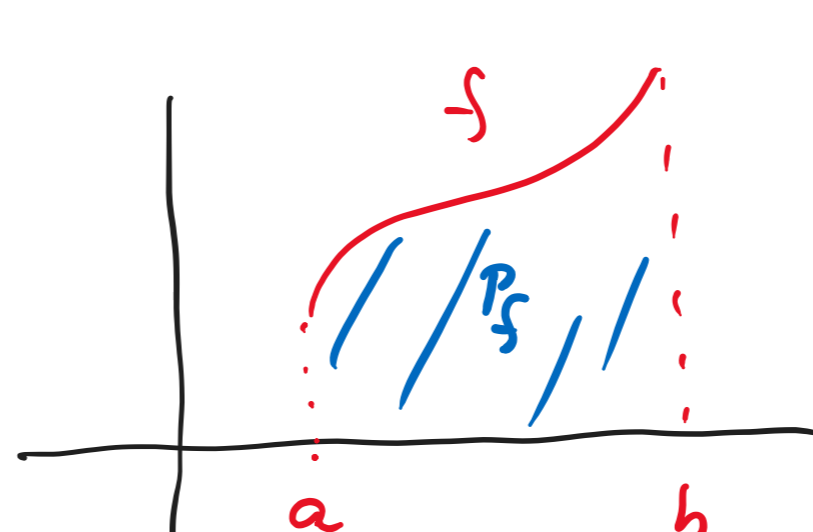
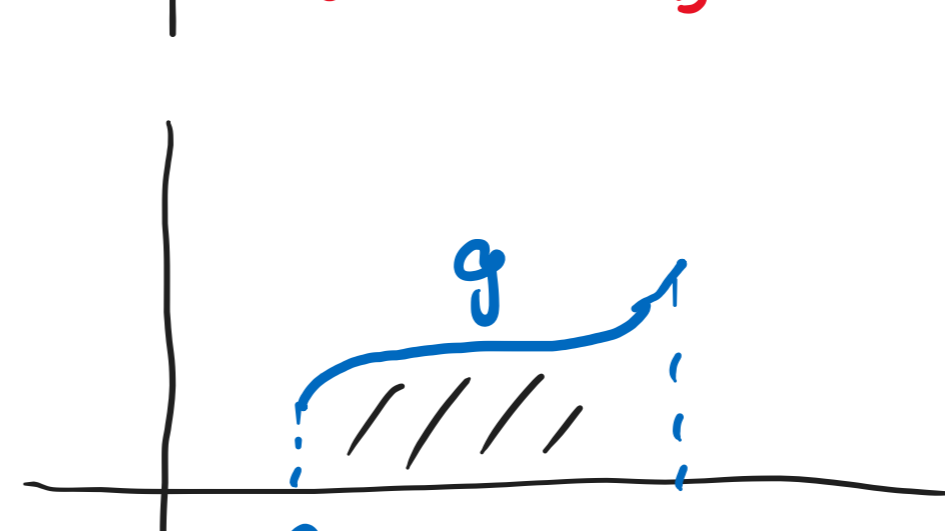


APLIKÁCIE URČ. INT.

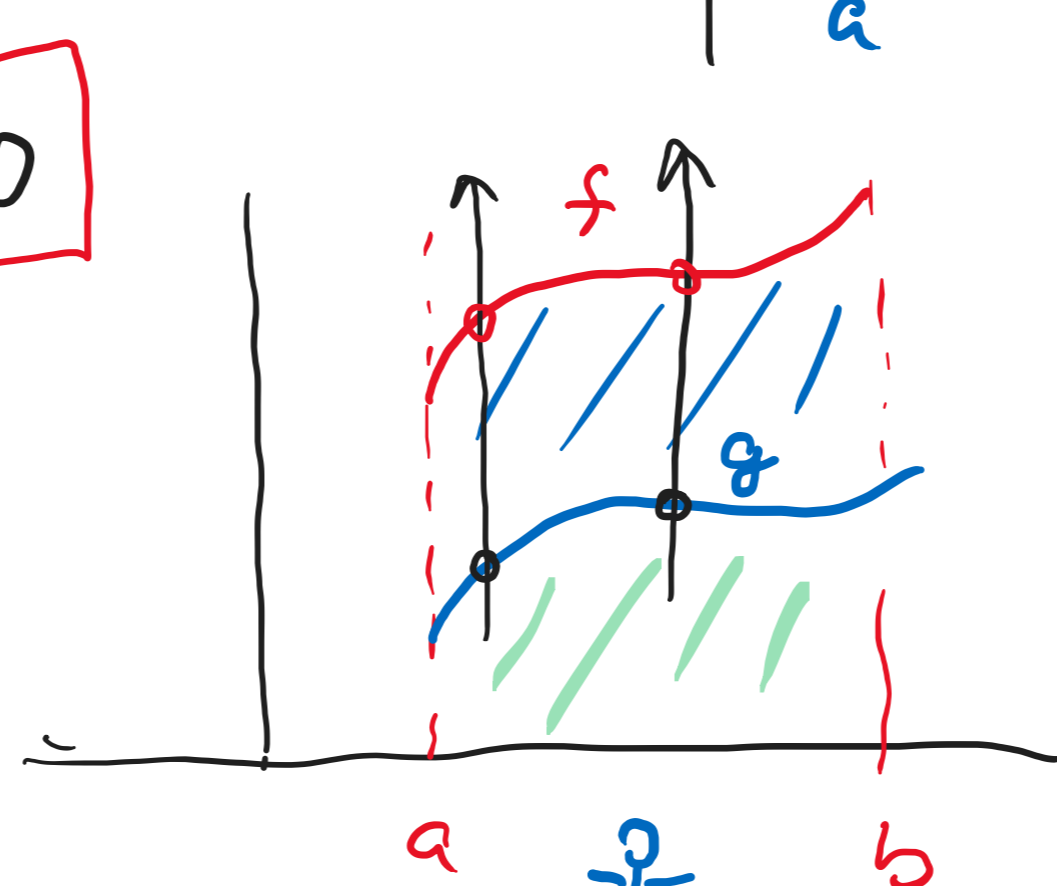
ak  $f(x) > 0$  na  $\langle a, b \rangle$   
 $P_f = \int_a^b f(x) dx$



ak  $g(x) > 0$  na  $\langle a, b \rangle$   
 $P_g = \int_a^b g(x) dx$



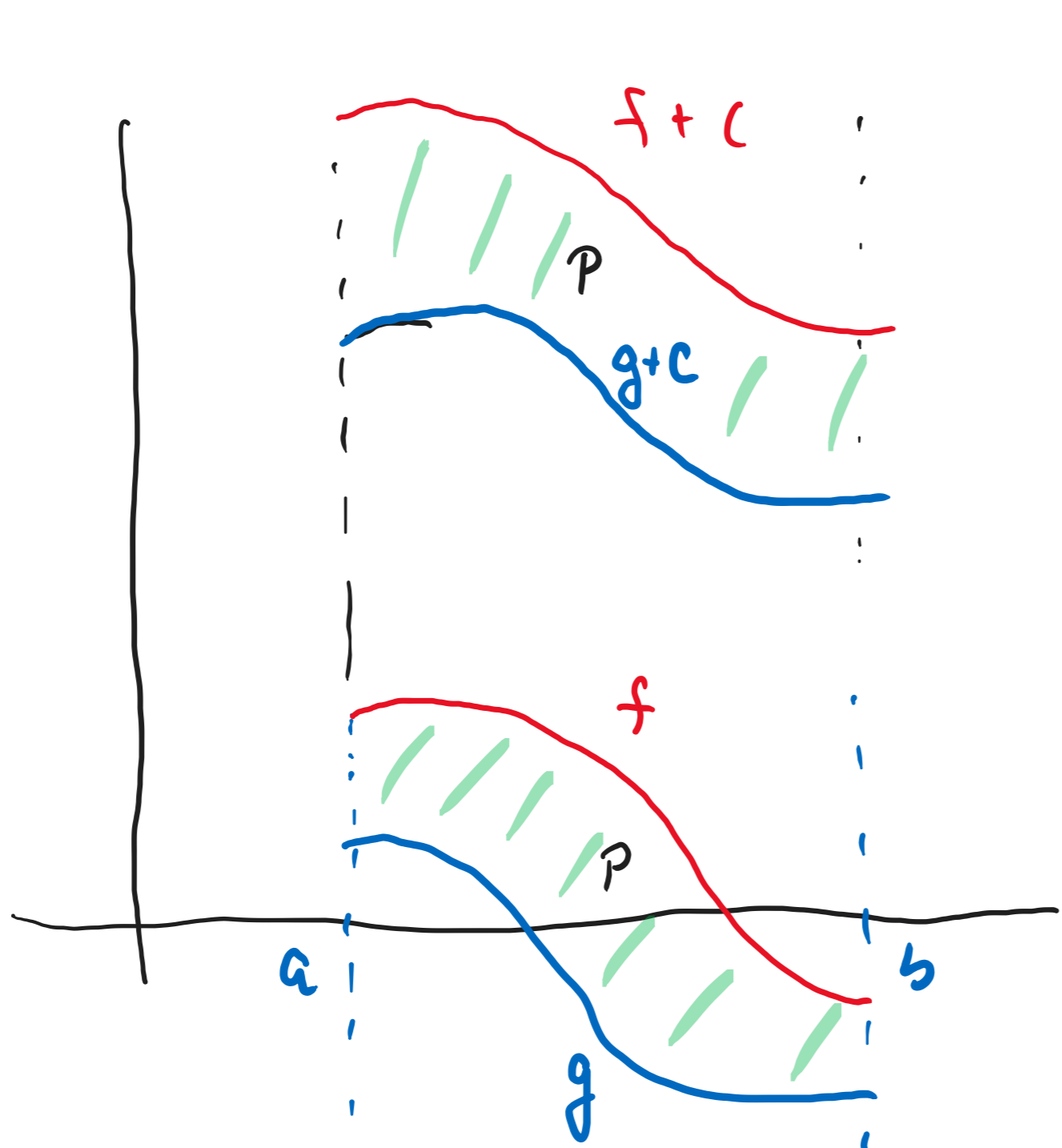
ak  $f(x) \geq g(x) > 0$



el. oblasť  $[x, y]$

$A = \{[x, y] \mid a \leq x \leq b, g(x) \leq y \leq f(x)\}$

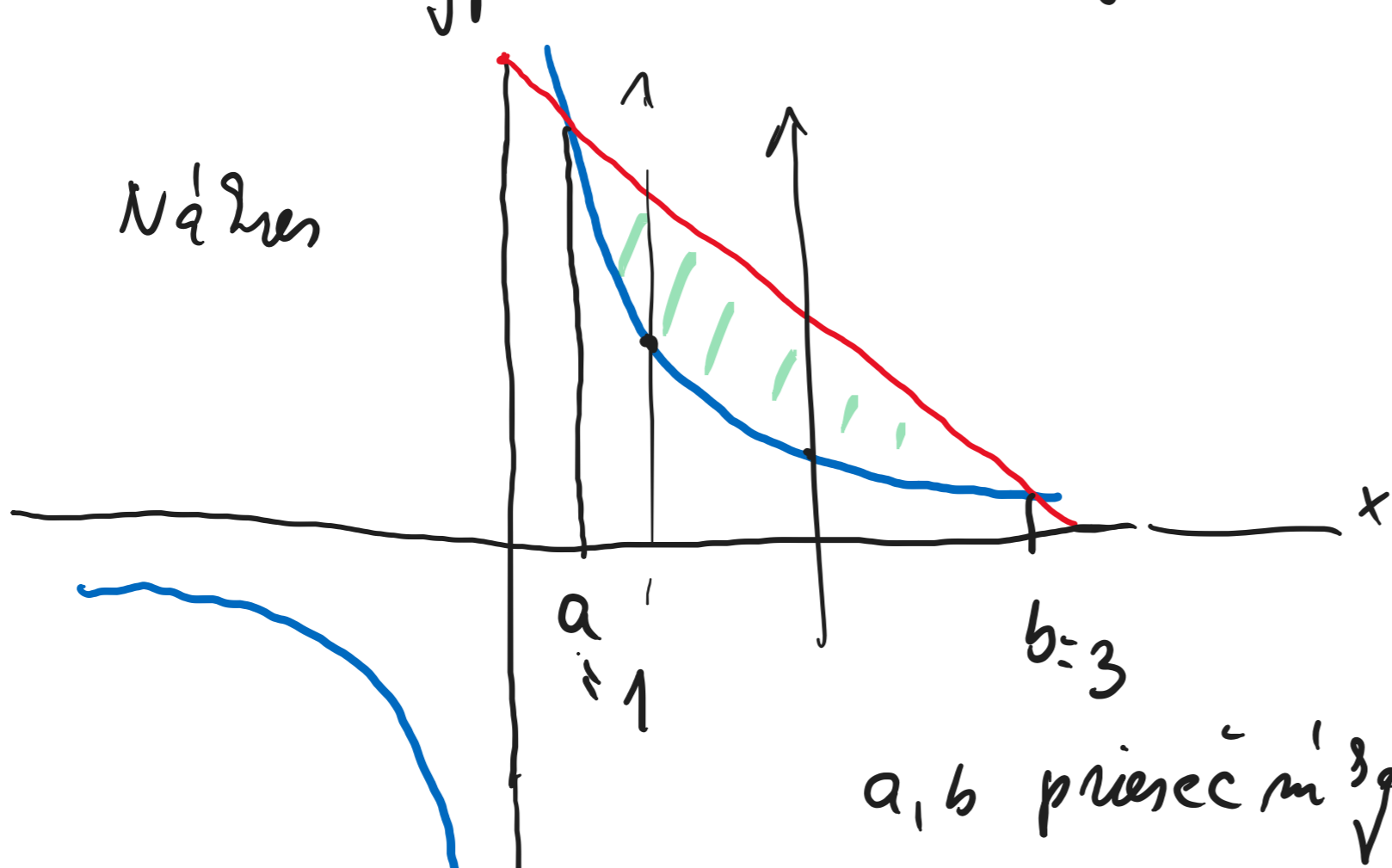
$P(A) = P_f - P_g = \int_a^b f(x) - g(x) dx$   
 HF - DF



$f(x) \geq g(x)$  na  $\langle a, b \rangle$

TRIK  
 roben  $c > 0$ , takže  $g(x) + c > 0$   
 $P = \int_a^b (f(x) + c) - (g(x) + c) dx$   
 $= \int_a^b f(x) - g(x) dx$   
 HF - DF

P1 Vypoč.  $P(A)$  ak  $A$  je obr. hranolmi  $y = \frac{3}{x}$  | DF



$y = 4 - x$  HF

$P = \int_a^b (4 - x - \frac{3}{x}) dx$

$\frac{3}{x} = 4 - x$

$3 = 4x - x^2$

$x^2 - 4x + 3 = 0$

$(x-1)(x-3)$

$P = \int_1^3 (4 - x - \frac{3}{x}) dx = 4x - \frac{x^2}{2} - 3 \ln x \Big|_1^3 = \heartsuit$

STANDARD PRE ZP = P=2 parabola, priamka

Pr.  $P(A) = ?$  ak  $A: y = x^2 + 2$  | DF

$y = 3x$  HF

Der.  $y' = 2x = 0 \Rightarrow x = 0 \Rightarrow y = 2$

$x^2 + 2 = 3x$   
 $x^2 - 3x + 2 = 0$   
 $(x-1)(x-2)$

$A: 1 \leq x \leq 2$

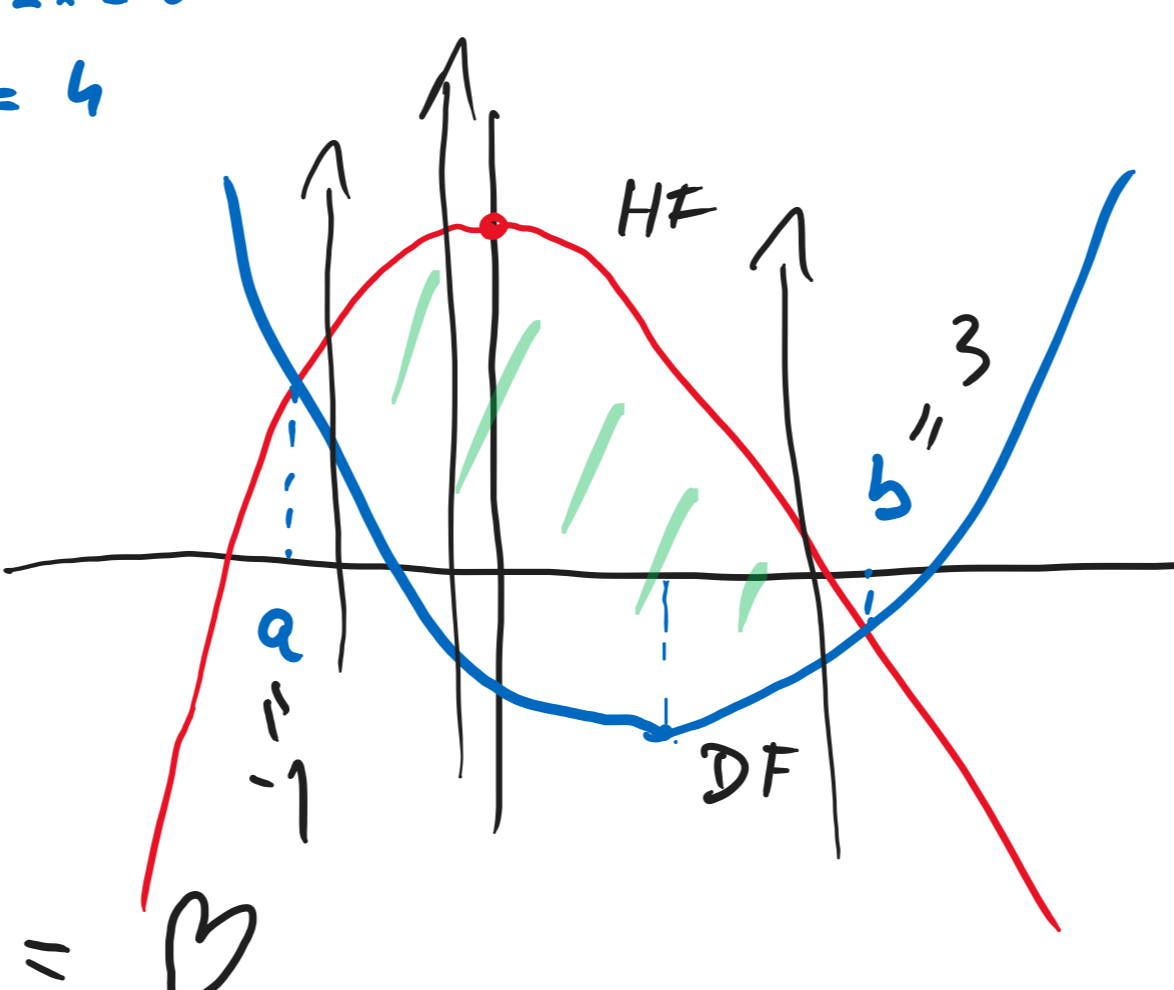
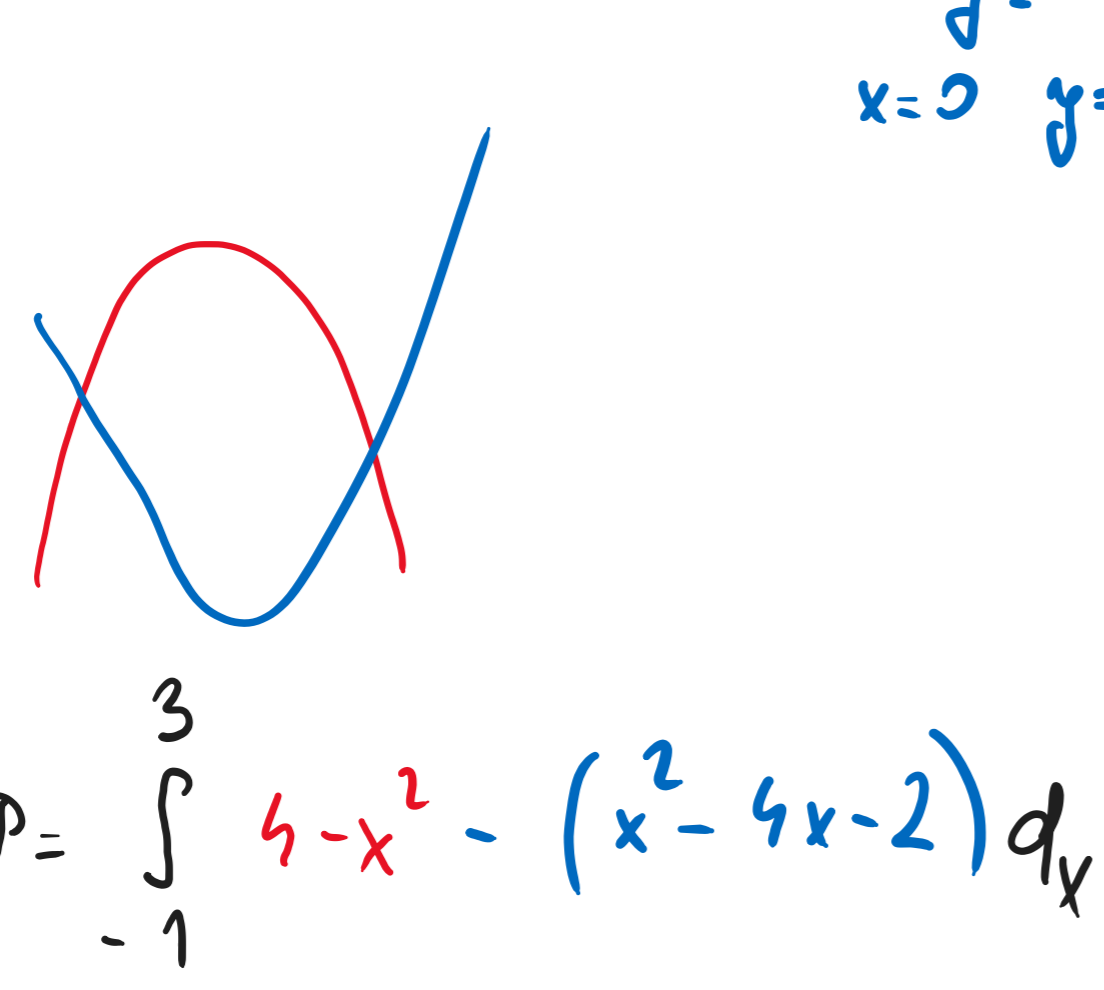
$x^2 + 2 \leq y \leq 3x$   
 DF HF

$P(A) = \int_1^2 (3x - (x^2 + 2)) dx = \frac{3x^2}{2} - \frac{x^3}{3} - 2x \Big|_1^2 = \heartsuit$

Pr.  $P(A) = ?$  ak  $A: y = 4 - x^2; y = x^2 - 4x - 2$

$y' = -2x = 0 \Rightarrow x = 0 \Rightarrow y = 4$

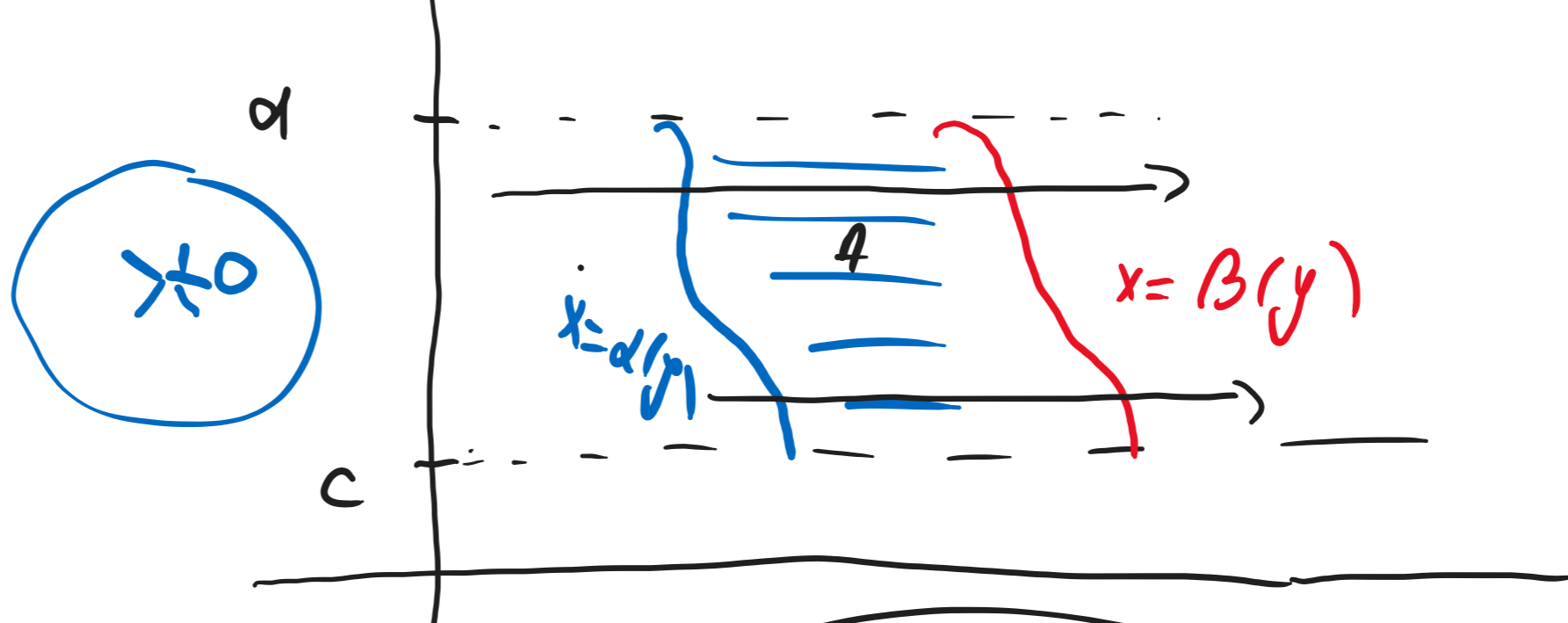
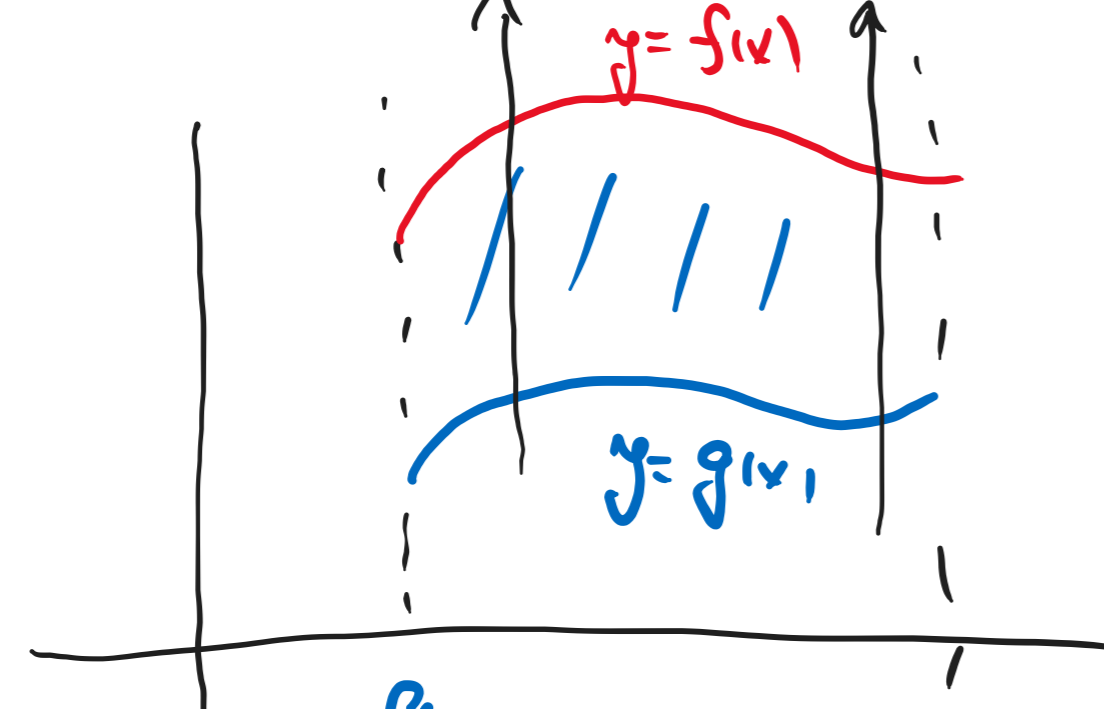
$y' = 2x - 4 = 0 \Rightarrow x = 2 \Rightarrow y < 0$



$4 - x^2 = x^2 - 4x - 2$   
 $2x^2 - 4x - 6 = 0$   
 $x^2 - 2x - 3 = 0$   
 $(x-3)(x+1) = 0$

$P = \int_{-1}^3 (4 - x^2 - (x^2 - 4x - 2)) dx = \heartsuit$

Viemu

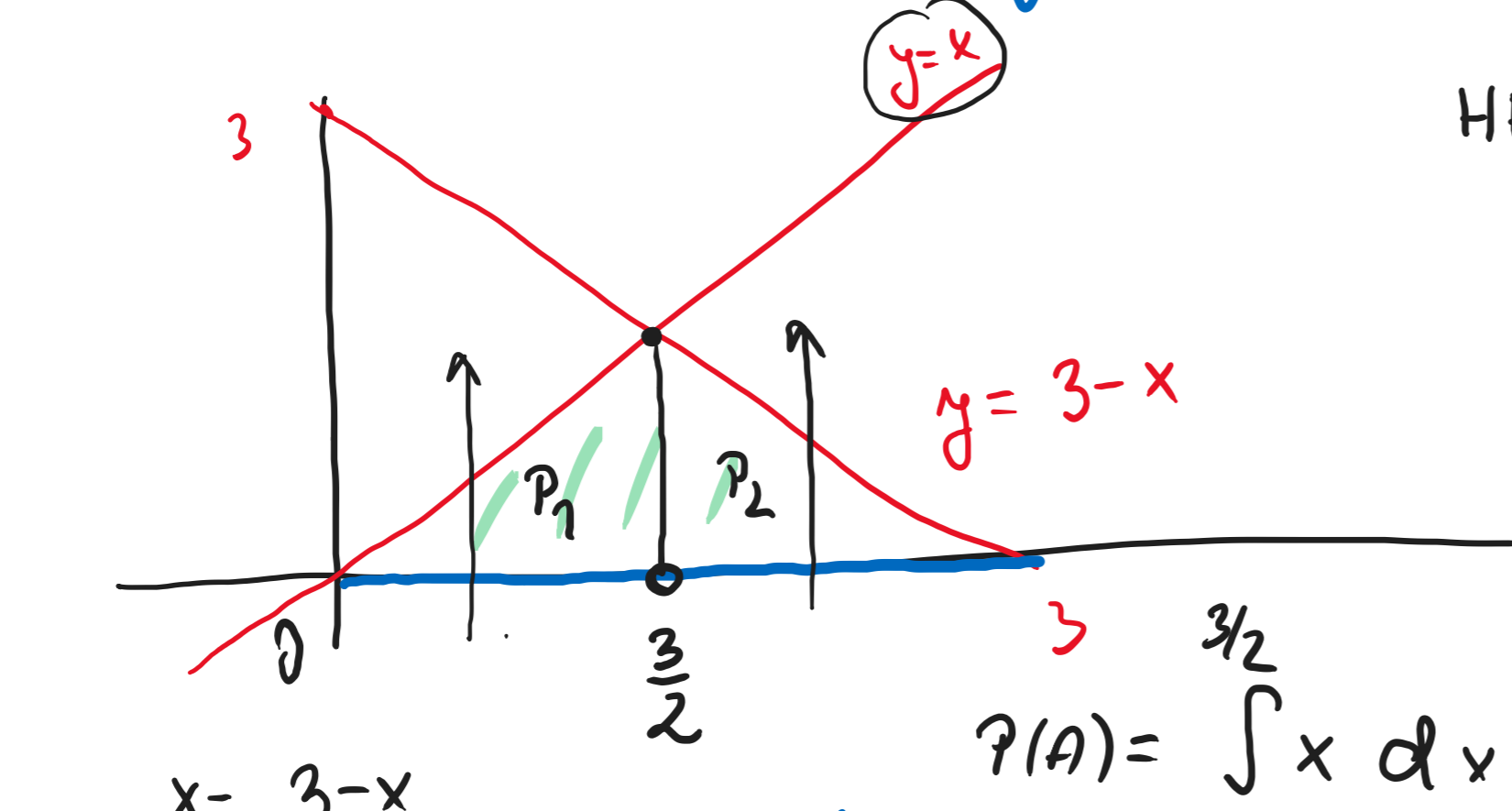


$a \leq x \leq b$   
 $g(x) \leq y \leq f(x)$   
 $[x, y]$

$A: c \leq y \leq d$   
 $a(y) \leq x \leq b(y)$   
 DF HF  
 LF PF

$P(A) = \int_c^d (b(y) - a(y)) dy$

Pr.  $P(A) = ?$  ak  $A: y = x, y = 3 - x, y = 0$

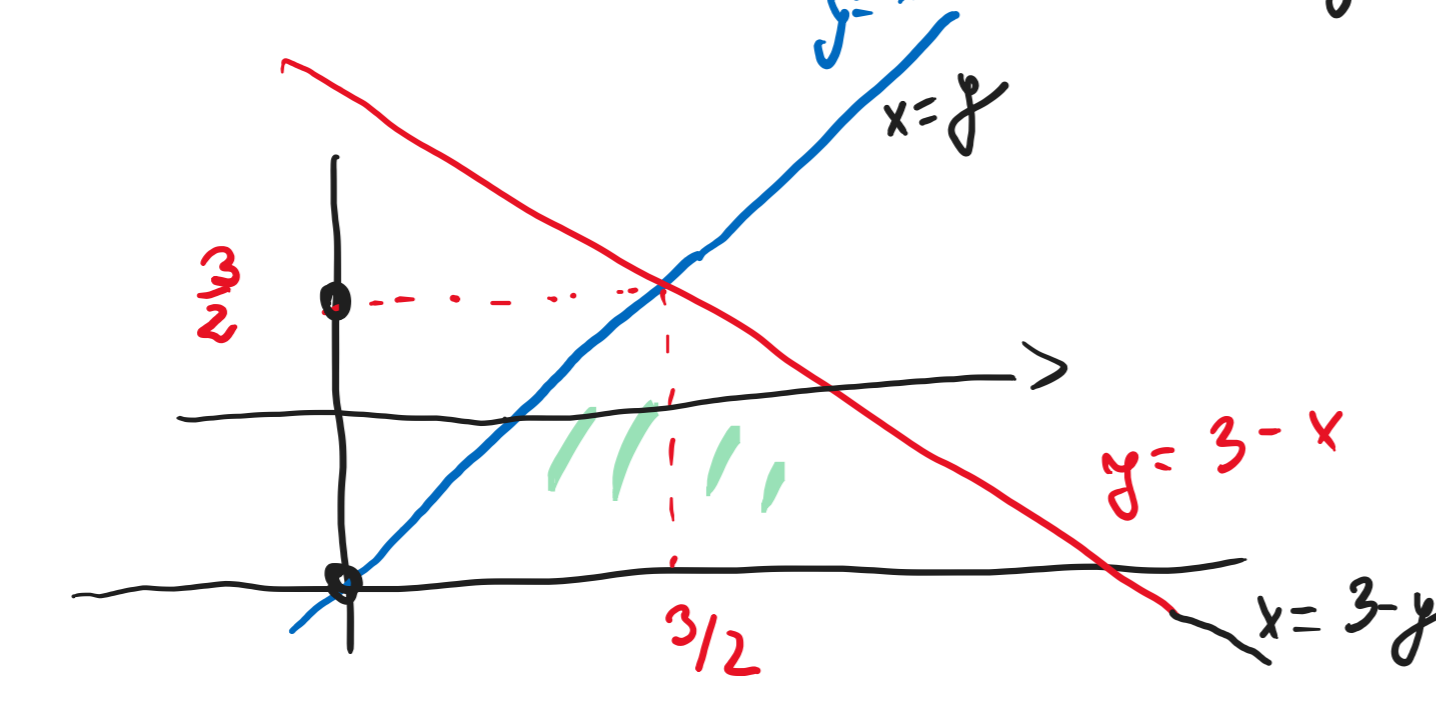


HF na memi!  $P = P_1 \cup P_2 \Rightarrow P(A) = P_1 + P_2$

$P_1: 0 \leq x \leq \frac{3}{2}$   $P_2: \frac{3}{2} \leq x \leq 3$

$0 \leq y \leq x$   $0 \leq y \leq 3 - x$

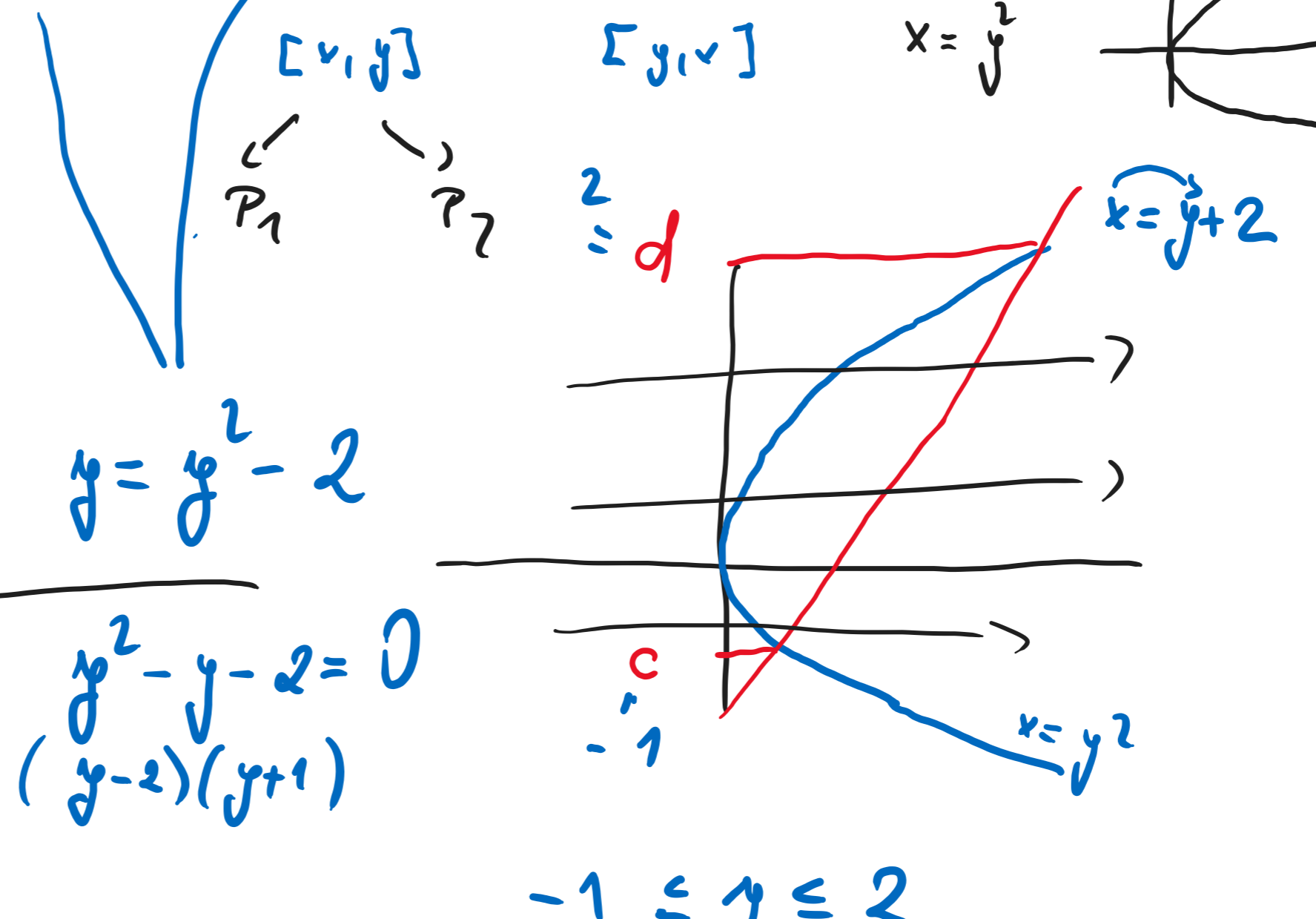
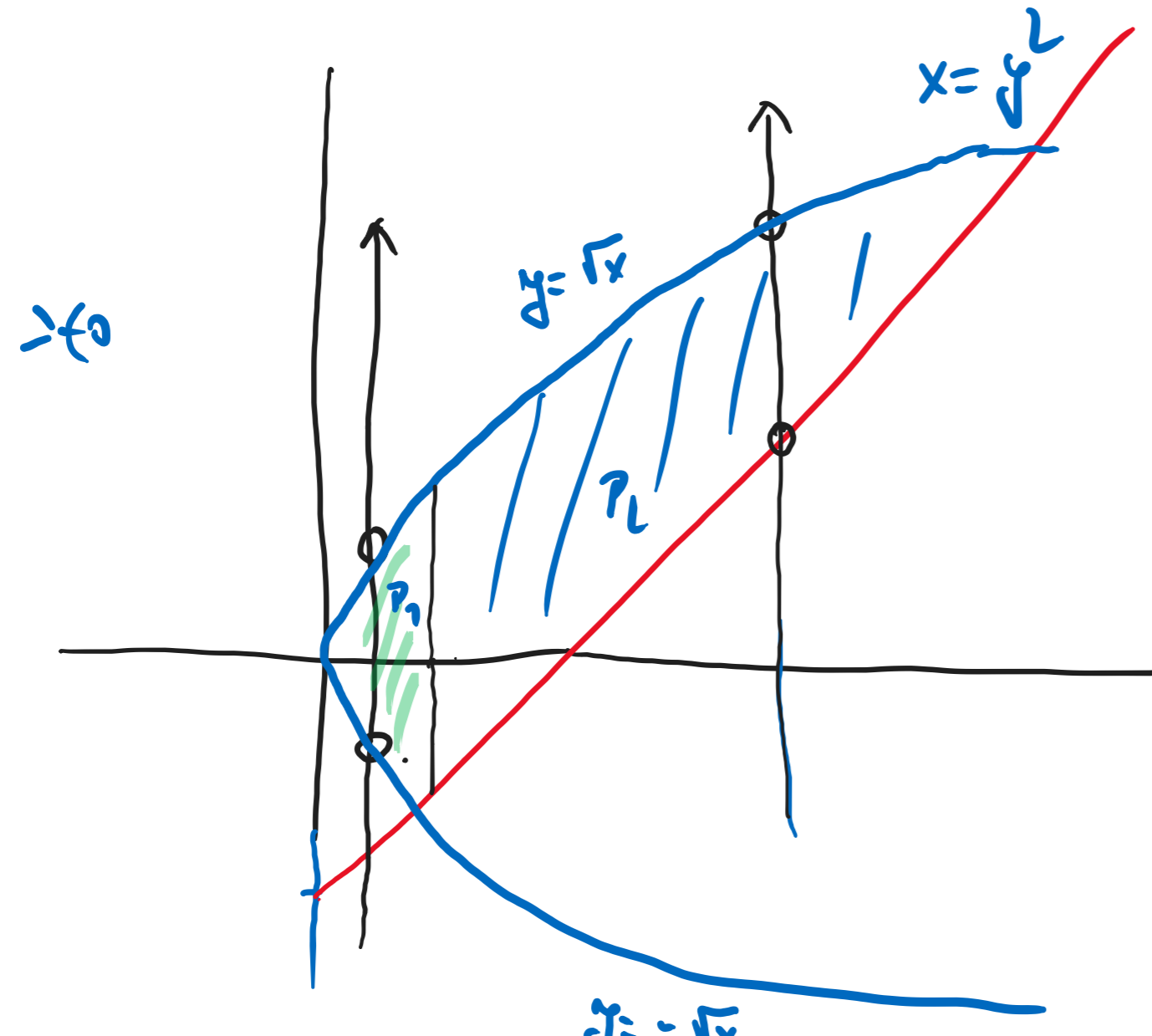
$P(A) = \int_0^{\frac{3}{2}} x dx + \int_{\frac{3}{2}}^3 (3 - x) dx = \heartsuit$



$[y, x]$   
 $0 \leq y \leq \frac{3}{2}$   
 $3 - y \leq x \leq y$

NETREBA DELET  
 +; ZATAM 1 U. I.  
 $P = \int_0^{\frac{3}{2}} (y - (3 - y)) dy = \dots$

Pr.  $P(A) = ?$  ak  $A: x = y^2; y = x - 2$



$y^2 - y - 2 = 0$   
 $(y-2)(y+1)$

$-1 \leq y \leq 2$

$P = \int_{-1}^2 (y+2) - y^2 dy = \frac{y^2}{2} + 2y - \frac{y^3}{3} \Big|_{-1}^2 = \dots$