OBJEM ROTAČNÉHO TELESA (okolo osi x)

TREBA VEDIEŤ

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{p_n} f(t_i)\,\Delta x_i$$

x_i - del. body

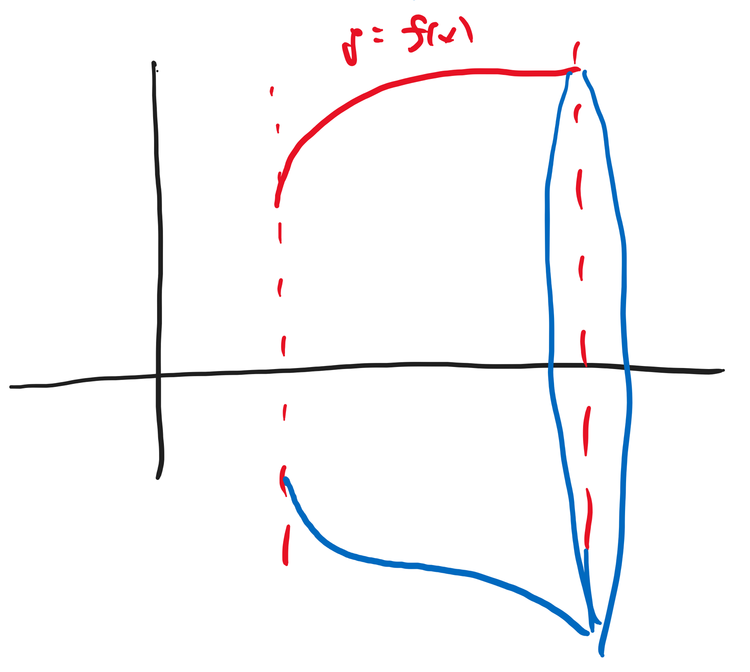
t_i - výb. body

delíme $\langle a,b\rangle$:

$$\lim_{n\to\infty} \sum_{i=1}^{p_n} \sin t_i\,\Delta x_i = \int_a^b \sin x\,dx$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{1+(f'(t_i))^2}\,\Delta x_i = \int_a^b \sqrt{1+(f'(x))^2}\,dx$$

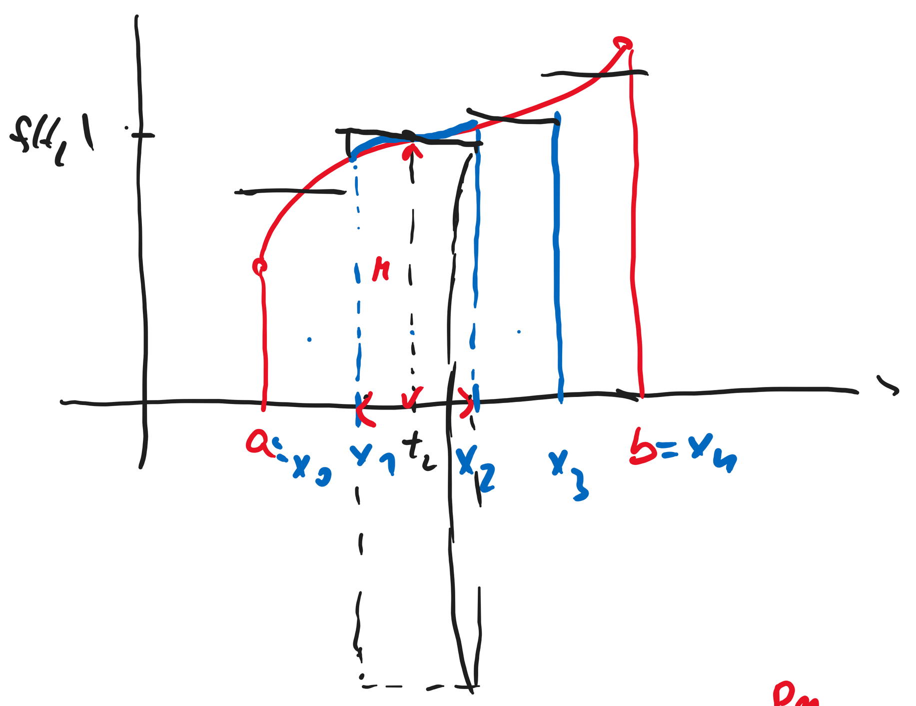
každej takejto limite odpovedá U.I



V=?

$f(x)>0$ na $\langle a,b\rangle$

↳ graf rotuje okolo osi x vznikne priestorové teleso (máme určiť jeho objem



nech $\{D_n\}$ je postupnosť delení $\langle a,b\rangle$ (ako pri def. U. I)

$$V = \sum_{i=1}^{p_n} V_i$$

V_i - objem i-teho seg.

fcia f na i-tom segm. nahradí úsečkou

$V_i \approx V(\text{valca})$

Rotáciou úsečky okolo osi x vznikne valec

$$\pi r^2 v = \pi f^2(t_i)\,\Delta x_i$$

$r = f(t_i)$, $v = \Delta x_i$

$$V \approx \pi \sum_{i=1}^{p_n} f^2(t_i)\,\Delta x_i$$

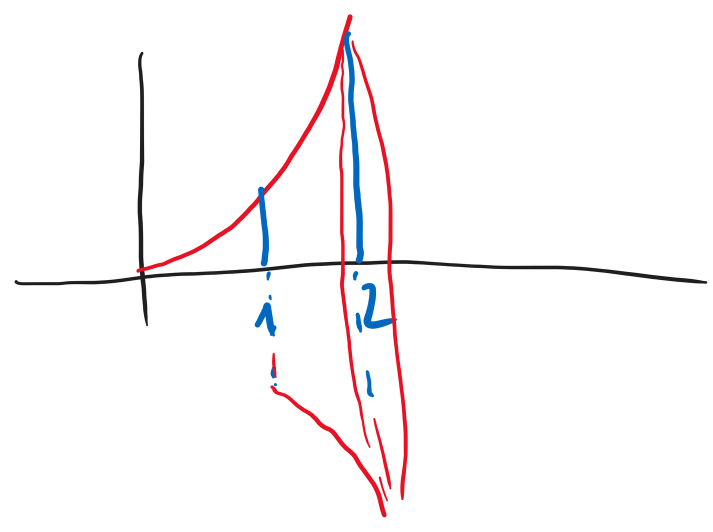
$n\to\infty$ $\quad v = \Delta x_i \to 0$

$n\to\infty$

$$V = \pi \lim_{n\to\infty} \sum_{i=1}^{p_n} f^2(t_i)\,\Delta x_i = \pi \int_a^b f^2(x)\,dx$$

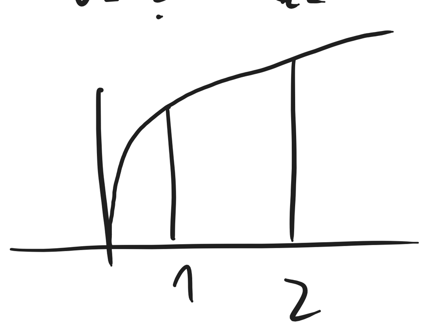
Vzorec pre objem rotačného telesa!!!

V rotač. telesa = ? ak rotuje $y=x^2$ na $\langle 1,2\rangle$



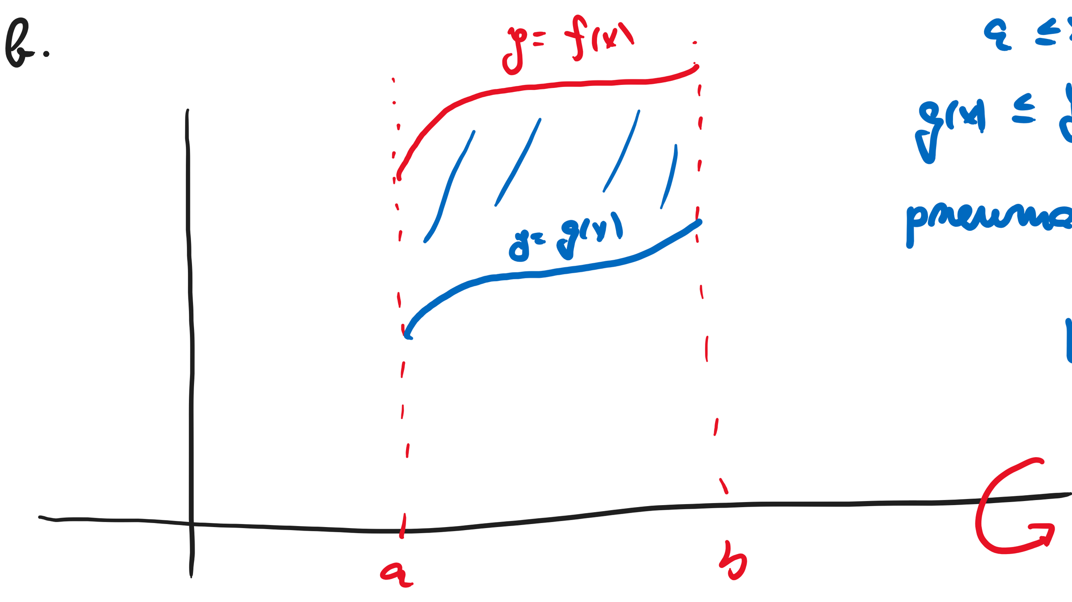
$$V = \pi \int_1^2 (x^2)^2\,dx = \pi \int_1^2 x^4\,dx = \pi \frac{x^5}{5}\Big|_1^2$$

Pr $V=?$ ak $y=\sqrt{x}$ na $\langle 1,2\rangle$



$$V = \pi \int_1^2 (\sqrt{x})^2\,dx = \pi \int_1^2 x\,dx = \quad \ldots$$

Zovšeob.

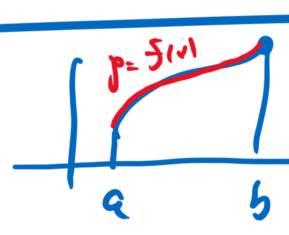


A: $a \le x \le b$ taká že $g(x) \le y \le f(x)$

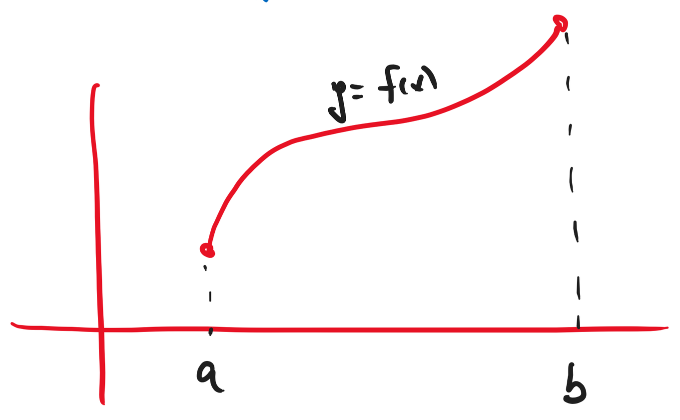
pravouholník

$$V = \pi \int_a^b f^2(x) - g^2(x)\,dx$$

$\int_1^{\infty} \sin x\,dx = ?$



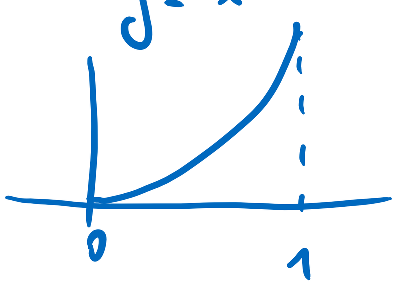
Primerne hodnota fci



$$l = \int_a^b \sqrt{1+(f'(x))^2}\,dx$$

PROBLEM $\sqrt{}$

Pr$_1$ $y = x^2$



$$l = \int_0^1 \sqrt{1+4x^2}\,dx$$

$y' = 2x$

$$\int \frac{\sqrt{1+4x^2}}{1}\,dx = \int \frac{1+4x^2}{\sqrt{1+4x^2}}\,dx = (Ax+B)\sqrt{1+4x^2} + \lambda \int \frac{1}{\sqrt{1+4x^2}}\,dx$$

ZP = P(A) = ? V = ?