Matematika 1 – 6.cvičenie

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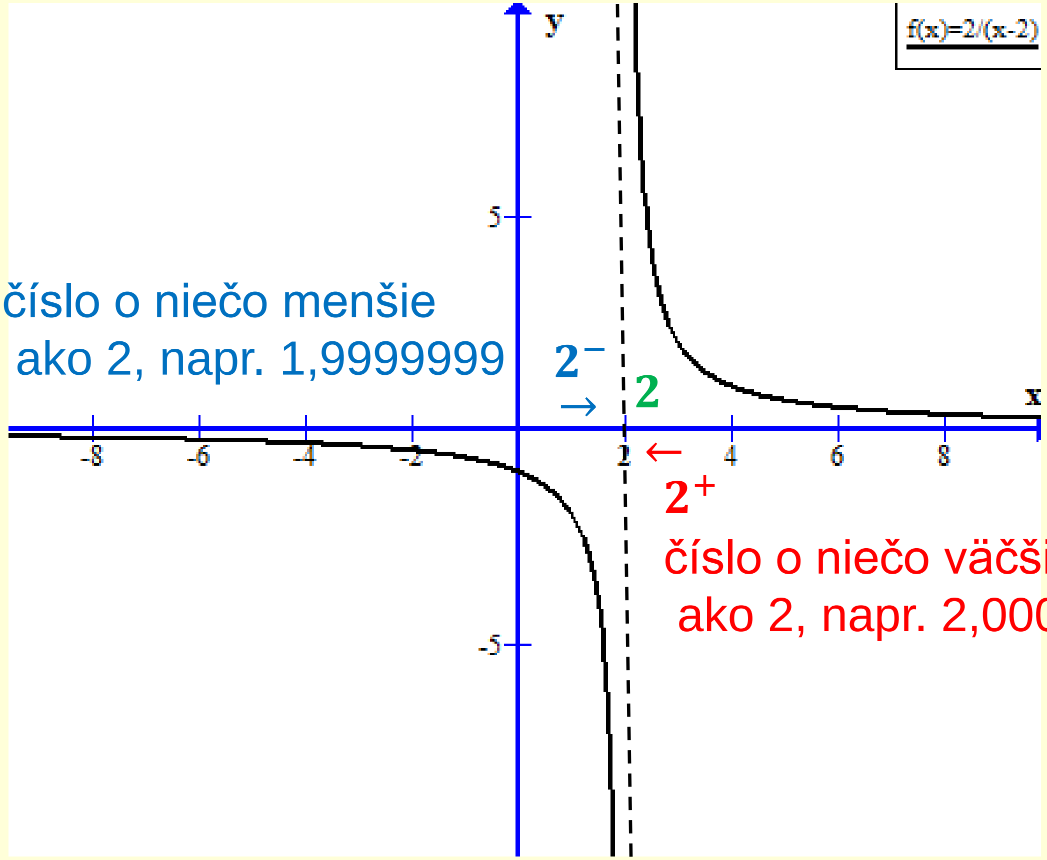
Asymptoty funkcie

Jednostranné limity:

limita pre *x* idúce k ***a*** sprava $\lim\limits_{x \to a^+} f(x)$

limita pre *x* idúce k ***a*** zľava $\lim\limits_{x \to a^-} f(x)$



číslo o niečo menšie ako 2, napr. 1,9999999

2^- $\rightarrow$ 2 $\leftarrow$ 2^+

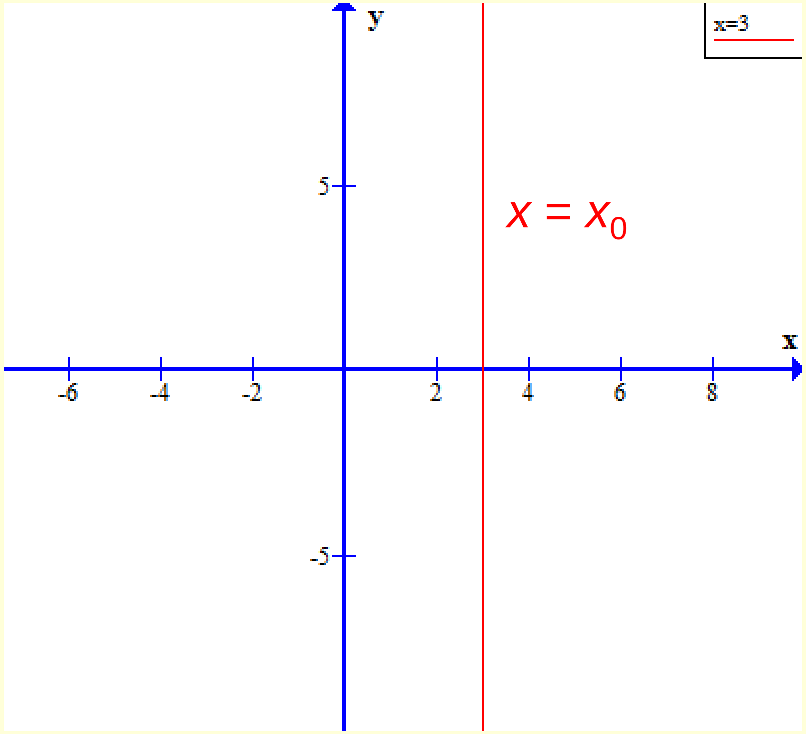
číslo o niečo väčšie ako 2, napr. 2,0000000001

Asymptota je priamka, ktorá opisuje správanie sa krivky.

Asymptota bez smernice (ABS): vypočítame jednostranné limity v bode nespojitosti x_0 (bod v ktorom funkcie nie je definovaná), ak vyjdú nevlastné čísla $\pm\infty$ (stačí jedna z limít rovná $\pm\infty$, potom funkcia má ABS)

$$\lim_{x \to x_0^+} f(x) = \pm\infty \qquad \lim_{x \to x_0^-} f(x) = \pm\infty$$

potom ABS je $x = x_0$

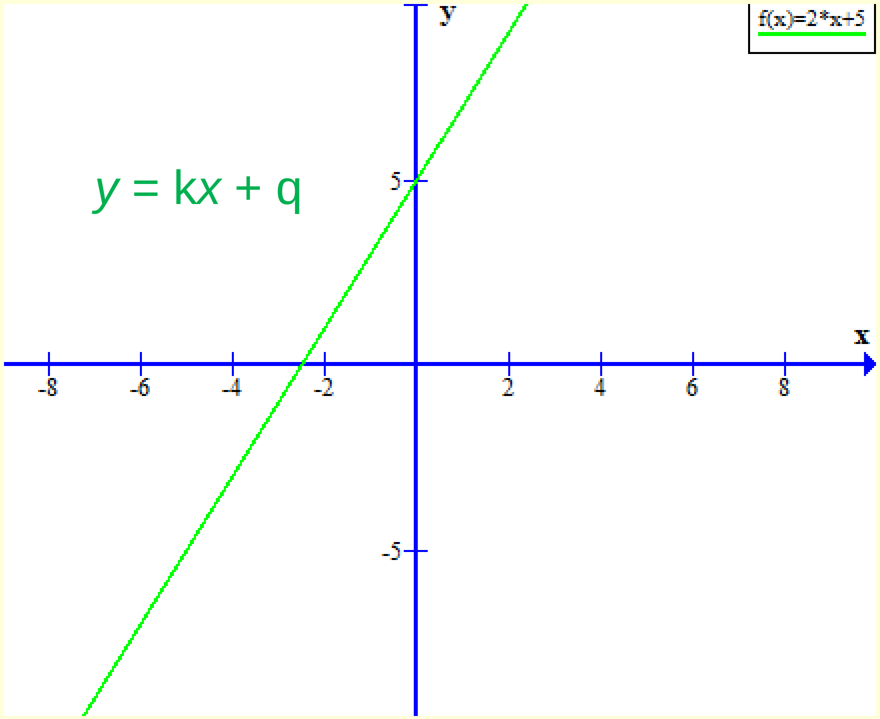


Asymptota so smernicou (ASS): priamka $y = kx + q$

$$k = \lim_{x \to \pm\infty} \frac{f(x)}{x} \qquad\qquad q = \lim_{x \to \pm\infty} [f(x) - kx]$$

vypočítame k, q, ak vyjdú vlastné čísla, potom

ASS je $y = k_1 x + q_1$, $y = k_2 x + q_2$



Pr. 1 - 47 / 12: Určte ASS a ABS funkcie

$$y = \frac{x^2}{4 - x^2}$$

$$\lim_{x \to x_0^+} f(x) = \lim_{x \to -2^+} \frac{x^2}{4 - x^2} = \lim_{x \to -2^+} \frac{1}{4 - x^2} . \lim_{x \to -2^+} x^2 = \frac{1}{0^+} . 4^- = \infty$$

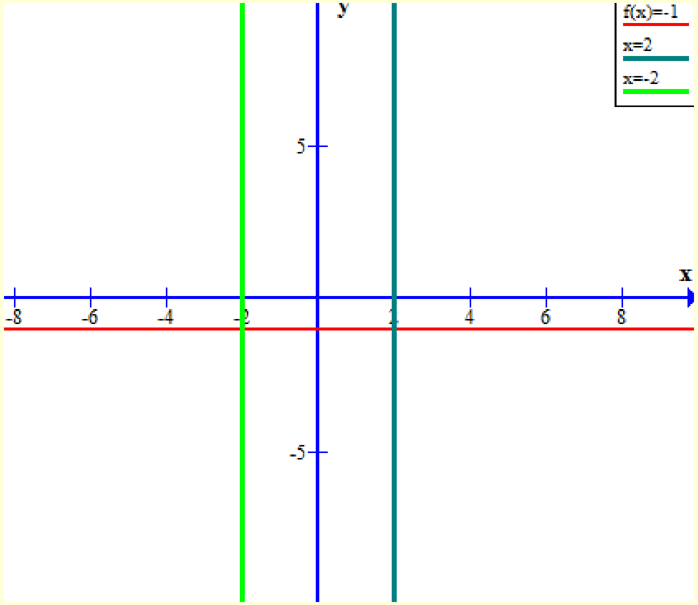
$$\lim_{x \to x_0^-} f(x) = \lim_{x \to -2^-} \frac{x^2}{4 - x^2} = \lim_{x \to -2^-} \frac{1}{4 - x^2} . \lim_{x \to -2^-} x^2 = \frac{1}{0^-} . 4^- = -\infty$$

ABS: $x = -2$

$k = 0$

$$q = \lim_{x \to \pm\infty} [f(x) - kx] = \lim_{x \to \pm\infty} \left[\frac{x^2}{4 - x^2} \right] = -1$$

ASS: $y = -1 \; pre \;\; x \to \underline{+}\infty$



ASS: $y = -1 \;\; pre \;\; x \to \underline{+}\infty$

ABS: $x = -2$

ABS: $x = 2$

Pr. 2 - 47 / 2: Určte ASS a ABS funkcie

$$y = x^3 + 3x^2 - 2$$

Pr. 3: Určte ASS a ABS funkcie

$$y = \frac{2x^2}{2x - 1}$$

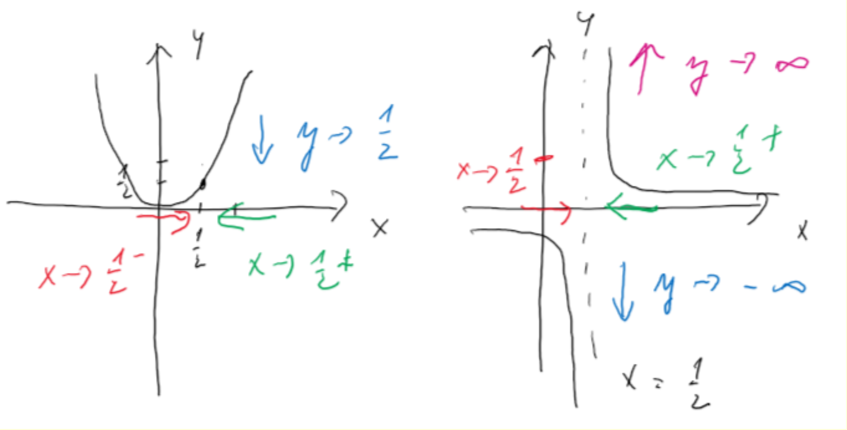
$2x - 1 \neq 0$

$x \neq \frac{1}{2}\,, \quad D(f) = R \; - \left\{\frac{1}{2}\right\}$

bod nespojitosti $\boldsymbol{x_0 = 0,5}$

$$\lim_{x \to x_0^+} f(x) = \lim_{x \to 0,5^+} \frac{2x^2}{2x - 1} = \lim_{x \to 0,5^+} \frac{1}{2x - 1} . \lim_{x \to 0,5^+} 2x^2 = \frac{1}{0^+} . (+\, číslo) = +\infty$$

$$\lim_{x \to x_0^-} f(x) = \lim_{x \to 0,5^-} \frac{2x^2}{2x - 1} = \lim_{x \to 0,5^-} \frac{1}{2x - 1} . \lim_{x \to 0,5^-} 2x^2 = \frac{1}{0^-} . (+\, číslo) = -\infty$$

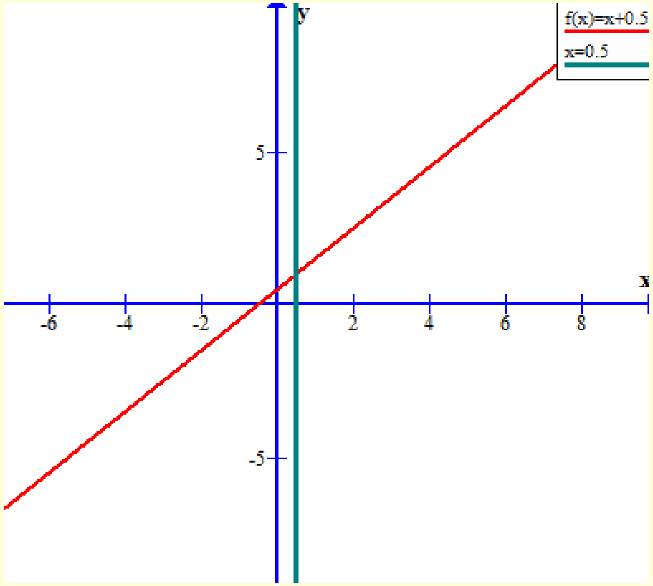


ABS: $x = 0,5$

$$k = \lim_{x\to\pm\infty} \frac{f(x)}{x} = \lim_{x\to\pm\infty} \frac{\frac{2x^2}{2x-1}}{x} = \lim_{x\to\pm\infty} \frac{2x^2}{2x^2 - x} = 1$$

$$q = \lim_{x\to\pm\infty} [f(x) - kx] = \lim_{x\to\pm\infty} \left[\frac{2x^2}{2x-1} - x\right] = \lim_{x\to\pm\infty} \left[\frac{2x^2 - 2x^2 + x}{2x-1}\right] = \frac{1}{2}$$

ASS: $y = x + 0{,}5 \quad pre\ \ x \to \pm\infty$



ASS: $y = x + 0{,}5 \quad pre\ \ x \to \pm\infty$

ABS: $x = 0{,}5$

Dú – 47 / 1, 5, 6, 8, 9,12,13,16

Derivácia funkcie

Označenie derivácie funkcie v bode x_0: $f'(x_0)$

Pravidlá pre výpočet derivácie funkcie**:**

$$[cf(x)]' = cf'(x) \qquad c \in R$$

$$[f(x) + g(x)]' = f'(x) + g'(x)$$

$$[f(x)g(x)]' = f'(x)g(x) + f(x)g'(x)$$

$$\left[\frac{f(x)}{g(x)}\right]' = \frac{f'(x)g(x) - f(x)g'(x)}{g^2(x)}.$$

$$[f(g(x))]' = f'(g(x))g'(x).$$

$$\left[f(x)^{g(x)}\right]' = \left[e^{g(x)\cdot \ln f(x)}\right]'$$

Napr.:

$(3x)'$

$(2x^2 + 5 - \ln x)'$

$(x \cdot \ln x)'$

$\left(\frac{3x}{\sin x}\right)'$

$(\ln \sin 2x)'$

$\left[(\sin x)^x = e^{x \ln \sin x}\right]'$

Derivácie elementárnych funkcií:

- $[c]' = 0$ $\quad [x]'=1$
- $[x^\alpha]' = \alpha x^{\alpha-1}, \alpha \in R$
- $[\sin x]' = \cos x$
- $[\cos x]' = -\sin x$
- $[\operatorname{tg} x]' = \dfrac{1}{\cos^2 x}$
- $[\operatorname{cotg} x]' = -\dfrac{1}{\sin^2 x}$
- $[\arcsin x]' = \dfrac{1}{\sqrt{1-x^2}}$
- $[\arccos x]' = -\dfrac{1}{\sqrt{1-x^2}}$
- $[\operatorname{arctg} x]' = \dfrac{1}{1+x^2}$
- $[\operatorname{arccotg} x]' = -\dfrac{1}{1+x^2}$
- $[e^x]' = e^x$
- $[a^x]' = a^x \ln a$
- $[\ln x]' = \dfrac{1}{x}$
- $[\log_a x]' = \dfrac{1}{x \ln a}$

Pr.1: Vypočítajte deriváciu funkcie

$$f(x) = \frac{\sin x}{2} + x.2^x + \frac{x}{\ln x} - \sqrt{x^3}$$

Pr. 2 - 25 /2: Vypočítajte deriváciu funkcie

$$f(x) = \sqrt[3]{x^4} + 5^x - \ln x$$

$$f'(x) = \left(x^{\frac{4}{3}}\right)' + (5^x)' - (\ln x)'$$

$$(x^n)' = nx^{n-1} \qquad (a^x)' = a^x.\ln a \qquad (\ln x)' = \frac{1}{x}$$

$$f'(x) = \frac{4}{3}x^{\frac{4}{3}-1} + 5^x.\ln 5 - \frac{1}{x}$$

$$f'(x) = \frac{4}{3}x^{\frac{1}{3}} + 5^x.\ln 5 - \frac{1}{x}$$

$$f'(x) = \frac{4}{3}\sqrt[3]{x} + 5^x.\ln 5 - \frac{1}{x}$$

Pr. 3: Vypočítajte deriváciu funkcie

$$f(x) = \operatorname{cotg}(5x - 1)\log_3 x \,.\, e^{2x} + \frac{5}{x} + \frac{2x}{x^3 + 4x} + \sin^3 x$$

Pr. 4: Vypočítajte deriváciu funkcie

$$f(x) = \text{tg}\,(3x+2) + 2x.e^{x} + \sin x$$

$$f'(x) = \text{tg}'\,(3x+2).(3x+2)' + (2x)'.e^{x} + 2x.(e^{x})' + (\sin x)'$$

$f(g(x))' = f'(g(x)).g(x)'$ $\quad$ $(ab)' = a'b + ab'$ $\quad$ $(\sin x)' = \cos x$

$$f'(x) = \frac{1}{\cos^2(3x+2)}.3 + 2.e^{x} + 2x.e^{x} + \cos x$$

$(tgx)' = \dfrac{1}{\cos^2 x}$ $\quad$ $(3x+2)' = 3.1.x^0 + 0 = 3$ $\quad$ $(2x)' = 2.1.x^0 = 2$ $\quad$ $(e^x)' = e^x$

$$f'(x) = \frac{1}{\cos^2(3x+2)}.3 + 2.e^{x} + 2x.e^{x} + \cos x$$

Pr. 5: Vypočítajte deriváciu funkcie

$$f(x) = \frac{x^2 + 2x + 3}{\operatorname{arctg} x} + \ln \sin 5x$$

$$f'(x) = \frac{(2x+2).\operatorname{arctg} x - (x^2+2x+3).\frac{1}{1+x^2}}{(\operatorname{arctg} x)^2} + \frac{1}{\sin 5x}(\cos 5x)\,.5$$

$$\left(\frac{a}{b}\right)' = \frac{a'b - ab'}{b^2} \qquad\qquad f\big(g(x)\big)' = f'(g(x)).\,g(x)'$$

$$f'(x) = \frac{(2x+2).\operatorname{arctg} x - (x^2+2x+3).\frac{1}{1+x^2}}{(\operatorname{arctg} x)^2} + 5\operatorname{cotg} 5x$$

Pr. 6: Vypočítajte deriváciu funkcie

$$f(x) = \sin\sqrt{1 + x^2}$$

Pr. 7: Vypočítajte deriváciu funkcie

$$f(x) = x.\arccos(x^2 + 5x) + \sqrt{2 - x^2 + 3^x}$$

Dú – str: 25, 26 / 1 - 35