PRÍKLAD: VYPOČÍTAJME z^2, z^3, AK $z = 3 - i$.
URČME REÁLNU, IMAGINÁRNU ZLOŽKU VÝSLEDKU.

$$z = 3 - i \Rightarrow z^2 = (3-i)^2 = 9 - 2\cdot 3\cdot i + \underbrace{i^2}_{-1} = 9 - 6i - 1 = 8 - 6i$$

$(A-B)^2 = A^2 - 2AB + B^2$

$8 \rightarrow \mathrm{Re}\, z^2$, $-6i \rightarrow \mathrm{Im}\, z^2$

$$z^3 = (3-i)^3 = 27 - 3\cdot 9\cdot i + 3\cdot 3\cdot (i)^2 - (i)^3 = 27 - 27i - 9 + i = 18 - 26i$$

$(A-B)^3 = A^3 - 3A^2B + 3AB^2 - B^3$

$18 \rightarrow \mathrm{Re}\, z^3$, $-26i \rightarrow \mathrm{Im}\, z^3$

PRÍKLAD VYPOČÍTAME EŠTE RAZ POSTUPNÝM NÁSOBENÍM.

$$z^2 = (3-i)^2 = (3-i)(3-i) = 9 - 3i - 3i - 1 = 8 - 6i$$

$$z^3 = (3-i)^3 = (3-i)^2\cdot(3-i) = (8-6i)\cdot(3-i) = 24 - 8i - 18i - 6 = 18 - 26i$$

PRÍKLAD: VYPOČÍTAJME

a) $\left(-\frac{1}{2} + 2i\right)\cdot 2i - 3(1+i) = -i - 4 - 3 - 3i = -7 - 4i$

b) $\frac{2-3i}{3-2i} + \frac{2}{13}(2+i) = \frac{(2-3i)}{(3-2i)}\cdot\frac{(3+2i)}{(3+2i)} + \frac{2}{13}(2+i) = \frac{6+4i-9i+6}{3^2+2^2} + \frac{4}{13} + \frac{2}{13}i =$

$= \frac{12-5i}{13} + \frac{4}{13} + \frac{2}{13}i = \frac{12}{13} + \frac{4}{13} - \frac{5}{13}i + \frac{2}{13}i = \frac{16}{13} - \frac{3}{13}i$

c) $\boxed{(\underbrace{-2}_{A} + \underbrace{3i}_{B})^2} + \boxed{\overbrace{(1+3i)(1-3i)}^{z\cdot\bar{z}}} = \boxed{\overbrace{4 - 12i - \cancel{9}}^{A^2+2AB+B^2}} + \boxed{1 + \cancel{9}} = 5 - 12i$

d) $\frac{5+2i}{-2+i} + \frac{2}{5}i(3-2i) - \frac{2}{5}(i)^5 = \frac{(5+2i)}{(-2+i)}\cdot\frac{(-2-i)}{(-2-i)} + \frac{6}{5}i + \frac{4}{5} - \frac{2}{5}(i)^4\cdot(i)^1 =$

$(i)^4 \rightarrow (i^2)^2 \rightarrow (-1)^2 \rightarrow 1$

$z\cdot\bar{z} = x^2 + y^2 \in \mathbb{R}$

$= \frac{-10-5i-4i+2}{(-2)^2+(1)^2} + \frac{6}{5}i + \frac{4}{5} - \frac{2}{5}i = \frac{-8-9i}{5} + \frac{6}{5}i + \frac{4}{5} - \frac{2}{5}i =$

$-\frac{8}{5} - \frac{9}{5}i + \frac{6}{5}i + \frac{4}{5} - \frac{2}{5}i =$

$= -\frac{4}{5} - \frac{5}{5}i = -\frac{4}{5} - i$

$= -\frac{4}{5} - i$

e) $\frac{(-4-7i)}{(-2+3i)}\cdot\boxed{\frac{(-2-3i)}{(-2-3i)}}^{=1} = \frac{(-4)(-2) - 4(-3i) - 7i(-2) - 7i(-3i)}{(-2)(-2) - 2(-3i) + 3i(-2) + 3i(-3i)} =$

$z \cdot \bar{z}$

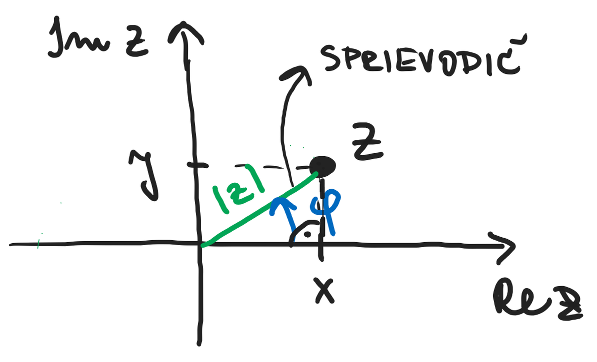
$-7i(-3i) \rightarrow (-7)(-3)(i\cdot i) = 21\cdot i^2 = -21$

$+3i(-3i) \rightarrow 3(-3)(i)^2 = -9(-1)$

$= \frac{8 + 12i + 14i - 21}{4 + \cancel{6i} - \cancel{6i} + 9} = \frac{-13}{13} + \frac{26}{13}i = -1 + 2i$

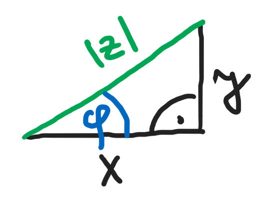
PREPIS KČ Z ALGEBRAICKÉHO TVARU DO GONIOMETRICKÉHO ALEBO EXPONENCIÁLNEHO TVARU

KČ MÔŽEME ZNÁZORNIŤ V GAUSSOVEJ ROVINE POMOCOU REÁLNEJ A IMAGINÁRNEJ ZLOŽKY (ALGEBR. TVAR).
EXISTUJE AJ INÁ MOŽNOSŤ ZNÁZORNENIA, POMOCOU MODULU A AMPLITÚDY (GEOM. A EXPONENC. TVAR)



$|z|$ – MODUL KČ

φ – AMPLITÚDA KČ – UHOL MEDZI SPRIEVODIČOM KČ A KLADNOU ČASŤOU REÁLNEJ OSI



PLATÍ $|z| = \sqrt{x^2 + y^2}$

$\cos\varphi = \frac{x}{|z|} \Rightarrow x = |z|\cos\varphi$

$\sin\varphi = \frac{y}{|z|} \Rightarrow y = |z|\sin\varphi$

} DOSADÍM DO ALG. TVARU KČ

$$z = x + iy = |z|\cos\varphi + i|z|\sin\varphi = |z|(\cos\varphi + i\sin\varphi)$$

GONIOMETRICKÝ TVAR KČ

PLATÍ: $\cos\varphi + i\sin\varphi = e^{i\cdot\varphi}$ => EULEROV VZŤAH

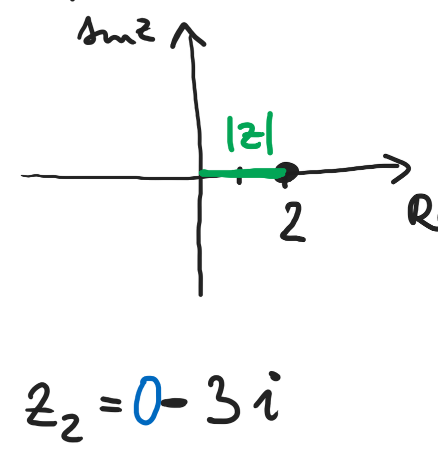
$$z = |z|(\cos\varphi + i\sin\varphi) = |z|\cdot e^{i\cdot\varphi}$$

EXPONENCIÁLNY TVAR KČ

PRI PREPISE KČ Z ALG. DO INÉHO TVARU SI BUDEME VŽDY POMÁHAŤ OBRÁZKOM.

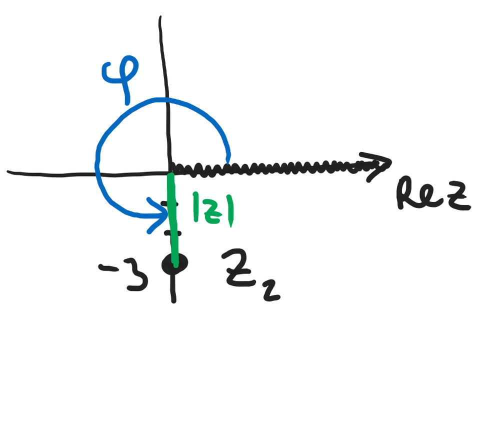
PRÍKLAD: PREPÍŠME DO GONIOM. A EXPON. TVARU KČ:

$z_1 = 2 + 0i$



$|z| = 2$
$\varphi = 0^\circ = 0\pi = 0$
} $z_1 = 2(\cos 0 + i\sin 0)$
$z_1 = 2\cdot e^{0\cdot i}$

$z_2 = 0 - 3i$



$|z| = 3$
$\varphi = 270^\circ = \frac{3}{2}\pi$
} $z_2 = 3\left(\cos\frac{3}{2}\pi + i\sin\frac{3}{2}\pi\right)$
$z_2 = 3\cdot e^{\frac{3}{2}\pi\cdot i}$