{2}=$$

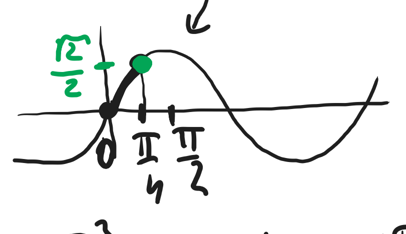
$$=\frac{-6\cdot 4}{30\cdot 20}+\frac{6\cdot 12\cdot\sqrt{3}}{5\cdot 20\cdot 2}+\frac{4\cdot 12\cdot\sqrt{2}}{5\cdot 30\cdot 2}\doteq\underline{\underline{0,809812}}$$

b) $$|f(x)-L_n(x)|\le\frac{M_{n+1}}{(n+1)!}\cdot|\omega_{n+1}(x)|\ ;\quad M_{n+1}=\max_{x\in\langle 0;\frac{\pi}{4}\rangle}|f'''(x)|$$

$$\omega_{n+1}(x)=(x-x_0)(x-x_1)\dots(x-x_n)$$

$n=2$

$$f(x)=\cos x\Rightarrow f'(x)=-\sin x\Rightarrow f''(x)=-\cos x\Rightarrow f'''(x)=\sin x$$



$$\Rightarrow M_3=\frac{\sqrt{2}}{2}$$

$$\omega_3\left(\frac{\pi}{5}\right)=\left(\frac{\pi}{5}-0\right)\left(\frac{\pi}{5}-\frac{\pi}{6}\right)\left(\frac{\pi}{5}-\frac{\pi}{4}\right)=\frac{\pi}{5}\cdot\frac{\pi}{30}\cdot\left(\frac{-\pi}{20}\right)=\frac{-\pi^3}{3000}\Rightarrow\left|\omega_3\left(\frac{\pi}{5}\right)\right|=\frac{\pi^3}{3000}$$

$$|f(x)-L_2(x)|\le\frac{\frac{\sqrt{2}}{2}}{3!}\cdot\frac{\pi^3}{3000}\doteq\underline{\underline{0,001218}}$$

c) $$|\underset{\frac{\pi}{5}}{f(x)}-\underset{\frac{\pi}{5}}{L_2(x)}|=|\underset{\text{Z KALKUL.}}{\cos\frac{\pi}{5}}-0,809812|\doteq 0,000795=\underline{\underline{7,95\cdot 10^{-4}}}$$