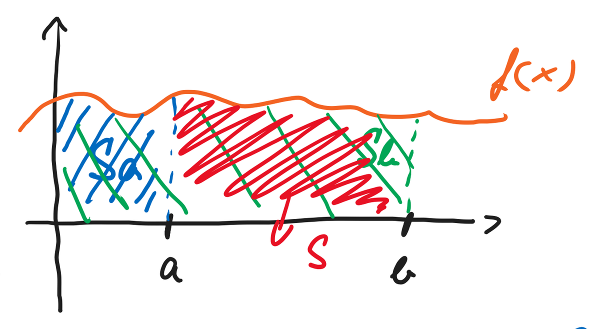
$\int f(x)\,dx = F(x) + c \qquad f(x) = F'(x)$



$\int_a^b f(x)\,dx = \left[F(x)\right]_a^b = F(b) - F(a)$

$F(a) = S_a \qquad S = S_b - S_a$

$F(b) = S_b \qquad S = F(b) - F(a)$

$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

$F(b) - F(a) = -(F(a) - F(b))$

Př: $\int_1^2 (16x^3 + 12x^2 - 7)\,dx = \left[16 \cdot \frac{x^4}{4} + 12 \cdot \frac{x^3}{3} - 7x\right]_1^2 = \left[4x^4 + 4x^3 - 7x\right]_1^2 =$

$(4 \cdot 2^4 + 4 \cdot 2^3 - 7 \cdot 2) - (4 \cdot 1^4 + 4 \cdot 1^3 - 7 \cdot 1) = (64 + 32 - 14) - (4 + 4 - 7) =$

$= 82 - 1 = 81$

Př: $\int_3^7 \frac{2}{2}\frac{x}{x^2-4}\,dx = \frac{1}{2}\int_3^7 \frac{2x}{x^2-4}\,dx = \left[\frac{1}{2}\ln|x^2-4|\right]_3^7 =$

$\left\{ \int \frac{f'}{f}\,dx = \ln|f| + c \right.$

$f = x^2 - 4$

$f' = (x^2-4)' = 2x$

$\ln a - \ln b = \ln\frac{a}{b}$

$r \ln a = \ln a^r$

$= \frac{1}{2}\ln|7^2 - 4| - \frac{1}{2}\ln|3^2 - 4| =$

$= \frac{1}{2}(\ln 45 - \ln 5) = \frac{1}{2}\ln\frac{45}{5} = \frac{1}{2}\ln 9 = \ln 9^{\frac{1}{2}} = \ln\sqrt{9} = \ln 3$

Př: $\int_1^e \frac{x^2+1}{x}\,dx = \int_1^e \left(x + \frac{1}{x}\right)dx = \left[\frac{x^2}{2} + \ln|x|\right]_1^e =$

$= \left(\frac{e^2}{2} + \ln e\right) - \left(\frac{1^2}{2} + \ln 1\right) = \frac{e^2}{2} + 1 - \frac{1}{2} = \frac{1}{2}(e^2 + 1)$

Př: $\int_0^{\frac{\pi}{2}} \sqrt{\sin x - \sin^3 x}\,dx = \int_0^{\frac{\pi}{2}} \sqrt{\sin x(\underbrace{1 - \sin^2 x}_{\cos^2 x})}\,dx =$

$\int_0^{\frac{\pi}{2}} \sqrt{\sin x} \cdot \sqrt{\cos^2 x}\,dx = \int_0^{\frac{\pi}{2}} \sqrt{\sin x} \cdot \cos x\,dx = \left| \begin{array}{ll} \sin x = t & \\ \cos x\,dx = dt & \\ x \to 0 & t \to 0 \\ x \to \frac{\pi}{2} & t \to 1 \end{array} \right| =$

$\int_0^1 \sqrt{t}\,dt = \int_0^1 t^{\frac{1}{2}}\,dt = \left[\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^1 = \frac{2}{3}\sqrt{1^3} - \frac{2}{3}\sqrt{0^3} = \frac{2}{3}$

Př: $\int_0^7 \frac{x - 1}{\sqrt[3]{x+1}}\,dx = \left| \begin{array}{ll} x+1 = t^3 & \\ x = t^3 - 1 & \\ dx = 3t^2\,dt & \\ x \to 0 & t \to 1 \\ x \to 7 & t \to \sqrt[3]{8} = 2 \end{array} \right| = \int_1^2 \frac{t^3 - 1 - 1}{t} 3t^2\,dt =$

$= 3\int_1^2 (t^4 - 2t)\,dt = 3\left[\frac{t^5}{5} - 2\frac{t^2}{2}\right]_1^2 = 3\left[\left(\frac{2^5}{5} - 2^2\right) - \left(\frac{1^5}{5} - 1^2\right)\right] =$

$= 3\left[\left(\frac{32}{5} - 4\right) - \left(\frac{1}{5} - 1\right)\right] = \frac{48}{5}$

Př: $\int_0^1 x e^{1+x^2}\,dx = \left| \begin{array}{ll} 1+x^2 = t & \\ 2x\,dx = dt & \\ x \to 0 & t \to 1 \\ x \to 1 & t \to 2 \end{array} \right| = \int_1^2 e^t \frac{1}{2}\,dt = \left[\frac{1}{2}e^t\right]_1^2 = \frac{1}{2}(e^2 - e) = \frac{1}{2}e(e-1)$

$x\,dx = \frac{1}{2}\,dt$

$\int_a^b u(x)v'(x)\,dx = \left[u(x)v(x)\right]_a^b - \int_a^b u'(x)v(x)\,dx$

Př: $\int_0^{\frac{\pi}{2}} x \sin x\,dx = \left| \begin{array}{ll} u = x & v' = \sin x \\ u' = 1 & v = -\cos x \end{array} \right| =$

$= \left[-x\cos x\right]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} (-\cos x)\,dx = \left[-x\cos x\right]_0^{\frac{\pi}{2}} + \left[\sin x\right]_0^{\frac{\pi}{2}} =$

$= \underbrace{\left(-\frac{\pi}{2}\cos\frac{\pi}{2} - (-0 \cdot \cos 0)\right)}_{0} + \underbrace{\left(\sin\frac{\pi}{2} - \sin 0\right)}_{1} = 1$

Př: $\int_2^e (2x - 3)\ln x\,dx = \left| \begin{array}{ll} u = \ln x & v' = 2x - 3 \\ u' = \frac{1}{x} & v = 2 \cdot \frac{x^2}{2} - 3x \end{array} \right| =$

$= \left[(x^2 - 3x)\ln x\right]_2^e - \int_2^e (x - 3)\,dx = \left[(x^2 - 3x)\ln x\right]_2^e - \left[\frac{x^2}{2} - 3x\right]_2^e =$

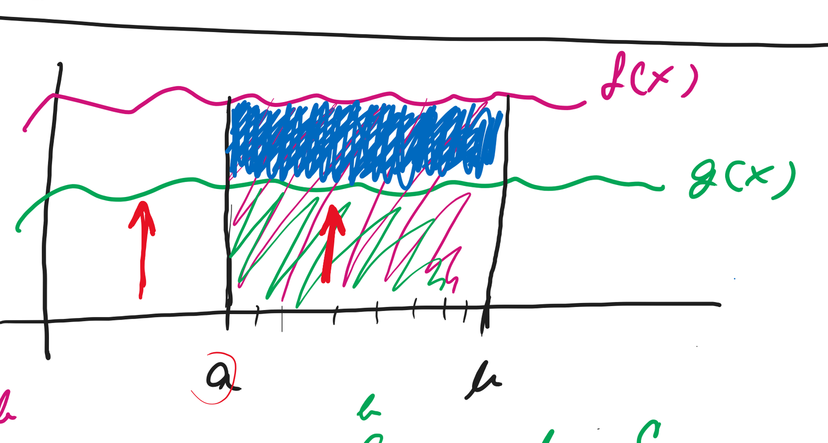
$\left[(e^2 - 3e)\ln e - (2^2 - 3 \cdot 2)\ln 2\right] - \left[\left(\frac{e^2}{2} - 3 \cdot e\right) - \left(\frac{2^2}{2} - 3 \cdot 2\right)\right] =$

$e^2 - 3e - (-2)\ln 2 - \left(\frac{e^2}{2} - 3e - (-4)\right) =$

$= e^2 - \cancel{3e} + 2\ln 2 - \frac{e^2}{2} + \cancel{3e} - 4 = \frac{e^2}{2} + \ln 4 - 4$

$a \leq x \leq b$

$g(x) \leq y \leq f(x)$



$S = \int_a^b (f(x) - g(x))\,dx$

$\int_a^b f(x)\,dx = S_f \qquad \int_a^b g(x)\,dx = S_g$

$V = \pi \int_a^b \left(f^2(x) - g^2(x)\right)dx$

$S = S_f - S_g$

$S = \int_a^b f(x)\,dx - \int_a^b g(x)\,dx = \int_a^b (f(x) - g(x))\,dx$