Algebrické rovnice

streda 24. septembra 2025 7:30

PR 3. $x^3 - 5x^2 + 6x = 0$

$$x^3 - 5x^2 + 6x = x(\underbrace{x^2 - 5x + 6}_{=0}) = x(x-2)(x-3)$$

VIETOVE VZŤAHY 1. $x^2 + bx + c = 0$ resp. pomocou diskriminantu

$x_1 \cdot x_2 = c = 6$

$x_1 + x_2 = -b = 5$ ✓ $\Rightarrow$ $x_1 = 2$, $x_2 = 3$

PR4 $x^3 - 27 = 0$

$x^3 - 3^3 = 0$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$$x^3 - 3^3 = (x-3)(\underbrace{x^2 + 3x + 3^2}_{\neq 0})$$

PR5 $x^3 + 1 = 0$

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$x^3 + 1 = (x+1)(x^2 - x + 1)$$

PR6 $\underbrace{x^3 - 3x^2}\underbrace{+ 4x - 12} = 0$

$x^2(x-3) + 4(x-3) = 0$

$(x^2+4)(x-3) = 0$

PR7 $x^3 + 3x^2 - 4 = 0$

korene sú delitele -4, t.j. $\alpha \in \{\pm 1, \pm 2, \pm 4\}$

HORNEROVA SCHÉMA

$(x-1)$

	1	3	0 !	−4
1		+ 1	+ 4	4
	1	4	4	0

$P(1) = 0 \Rightarrow \alpha = 1$ je koreň

1 je koreň

1	4	4	$\boxed{0}$ – 1 je koreň

$\underbrace{1 \quad 4 \quad 4}_{x^2+4x+4}$

$$x^3+3x^2-4 = (x-1)(x^2+4x+4) = (x-1)(x+2)^2$$

$$(a+b)^2 = a^2+2ab+b^2$$

PR 8 $\quad x^4-2x^3-5x^2+6x = 0$

chýba absolútny člen

$$x^4-2x^3-5x^2+6x = x(x^3-2x^2-5x+6) \Rightarrow x=0 \text{ je koreň}$$

riešime rovnicu

$$x^3-2x^2-5x+6=0$$

delitele 6 : $\pm 1, \pm 2, \pm 3, \pm 6$

	1	-2	-5	6	
1		1	-1	-6	
	1	-1	-6	$\boxed{0}$	1 je koreň
~~1~~		~~1~~	~~0~~		x = 1
	~~1~~	~~0~~	~~-6~~	$\neq 0$	nie je dvojnásobný koreň
-2		-2	6		
	1	-3	0 = P(-2)		

$$P(x) = (x-1)(x^2-x-6)$$

$$P(x) = (x-1)(x+2)(x-3)$$

$$x-(-2) = x+2$$

PR 9. $\quad 4x^4-5x^2+1=0 \qquad a_n=4 \qquad a_0=1$

koreň $\boxed{\alpha = \frac{p}{q}}$ ✓ $\qquad$ p delí 1 $\qquad p=\pm 1$

q delí 4 $\qquad q = \pm 1, \pm 2, \pm 4$

$$\Rightarrow \alpha \in \left\{\pm 1, \pm \frac{1}{2}, \pm \frac{1}{4}\right\}$$

a nasleduje overovanie pomocou HORNEROVEJ SCHÉMY

INÝ POSTUP: $\boxed{t = x^2}$ substitúcia

$$4t^2-5t+1=0 \qquad (au^2+bu+c=0)$$

$$D = b^2 - 4ac = (-5)^2 - 4 \cdot 4 \cdot 1 = 9$$

$$\sqrt{D} = 3$$

$$t_{1,2} = \frac{-b \pm \sqrt{D}}{2a} = \frac{5 \pm 3}{2 \cdot 4} = \left\langle \begin{array}{l} 1 \\ \frac{1}{4} \end{array} \right.$$

$$x^2 = t \Rightarrow x = \pm\sqrt{t}$$

$x^2 = 1$	$x^2 = \frac{1}{4}$
$x = \pm\sqrt{1}$	$x = \pm\sqrt{\frac{1}{4}}$
$x = \pm 1$	$x = \pm \frac{1}{2}$

$$P(x) = 4(x-1)(x+1)\left(x-\frac{1}{2}\right)\left(x+\frac{1}{2}\right)$$

PR10 $\quad x^4 + 4x^3 + 7x^2 + 6x + 2 = 0$

$$\alpha \in \{\pm 1, \pm 2\}$$

$\alpha > 0 : P(\alpha) > 0 \neq 0$ nie je koreň

$$\Rightarrow \alpha < 0 \Rightarrow \alpha \in \{-1, -2\}$$

	1	4	7	6	2
-1		-1	-3	-4	-2
	1	3	4	2	0
-1		-1	-2	-2	
	1	2	2	0	

$$P(x) = (x+1)^2 \overbrace{(x^2 + 2x + 2)}^{(x+1)^2 + 1}$$

$$D = -4$$

už nemá reálne korene