Matematika 1 – 5.cvičenie

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Limita funkcie

d) goniometrická funkcia, dosadíme, ak **neurčitosť** $\frac{0}{0}$, použijeme vzorec

$$\lim_{x \to 0} \frac{\sin x}{x} = 1 \qquad \lim_{\otimes \to 0} \frac{\sin \otimes}{\otimes} = 1$$

Pr. 1 – 17 / 22:

$$\lim_{x \to 0} \frac{\operatorname{tg} 2x}{\sin 5x} \qquad \frac{2}{5}$$

Pr. 2 – 17 / 19:

$$\lim_{x \to 0} \frac{\operatorname{tg} 5x}{x} \qquad 5$$

Pr. 3:

$$\lim_{x\to 0}\frac{\sin 3x}{x} \qquad 3$$

$$\lim_{x\to 0}\frac{\sin 3.0}{0}=\frac{0}{0}$$

$$\lim_{x\to 0}\frac{\sin 3x}{x} \overset{\frac{0}{0}}{=} \lim_{x\to 0}\frac{\sin 3x}{x}\,\frac{3}{3} = \lim_{x\to 0}\frac{\sin 3x}{3\,x}\,\frac{3}{1} = 3$$

Dú : 17 / 20, 21

e) dosadíme, ak **neurčitosť** $\frac{\infty}{\infty}$, delíme čitateľa aj menovateľa najvyššou mocninou v menovateli (ak v čitateli a menovateli polynómy)

Platí: $\infty.\infty = \infty, \quad -\infty.-\infty = \infty, \quad -\infty.\infty = -\infty, \quad c.\infty = \infty$

$$\infty + \infty = \infty, \quad -\infty - \infty = -\infty, \quad \infty + c = \infty, \quad \sqrt{\infty} = \infty$$

$$\lim_{x\to\infty} \frac{1}{x} = 0, \lim_{x\to\infty} \frac{1}{x^n} = 0, \lim_{x\to\infty} x = \infty$$

Pr. 4 – 17 / 29:

$$\lim_{x\to\infty} \frac{x^3 + x}{x^4 - 3x^2 + 1} \qquad 0$$

Pr. 5 – 17 / 30:

$$\lim_{x\to\infty} \frac{2x^4 - 5x}{10x^2 - 3x + 1} \qquad \infty$$

Pr. 6:

$$\lim_{x\to\infty} \frac{x^2+x+1}{3x^2+1} \qquad \frac{1}{3}$$

$$\lim_{x\to\infty} \frac{\infty+\infty+1}{\infty+1} = \frac{\infty}{\infty}$$

$$\lim_{x\to\infty} \frac{x^2+x+1}{3x^2+1} \overset{\frac{\infty}{\infty}}{=} \lim_{x\to\infty} \frac{\frac{x^2}{x^2}}{\frac{3x^2}{x^2}} = \lim_{x\to\infty} \frac{1}{3} = \frac{1}{3}$$

najvyššia mocnina, delíme x^2

členy s nižšími mocninami

$$\lim_{x\to\infty} \frac{\frac{x}{x^2}+\frac{1}{x^2}}{\frac{1}{x^2}} = 0$$

preto v limite stačí deliť najvyššou mocninou len prvé členy polynómov (výrazy v čitateli a menovateli)

$$\lim_{x\to\infty} \frac{4x^2-5x+2}{2x^2+3}$$ Dú $\qquad 2$

$$\lim_{x\to\infty} \frac{2x+5}{3x-1}$$ Dú $\qquad \frac{2}{3}$

g) dosadíme, ak **neurčitosť** $\infty - \infty$, rozšírime vhodnou jednotkou (pomocou vzorca $a^2 - b^2 = (a-b)(a+b)$) a potom delíme čitateľa aj menovateľa najvyššou mocninou v menovateli

Pr. 7:

$$\lim_{x \to \infty}\left(\sqrt{x^2 + 5x + 6} - \sqrt{x^2 - 3}\right)$$

$$\frac{5}{2}$$

Pr. 8 – 17 / 34:

$$\lim_{x \to \infty} \left(\sqrt{x-2} - \sqrt{x} \right) \qquad 0$$

$$\lim_{x\to\infty} \sqrt{\infty} - \sqrt{\infty} = \infty - \infty$$

$$\lim_{x\to\infty} \left(\sqrt{x-2} - \sqrt{x}\right) \overset{\infty - \infty}{=} \lim_{x\to\infty} \overset{(a-b)}{\left(\sqrt{x-2} - \sqrt{x}\right)} \overset{(a+b)}{\frac{\sqrt{x-2}+\sqrt{x}}{\sqrt{x-2}+\sqrt{x}}} = \lim_{x\to\infty} \frac{\overset{a^2-b^2}{x-2-x}}{\sqrt{x-2}+\sqrt{x}} =$$

$$\lim_{x\to\infty} \frac{-\frac{2}{\sqrt{x}}}{\sqrt{\frac{x}{x}-\frac{2}{x}}+\sqrt{\frac{x}{x}}} = \lim_{x\to\infty} \frac{0}{\sqrt{1-0}+\sqrt{1}} = 0$$

najvyššia mocnina je $\sqrt{x}$

Dú : 17 / 35, 36

h) dosadíme, ak **neurčitosť 1**$^{\infty}$, použijeme jeden zo vzorcov

$$\lim_{x\to\infty}\left(1 \pm \frac{1}{x}\right)^{x} = e^{\pm 1}$$

$$\lim_{x\to\infty}\left(1 + \frac{k}{f(x)}\right)^{f(x)} = e^{k}\ , \quad f(x) \to \infty$$

Pr. 9 – 22 / 21:

$$\lim_{x\to\infty}\left(1 + \frac{2}{x+3}\right)^{x}$$

Pr. 10:

$$\lim_{x\to\infty}\left(1+\frac{3}{x-5}\right)^{2x} \qquad e^{6}$$

$$\left(1+\frac{3}{\infty}\right)^{\infty}=\mathbf{1}^{\infty}$$

$$\lim_{x\to\infty}\left(1+\frac{3}{x-5}\right)^{2x} \overset{\mathbf{1}^{\infty}}{=} \lim_{x\to\infty}\left(1+\frac{3}{x-5}\right)^{2x\frac{(x-5)}{x-5}} = \lim_{x\to\infty}\left[\left(1+\frac{3}{x-5}\right)^{x-5}\right]^{\frac{2x}{x-5}} =$$

$$e^{3.\lim\limits_{x\to\infty}\frac{2x}{x-5}} = e^{3.\frac{2}{1-0}} = e^{6}$$

$$\lim_{x\to\infty}\left(1+\frac{3}{x+1}\right)^{x-1} \quad \text{Dú} \qquad e^{3}$$

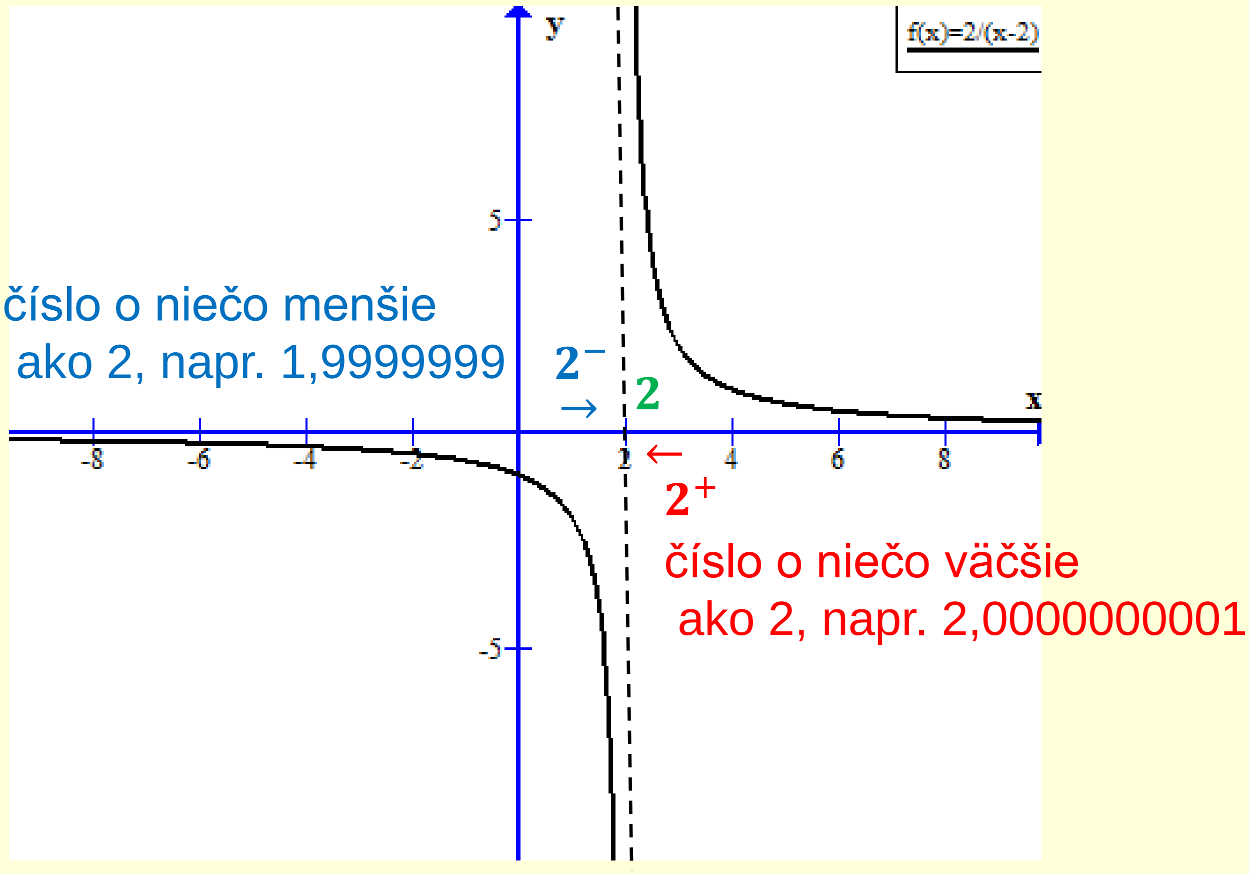
$$\lim_{x\to\infty}\left(1-\frac{1}{x+1}\right)^{x-1} \quad \text{Dú} \qquad e^{-1}$$

Asymptoty funkcie

Jednostranné limity:

limita pre *x* idúce k ***a*** sprava $\lim_{x \to a^+} f(x)$

limita pre *x* idúce k ***a*** zľava $\lim_{x \to a^-} f(x)$

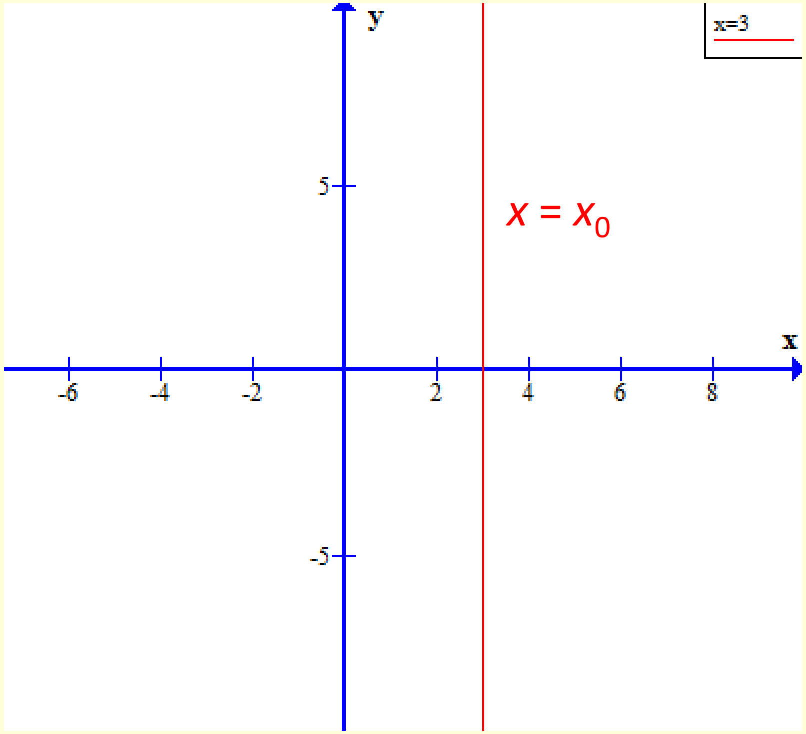


Asymptota je priamka, ktorá opisuje správanie sa krivky.

Asymptota bez smernice (ABS): vypočítame jednostranné limity v bode nespojitosti x_0 (bod v ktorom funkcie nie je definovaná), ak vyjdú nevlastné čísla $\pm\infty$ (stačí jedna z limít rovná $\pm\infty$, potom funkcia má ABS)

$$\lim_{x \to x_0^+} f(x) = \pm\infty \qquad \lim_{x \to x_0^-} f(x) = \pm\infty$$

potom ABS je $x = x_0$

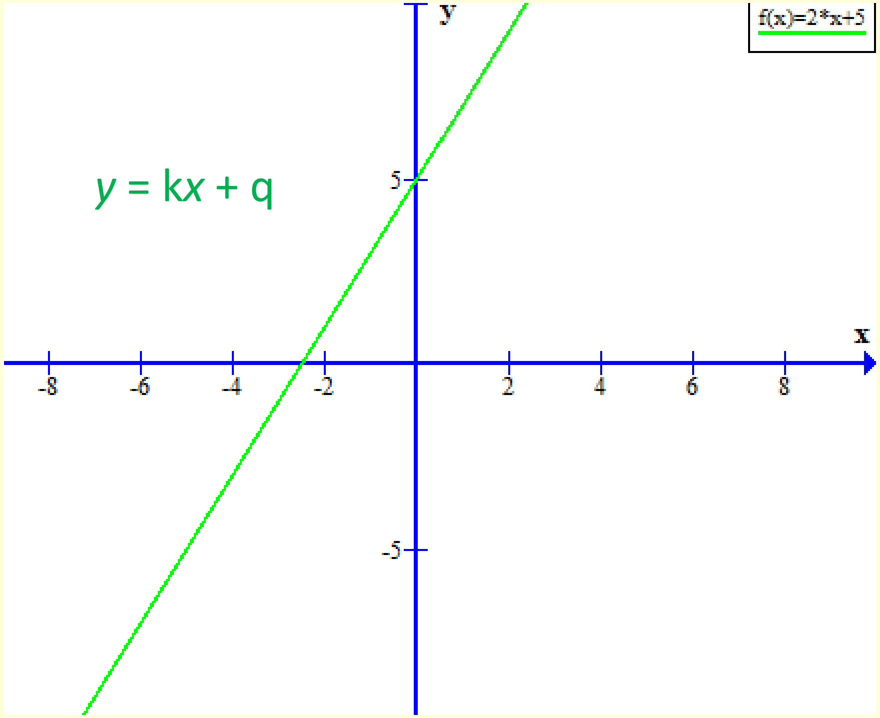


Asymptota so smernicou (ASS): priamka y = kx + q

$$k = \lim_{x \to \pm\infty} \frac{f(x)}{x} \qquad\qquad q = \lim_{x \to \pm\infty} [f(x) - kx]$$

vypočítame k, q, ak vyjdú vlastné čísla, potom

ASS je $y = k_1 x + q_1$, $y = k_2 x + q_2$



Pr. 1: Určte ASS a ABS funkcie

$$y = \frac{3(x-1)^2}{x-2}$$

1. Určiť D(f) a bod nespojitosti.

$$x - 2 \neq 0$$
$$x \neq 2\,,\quad D(f) = R\ - \{2\}$$

bod nespojitosti $x_0 = 2$

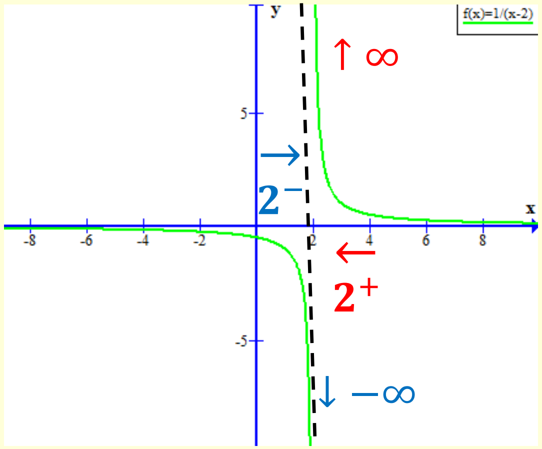
2. Určiť ABS v bode nespojitosti

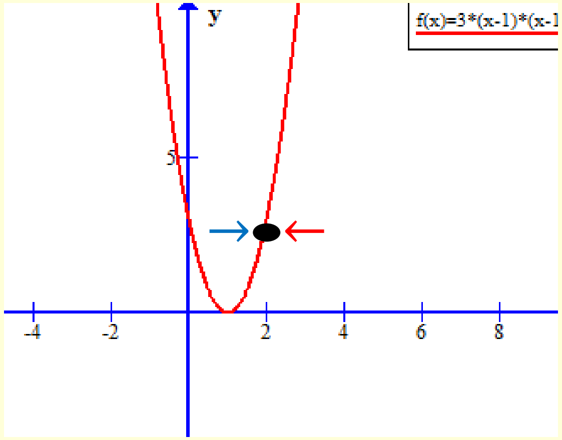
$$\frac{1}{2^+ - 2} = \frac{1}{0^+} \qquad 3(2^+ - 1)^2 = +\,\check{c}\acute{\imath}slo$$

$$\lim_{x\to x_0^+} f(x) = \lim_{x\to 2^+} \frac{3(x-1)^2}{x-2} = \lim_{x\to 2^+} \frac{1}{x-2} \cdot \lim_{x\to 2^+} 3(x-1)^2 = +\infty$$

$$\lim_{x\to x_0^-} f(x) = \lim_{x\to 2^-} \frac{3(x-1)^2}{x-2} = \lim_{x\to 2^-} \frac{1}{x-2} \cdot \lim_{x\to 2^-} 3(x-1)^2 = -\infty$$

$$\frac{1}{2^- - 2} = \frac{1}{0^-} \qquad 3(2^- - 1)^2 = +\,\check{c}\acute{\imath}slo$$





ABS: $x = 2$

3. Určiť ASS

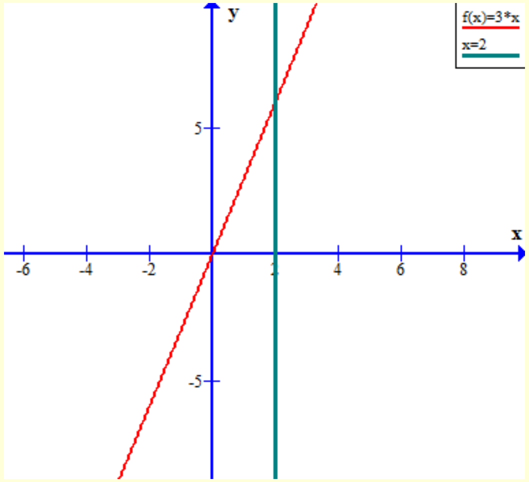
$$k = \lim_{x\to\infty} \frac{f(x)}{x} = \lim_{x\to\infty} \frac{3(x-1)^2}{x(x-2)} = \lim_{x\to\infty} \frac{3x^2-6x+3}{x^2-2x} = 3$$

$$k = \lim_{x\to-\infty} \frac{f(x)}{x} = \lim_{x\to-\infty} \frac{3(x-1)^2}{x(x-2)} = \lim_{x\to-\infty} \frac{3x^2-6x+3}{x^2-2x} = 3$$

$$q = \lim_{x\to\infty}[f(x)-kx] = \lim_{x\to\infty}\left[\frac{3(x-1)^2}{(x-2)} - 3x\right] = \lim_{x\to\infty}\left[\frac{3x^2-6x+3-3x^2+6x}{(x-2)}\right]$$

$$= \lim_{x\to\infty}\left[\frac{3}{(x-2)}\right] = 0 \qquad q = \lim_{x\to-\infty}\left[\frac{3}{(x-2)}\right] = 0$$

ASS: $y = 3x \quad pre \;\; x \to \pm\infty$



ASS: $y = 3x \qquad pre \;\; x \to \pm\infty$

ABS: $x = 2$

Pr.2 – 47/ 2: Určte ASS a ABS funkcie

$$y = x^3 + 3x^2 - 2$$

Pr.3: Určte ASS a ABS funkcie

$$y = \frac{2x^2}{2x-1}$$

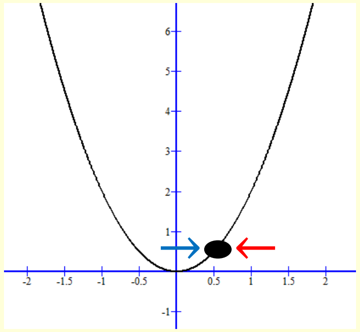
$$2x - 1 \neq 0$$

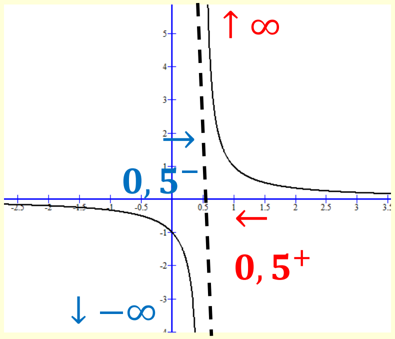
$$x \neq \frac{1}{2}, \quad D(f) = R - \left\{\frac{1}{2}\right\}$$

bod nespojitosti $\boldsymbol{x_0 = 0,5}$

$$\lim_{x \to x_0^+} f(x) = \lim_{x \to 0,5^+} \frac{2x^2}{2x-1} = \lim_{x \to 0,5^+} \frac{1}{2x-1} . \lim_{x \to 0,5^+} 2x^2 = \frac{1}{0^+} . (+\, \check{c}\acute{\imath}slo) = +\infty$$

$$\lim_{x \to x_0^-} f(x) = \lim_{x \to 0,5^-} \frac{2x^2}{2x-1} = \lim_{x \to 0,5^-} \frac{1}{2x-1} . \lim_{x \to 0,5^-} 2x^2 = \frac{1}{0^-} . (+\, \check{c}\acute{\imath}slo) = -\infty$$



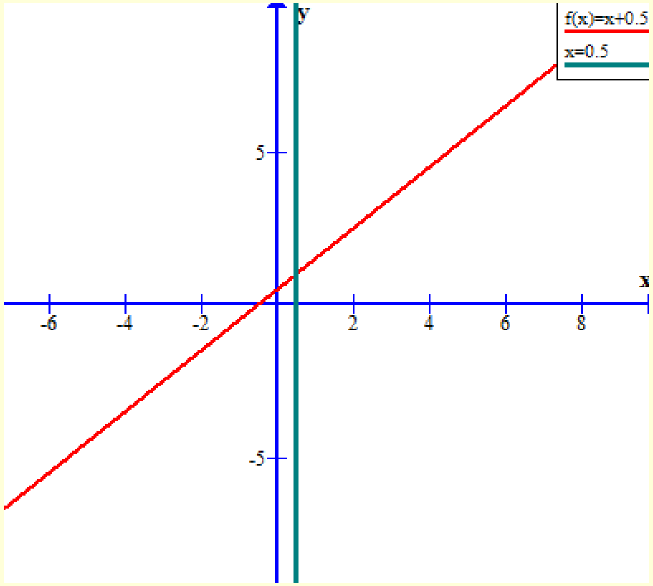


ABS: $x = 0,5$

$$k = \lim_{x\to\pm\infty} \frac{f(x)}{x} = \lim_{x\to\pm\infty} \frac{\frac{2x^2}{2x-1}}{x} = \lim_{x\to\pm\infty} \frac{2x^2}{2x^2 - x} = 1$$

$$q = \lim_{x\to\pm\infty} [f(x) - kx] = \lim_{x\to\pm\infty} \left[\frac{2x^2}{2x-1} - x\right] = \lim_{x\to\pm\infty} \left[\frac{2x^2 - 2x^2 + x}{2x-1}\right] = \frac{1}{2}$$

ASS: $y = x + 0{,}5 \quad pre \;\; x \to \pm\infty$



ASS: $y = x + 0{,}5 \quad pre \;\; x \to \pm\infty$

ABS: $x = 0{,}5$