Renta odložená, prerušená, nekonečná, so spojitým úrokovaním

streda 19. marca 2025 10:00

ODLOŽENÁ RENTA

PR 1

$$ {}_5A_{10} = 10\,000 $$
$$ i = 0{,}06 $$
$$ n = 10 $$
$$ t = 5 $$
$$ R = ? $$

$$ {}_tA_n = (1+i)^{-t} \cdot R \cdot \frac{1-(1+i)^{-n}}{i} $$

$$ 10\,000 = (1+0{,}06)^{-5} \cdot R \cdot \frac{1-(1+0{,}06)^{-10}}{0{,}06} $$

$$ R = 10\,000 \cdot 1{,}06^{5} \cdot \frac{0{,}06}{1-1{,}06^{-10}} $$

$$ \underline{R = 1818{,}21977}; $$

OTÁZKA: AKÁ BY BOLA VÝŠKA SPLÁTKY, AK BY SME SPLÁCANIE NEODLOŽILI?

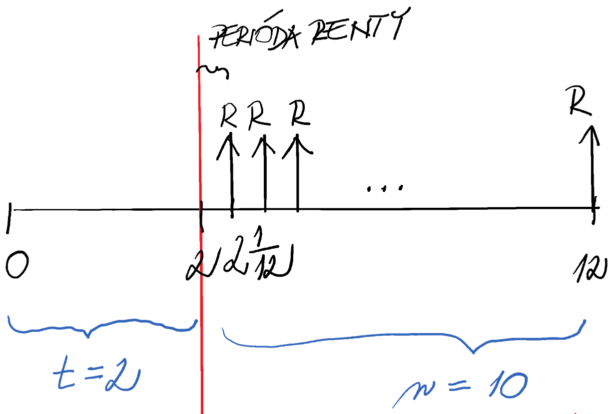
$$ R^{*} = R \cdot 1{,}06^{-5} = \frac{R}{1{,}06^{5}} = \underline{1358{,}67958}; $$

VÝŠKA SPLÁTKY

VÝŠKA SPLÁTKY PRI ODLOŽENEJ RENTE

PR 2

$$ {}_2A_{10} = 3058{,}25217 $$
$$ j = 0{,}1 $$
$$ n = 10 $$
$$ t = 2 $$
$$ p = 12 $$
$$ m = 4 $$
$$ R = ? $$



$$ {}_tA_n = R\left(1+\frac{j}{m}\right)^{-m \cdot t} \frac{1-\left(1+\frac{j}{m}\right)^{-mn}}{\left(1+\frac{j}{m}\right)^{\frac{m}{p}}-1} $$

$$ 3058{,}25217 = R\left(1+\frac{0{,}1}{4}\right)^{-4 \cdot 2} \frac{1-\left(1+\frac{0{,}1}{4}\right)^{-4 \cdot 10}}{\left(1+\frac{0{,}1}{4}\right)^{\frac{4}{12}}-1} $$

$$ R = 3058{,}25217 \cdot 1{,}025^{8} \cdot \frac{1{,}025^{\frac{4}{12}}-1}{1-1{,}025^{-40}} $$

$$ \underline{R = 49{,}07234}; $$

VEČNÁ RENTA

$$A_\infty = \lim_{n\to\infty} R \frac{1-(1+i)^{-n}}{i} \qquad (1+i)^{-n} = \frac{1}{(1+i)^{n}} \to 0$$

$$\underbrace{(1+i)}_{>1}, \quad (1+i)^{n} \to \infty$$

$$A_\infty = \frac{R}{i}$$

$$i = 0{,}1 \qquad \frac{1}{1{,}1^{n}} \to 0, \quad 1{,}1^{n} \to \infty \qquad \frac{1}{2^{n}}$$

$p > 1$

$$A_\infty = \lim_{n\to\infty} R \frac{1-\left(1+\frac{j}{m}\right)^{-m\cdot n}}{\left(1+\frac{j}{m}\right)^{\frac{m}{p}}-1} \qquad \frac{1}{\left(1+\frac{j}{m}\right)^{m\cdot \boxed{n}}} \to 0, \quad \left(1+\frac{j}{m}\right)^{m\cdot n} \to \infty$$

$$A_\infty = \frac{R}{\left(1+\frac{j}{m}\right)^{\frac{m}{p}}-1}$$

$$S_\infty = \lim_{n\to\infty} S_n = \lim_{n\to\infty} R \frac{\left(1+\frac{j}{m}\right)^{m\cdot n}-1}{\left(1+\frac{j}{m}\right)^{\frac{m}{p}}-1} = \infty$$

$$S_\infty = \infty$$

RENTA SO SPOJITÝM ÚROKOVANÍM

$$A_n = \lim_{m\to\infty} R \frac{1-\left(1+\frac{j}{m}\right)^{-m\cdot n}}{\left(1+\frac{j}{m}\right)^{\frac{m}{p}}-1} = \qquad 1^{\infty}$$

$$\left[\lim_{x\to\infty}\left(1+\frac{k}{x}\right)^{x} = e^{k}\right] \qquad \boxed{\begin{matrix} k = j \\ x = m \end{matrix}} \qquad \left(a^{b}\right)^{c} = a^{b\cdot c}$$

$$= \lim_{m\to\infty} R \frac{1-\Big[\underbrace{\left(1+\frac{j}{m}\right)^{m}}_{\to e^{j}}\Big]^{-n}}{\Big[\underbrace{\left(1+\frac{j}{m}\right)^{m}}_{\to e^{j}}\Big]^{\frac{1}{p}}-1} = \qquad \left(e^{j}\right)^{-n} = e^{-jn} \qquad \left(e^{j}\right)^{\frac{1}{p}} = e^{\frac{j}{p}}$$

$$= R \frac{1-\left(e^{j}\right)^{-n}}{\left(e^{j}\right)^{\frac{1}{p}}-1} = R \frac{1-e^{-jn}}{e^{\frac{j}{p}}-1}$$

$$S_n = \lim_{m\to\infty} R \frac{\overset{e^{\delta}}{\left(1+\frac{\delta}{m}\right)^{mn}} - 1}{\left(1+\frac{\delta}{m}\right)^{\frac{m}{p}} - 1} =$$

$$= \lim_{m\to\infty} R \frac{\left[\left(1+\frac{\delta}{m}\right)^{m}\right]^{n} - 1}{\left[\left(1+\frac{\delta}{m}\right)^{m}\right]^{\frac{1}{p}} - 1} =$$

$$= R \frac{(e^{\delta})^{n} - 1}{(e^{\delta})^{\frac{1}{p}} - 1} = R \frac{e^{\delta n} - 1}{e^{\frac{\delta}{p}} - 1}$$

VEČNÁ RENTA SO SPOJITÝM ÚROKOVANÍM

$$A_\infty = \lim_{n\to\infty} R \frac{1 - \overset{\to 0}{e^{-\delta n}}}{e^{\frac{\delta}{p}} - 1}$$

$$e^{-\delta n} = \frac{1}{\underset{\to\infty}{e^{\delta n}}} \to 0$$

$e = 2{,}7\ldots$

$e > 1$

$$A_\infty = \frac{R}{e^{\frac{\delta}{p}} - 1}$$

$$S_\infty = \lim \frac{\overset{\to\infty}{e^{\delta n}} - 1}{e^{\frac{\delta}{p}} - 1} = \infty$$