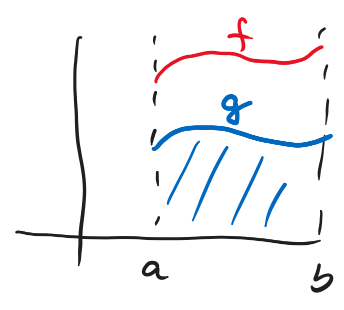
PDF <=> počet ≥ 300

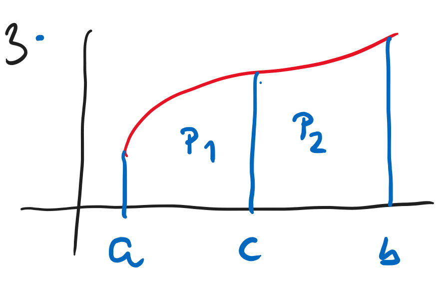
počet ≤ 80 => ∅

Vlastnosti $\int_a^b f(x)\,dx$

V1. Ak $f(x) \geq 0$, tak $\int_a^b f(x)\,dx \geq 0$ (plocha nemôže byť zápor.)

V2


Ak $f(x) \geq g(x)$ na $\langle a,b\rangle$, tak $\int_a^b f(x)\,dx \geq \int_a^b g(x)\,dx$

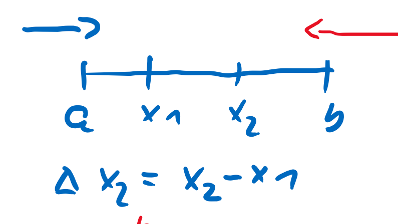
V3.


$\underbrace{\int_a^b f(x)\,dx}_{P} = \underbrace{\int_a^c f(x)\,dx}_{P_1} + \underbrace{\int_c^b f(x)\,dx}_{P_2}$

V4. $\int_a^b (f(x)+g(x))\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$

V5. $\alpha \in R$, $\int_a^b \alpha f(x)\,dx = \alpha \int_a^b f(x)\,dx$

V6 $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

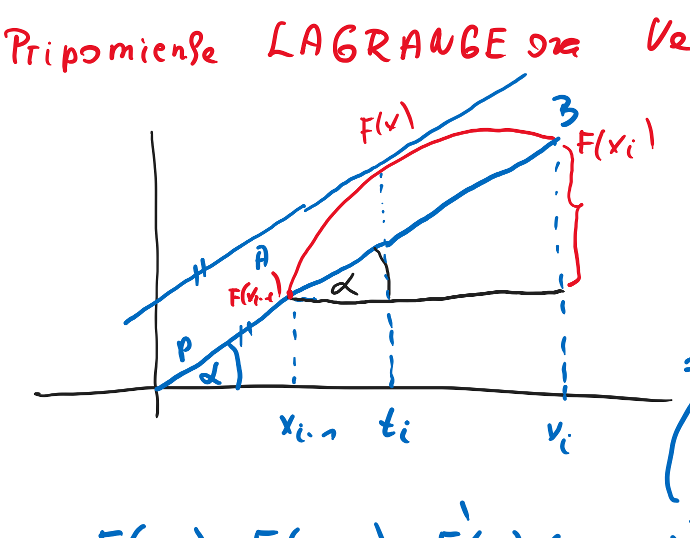


$\Delta x_2 = x_1 - x_2 = -(x_2 - x_1)$

$\Delta x_2 = x_2 - x_1$

Výpočet $\int_a^b f(x)\,dx$ <-> NEWTON – LEIBNIZ (N-L) (TZ)

Pripomienka LAGRANGEova Veta



pre $F(x)$ a interval $\langle x_{i-1}, x_i\rangle$

sečnica p

smernica $\to k_s = tg\,\alpha = \frac{F(x_i)-F(x_{i-1})}{x_i - x_{i-1}}$

presunom sečnice dostaneme dotyčnicu

$= k_d = F'(t_i)$

$F(x_i) - F(x_{i-1}) = F'(t_i)(x_i - x_{i-1}) = f(t_i)(x_i - x_{i-1}) = f(t_i)\Delta x_i$

Ak navyše F je PF k f (t.j. $F' = f$)

Úvahy k výpočtu U. I.

Def.

$a = x_0$, x_1, x_2, x_3, …, $b = x_{p_n}$

$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{p_n} f(t_i)\Delta x_i \overset{Lagrange}{=} \lim_{n\to\infty} \sum_{i=1}^{p_n} [F(x_i) - F(x_{i-1})]$

detail:

$F(x_1) - F(x_0)$
$F(x_2) - F(x_1)$
$F(x_3) - F(x_2)$
⋮
$F(x_{p_n}) - F(x_{p_n - 1})$

$\overset{výpočet}{=} \lim_{n\to\infty}(F(b) - F(a)) = F(b) - F(a) = F(x)\Big|_a^b$

ZÁZRAK N-L formule.

1. nech f je spoj na $\langle a,b\rangle$
2. nech F je PF (primitívna fcia) k f na $\langle a,b\rangle$

Potom

$\int_a^b f(x)\,dx = F(x)\Big|_a^b$ $(= F(b) - F(a))$

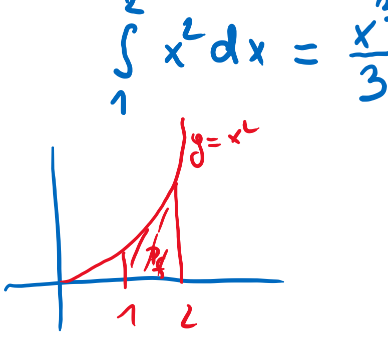
PODOBNOSŤ

NÁVRAT $\int f(x)\,dx = F(x) + c$ PREPOJENIE U.I. N.I

„TEORETICKY" HOTOVO K U.I. TREBA N.I. (toto poznám)

PRÍKLADY

① $\int_1^2 x^2\,dx = \frac{x^3}{3}(+c)\Big|_1^2 = \frac{8}{3} - \frac{1}{3} = \frac{7}{3}$ čo to je $= P_f$



$F(b) - F(a) = F(x)\Big|_a^b$

② $\int_0^1 x e^{x^2}\,dx = \frac{1}{2}e^{x^2}\Big|_0^1 = \frac{1}{2}(e-1)$

TREBA PF resp. N.I.

$\frac{1}{2}\int 2x e^{x^2}\,dx = \left| \begin{matrix} x^2 = t \\ 2x\,dx = dt \end{matrix} \right| = \frac{1}{2}\int e^t\,dt = \frac{1}{2}e^t + c = \frac{1}{2}e^{x^2} + c$ (c prečiarknuté)

AK ☺, TAK FINITO

SUBSTITÚCIA V URČITOM INT.

✓ $\int_a^b f(x)\,dx = F(x)\Big|_a^b$ $(F' = f)$

Ak F je PF k f (t.j. $F' = f$), tak očividne $F(\varphi(x))$ je PF k $f(\varphi(x))\varphi'(x)$ (lebo $(F(\varphi(x)))' = F'(\varphi(x))\varphi'(x) = f(\varphi(x))\varphi'(x)$)

a teda N-L

$\int_a^b f(\varphi(x))\varphi'(x)\,dx \overset{N-L}{=} F(\varphi(x))\Big|_a^b = F(\varphi(b)) - F(\varphi(a)) = F(t)\Big|_{\varphi(a)}^{\varphi(b)} \overset{N-L}{=} \int_{\varphi(a)}^{\varphi(b)} f(t)\,dt$

Veta o subst.

Zapíšeme ako mat. vetu + prakt. subst. sch.

Veta (subst. pre U.I.)

nech $\varphi(x)$, $\varphi'(x)$ sú spoj na $\langle a,b\rangle$

nech $f(t)$ je spoj na $\langle \varphi(a), \varphi(b)\rangle$. Potom

subst. sch.

$\int_a^b f(\varphi(x))\varphi'(x)\,dx = \left| \begin{matrix} \varphi(x) = t \\ \varphi'(x)\,dx = dt \\ \text{ak } x = a \Rightarrow t = \varphi(a) \\ x = b \Rightarrow t = \varphi(b) \end{matrix} \right| = \int_{\varphi(a)}^{\varphi(b)} f(t)\,dt$

Pr.

$\frac{1}{2}\int_0^{\sqrt{\frac{\pi}{2}}} 2x \sin(x^2)\,dx = \left| \begin{matrix} x^2 = t \\ 2x\,dx = dt ✓ \\ x = 0 \Rightarrow t = 0 \\ x = \sqrt{\frac{\pi}{2}} \Rightarrow t = \frac{\pi}{2} \end{matrix} \right| = \frac{1}{2}\int_0^{\pi/2} \sin t\,dt = -\frac{\cos t}{2}\Big|_0^{\pi/2}$

$= -\left(\frac{0-1}{2}\right) = 1/2$ ✓

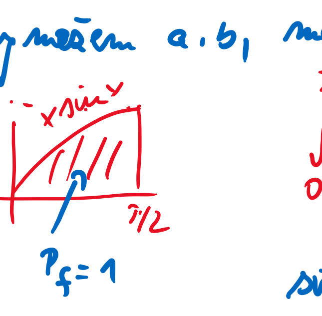
PER PARTES pre U. I

Veta nech $u(x)$, $u'(x)$, $v(x)$, $v'(x)$ sú spoj na $\langle a,b\rangle$

Potom

$\int_a^b u'(x)v(x)\,dx = u(x)v(x)\Big|_a^b - \int_a^b u(x)v'(x)\,dx$

(ak vymažem a, b, mám vetu pre neurčitý integrál …) POZOR NA HRANICE



$\int_0^{\pi/2} x \sin x\,dx = \left| \begin{matrix} u' = \sin x & v = x \\ u = -\cos x & v' = 1 \end{matrix} \right| = -x\cos x\Big|_0^{\pi/2} + \int_0^{\pi/2} \cos x\,dx$

$\sin x\Big|_0^{\pi/2} = 1$

$\int x \sin x\,dx \ldots = F(x) + c$ (c prečiarknuté)

$\int_a^b x \sin x\,dx = F(x)\Big|_a^b$