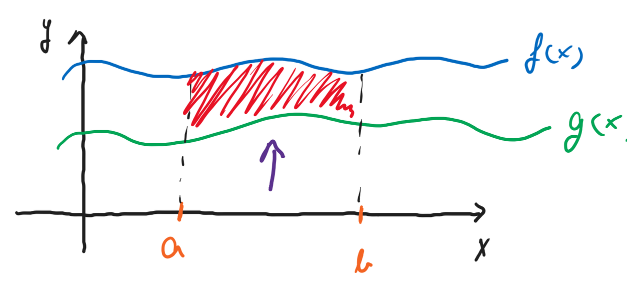


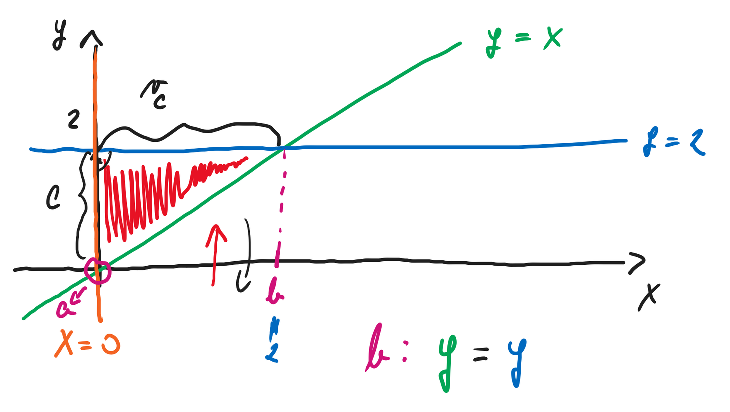
$a \le x \le b$

$g(x) \le y \le f(x)$

$$S = \int_a^b (f(x) - g(x))\,dx$$

$$V = \pi \int_a^b (f^2(x) - g^2(x))\,dx$$

Pr: $\boxed{y = x}$ $\boxed{y = 2}$ $x = 0$



$S = \frac{c \cdot v_c}{2} = \frac{|y||x|}{2} =$

$\frac{2 \cdot 2}{2} = 2$

okolo o_x:

$0 \le x \le 2$ ($0 = a$, $2 = b$)

$x \le y \le 2$ ($x \to g(x)$, $2 \to f(x)$)

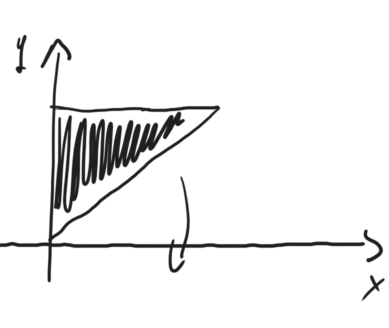
b: $y = y$

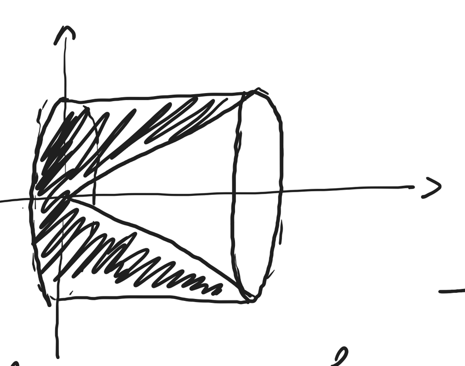
$x = 2$

$$S = \int_a^b (f(x) - g(x))\,dx = \int_0^2 (2 - x)\,dx = \left[2x - \frac{x^2}{2}\right]_0^2 =$$

$$= \left(2 \cdot 2 - \frac{2^2}{2}\right) - \left(2 \cdot 0 - \frac{0^2}{2}\right) = 2$$

V:





$V = V_{valca} - V_{kužeľa} =$

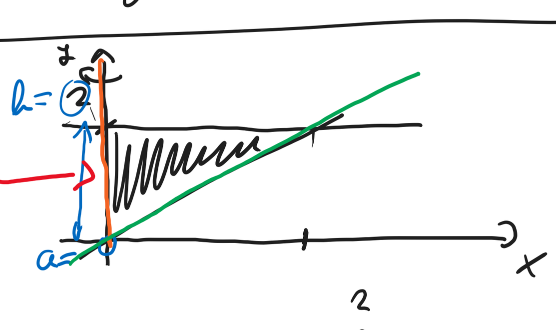
$= \pi R^2 \cdot v - \frac{1}{3}\pi R^2 \cdot v =$

$= \pi \cdot 2^2 \cdot 2 - \frac{1}{3}\pi \cdot 2^2 \cdot 2 = \frac{16}{3}\pi$

$$V = \pi \int_a^b (f^2(x) - g^2(x))\,dx = \pi \int_0^2 (2^2 - x^2)\,dx = \pi \int_0^2 (4 - x^2)\,dx =$$

$$\pi \left[4x - \frac{x^3}{3}\right]_0^2 = \pi\left[\left(4 \cdot 2 - \frac{2^3}{3}\right) - \left(4 \cdot 0 - \frac{0^3}{0}\right)\right] = \pi\left(8 - \frac{8}{3}\right) = \frac{16}{3}\pi$$

okolo o_y: $b = 2$



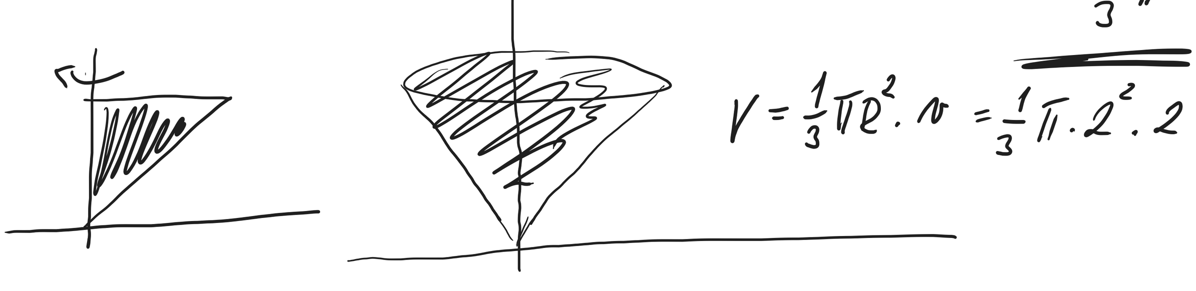
$0 \le y \le 2$ ($0 = a$, $2 = b$)

$0 \le x \le y$ ($0 \to \psi(y)$, $y \to \varphi(y)$)

$$S = \int_a^b (\varphi(y) - \psi(y))\,dy = \int_0^2 (y - 0)\,dy = \left[\frac{y^2}{2}\right]_0^2 = \left(\frac{2^2}{2} - \frac{0^2}{2}\right) = 2$$

$$V = \pi \int_a^b (\varphi^2(y) - \psi^2(y))\,dy = \pi \int_0^2 (y^2 - 0^2)\,dy = \pi\left[\frac{y^3}{3}\right]_0^2 = \pi\left(\frac{2^3}{3} - \frac{0^3}{3}\right) =$$

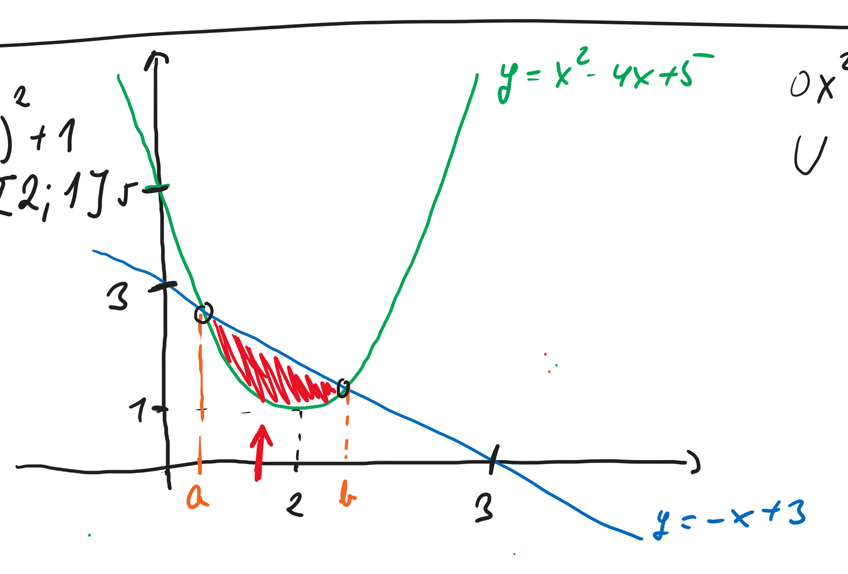
$$= \frac{8}{3}\pi$$



$V = \frac{1}{3}\pi R^2 \cdot v = \frac{1}{3}\pi \cdot 2^2 \cdot 2 = \frac{8}{3}\pi$

Pr: $\boxed{y = x^2 - 4x + 5} = (x-2)^2 + 1$ $\boxed{y = -x + 3}$

$[2; 1]$



$0x^2 - 4x + 5$

$\cup$ $b^2 - 4ac$

$16 - 20 < 0$

$1 \le x \le 2$ ($1 = a$, $2 \to b$)

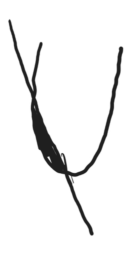
$x^2 - 4x + 5 \le y \le -x + 3$ ($x^2 - 4x + 5 \to g(x)$, $-x + 3 \to f(x)$)

$y = y$

$x^2 - 4x + 5 = -x + 3$

$x^2 - 3x + 2 = 0$

$(x-2)(x-1) = 0$ $\quad x_1 = 1 \to a$, $x_2 = 2 \to b$

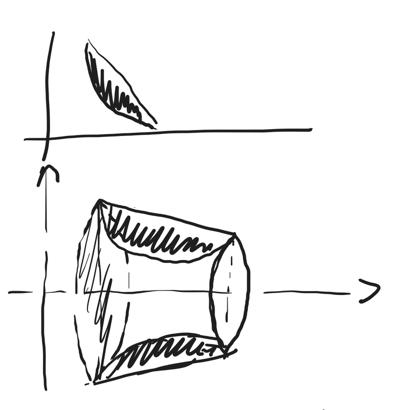


$$S = \int_a^b (f(x) - g(x))\,dx = \int_1^2 ((-x+3) - (x^2 - 4x + 5))\,dx =$$

$$= \int_1^2 (-x + 3 - x^2 + 4x - 5)\,dx = \int_1^2 (-x^2 + 3x - 2)\,dx = \left[-\frac{x^3}{3} + 3 \cdot \frac{x^2}{2} - 2x\right]_1^2 =$$

$$= \left(-\frac{2^3}{3} + 3 \cdot \frac{2^2}{2} - 2 \cdot 2\right) - \left(-\frac{1^3}{3} + 3 \cdot \frac{1^2}{2} - 2 \cdot 1\right) = \frac{1}{6}$$

$$V = \pi \int_a^b (f^2(x) - g^2(x))\,dx =$$



$$= \pi \int_1^2 ((-x+3)^2 - (x^2 - 4x + 5)^2)\,dx =$$

$$= \pi \int_1^2 (x^2 - 6x + 9) - (x^4 - 4x^3 + 5x^2 - 4x^3 + 16x^2 - 20x + 5x^2 - 20x + 25)\,dx =$$

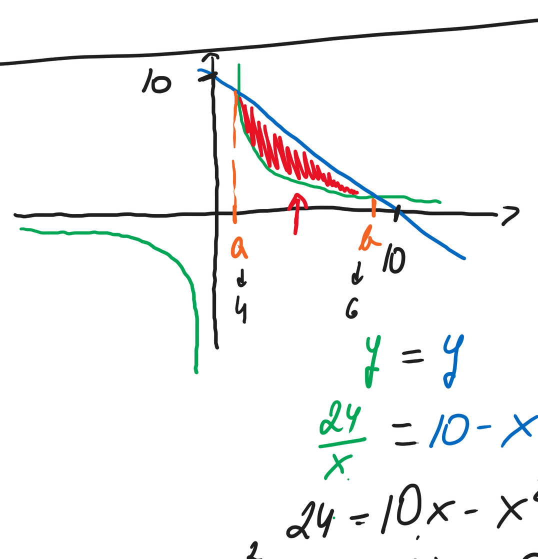
$$= \pi \int_1^2 (-x^4 + 8x^3 - 25x^2 + 34x - 16)\,dx = \pi\left[-\frac{x^5}{5} + 8 \cdot \frac{x^4}{4} - 25 \cdot \frac{x^3}{3} + \frac{34x^2}{2} - 16x\right]_1^2 =$$

$$= \pi\left[\left(-\frac{2^5}{5} + 2 \cdot 2^4 - 25 \cdot \frac{2^3}{3} + 17 \cdot 2^2 - 16 \cdot 2\right) - \left(-\frac{1^5}{5} + 2 \cdot 1^4 - 25 \cdot \frac{1^3}{3} + 17 \cdot 1^2 - 16 \cdot 1\right)\right] =$$

32, $\frac{200}{3}$, 68, 32, $\frac{25 \cdot 5}{3}$

$$= -\frac{76}{15} - \left(-\frac{83}{15}\right) = \frac{7}{15}\pi$$

Pr: $\boxed{y = \frac{24}{x}}$ $\boxed{y = 10 - x}$



$4 \le x \le 6$ ($4 = a$, $6 \to b$)

$\frac{24}{x} \le y \le 10 - x$ ($\frac{24}{x} \to g(x)$, $10 - x \to f(x)$)

$y = y$

$\frac{24}{x} = 10 - x$

$24 = 10x - x^2$

$x^2 - 10x + 24 = 0$

$(x-6)(x-4) = 0$ $\quad x_1 = 4 \to a$, $x_2 = 6 \to b$

$$S = \int_a^b (f(x) - g(x))\,dx =$$

$$= \int_4^6 \left(10 - x - \frac{24}{x}\right)dx = \left[10x - \frac{x^2}{2} - 24\ln|x|\right]_4^6 =$$

$$= \left(10 \cdot 6 - \frac{6^2}{2} - 24\ln 6\right) - \left(10 \cdot 4 - \frac{4^2}{2} - 24\ln 4\right) = 10 + 24\ln\frac{4}{6} =$$

$$= 10 + 24\ln\frac{2}{3}$$

$$V = \pi \int_a^b (f^2(x) - g^2(x))\,dx = \pi \int_4^6 \left((10-x)^2 - \left(\frac{24}{x}\right)^2\right)dx =$$

$$= \pi \int_4^6 \left(100 - 20x + x^2 - \frac{576}{x^2}\right)dx = \pi\left[100x - 20\frac{x^2}{2} + \frac{x^3}{3} + \frac{576}{x}\right]_4^6$$

$$= \pi\left[\left(100 \cdot 6 - 10 \cdot 6^2 + \frac{6^3}{3} + \frac{576}{6}\right) - \left(100 \cdot 4 - 10 \cdot 4^2 + \frac{4^3}{3} + \frac{576}{4}\right)\right]$$

$240 \cdot 3 = 720$, 216, $\frac{288}{3}$, $240 \to \frac{1440}{6}$, 64, $\frac{128}{6}$, $\frac{288}{2}$, $\frac{864}{6}$

$$= \pi\left(\frac{1224}{3} - \frac{2432}{6}\right) = \frac{16}{6}\pi = \frac{8}{3}\pi$$

$\frac{2448}{6}$