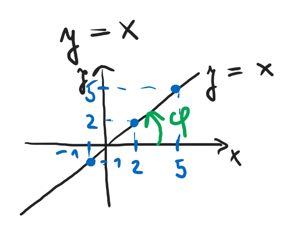
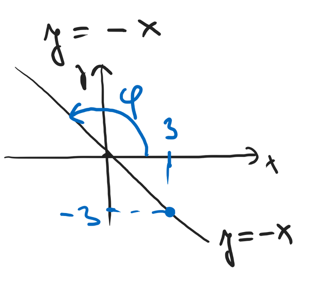
ČO UŽ VIEME ZO STREDNEJ ŠKOLY :)



$y = 1 . x$

$k = tg\,\varphi = 1$

$\varphi = \frac{\pi}{4}$



$y = -1 . x$

$k = tg\,\varphi = -1$

$\varphi = \frac{3}{4}\pi$

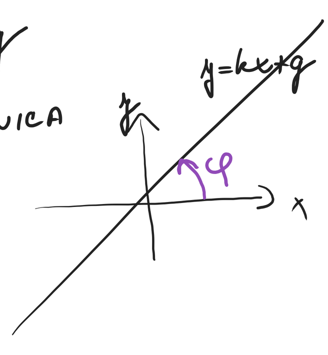
$y = kx + q$

↓

SMERNICA

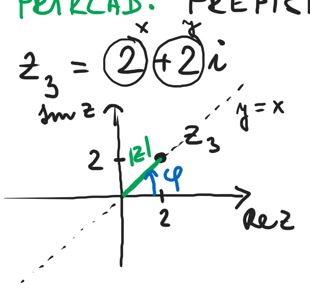
↓

$k = tg\,\varphi$



PRÍKLAD: PREPÍŠME DO GONIOM. A EXPON. TVARU KČ:

$z_3 = 2 + 2i$

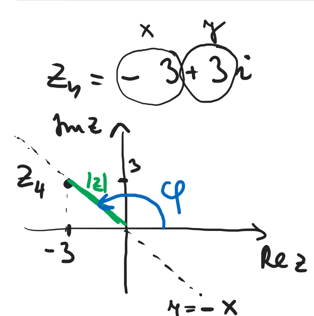


$|z| = \sqrt{x^2 + y^2} = \sqrt{2^2 + 2^2} = \sqrt{8} = 2\sqrt{2}$

$\varphi = \frac{\pi}{4}$ (VIDÍM Z OBRÁZKA)

$\Rightarrow \begin{cases} z_3 = 2\sqrt{2}\left(\cos\frac{\pi}{4} + i \sin\frac{\pi}{4}\right) & \rightarrow \text{GONIOM. TVAR} \\ z_3 = 2\sqrt{2} . e^{\frac{\pi}{4} . i} & \rightarrow \text{EXPON. TVAR} \end{cases}$

$z_4 = -3 + 3i$



$|z| = \sqrt{(-3)^2 + 3^2} = \sqrt{18} = 3\sqrt{2}$

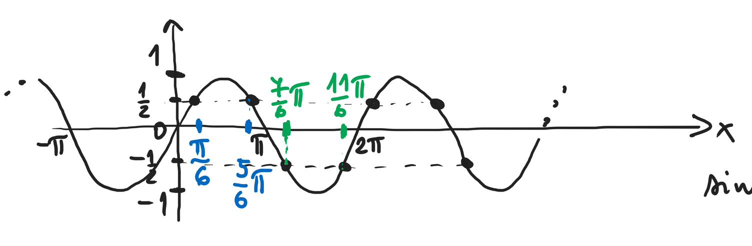
$\varphi = \frac{3}{4}\pi$ (VIDÍM Z OBRÁZKA)

$\Rightarrow \begin{cases} z_4 = 3\sqrt{2}\left(\cos\frac{3}{4}\pi + i \sin\frac{3}{4}\pi\right) \\ z_4 = 3\sqrt{2} . e^{\frac{3}{4}\pi . i} \end{cases}$

NÁVRAT K STREDNEJ ŠKOLE

ČO UŽITOČNÉ VIEME O GONIOM. F-CIÁCH (sin x, cos x)

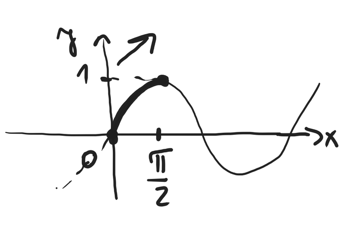
$y = \sin x$



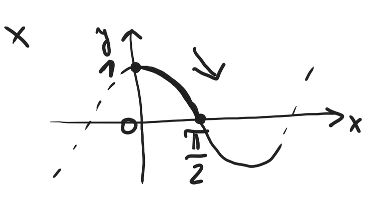
$\sin\frac{\pi}{6} = \sin\frac{5}{6}\pi = \dots = \frac{1}{2}$

$\sin\frac{7}{6}\pi = \sin\frac{11}{6}\pi = \dots = -\frac{1}{2}$

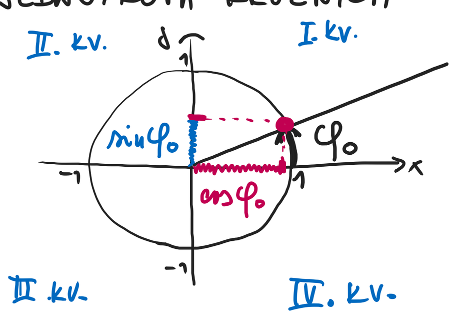
$y = \sin x$



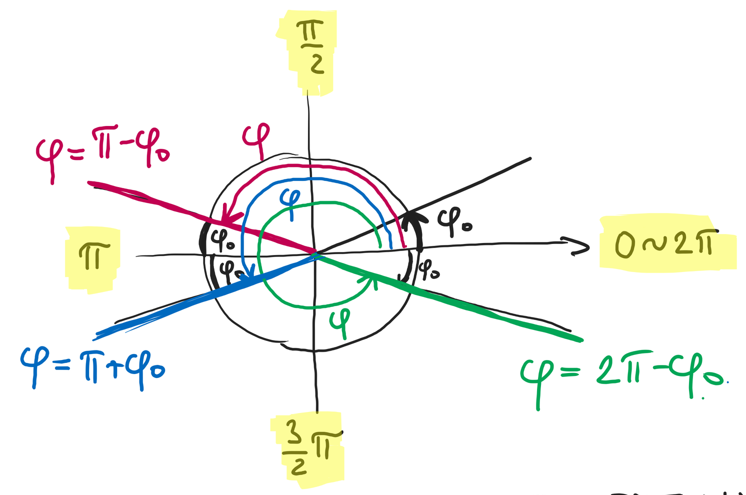
$y = \cos x$



JEDNOTKOVÁ KRUŽNICA

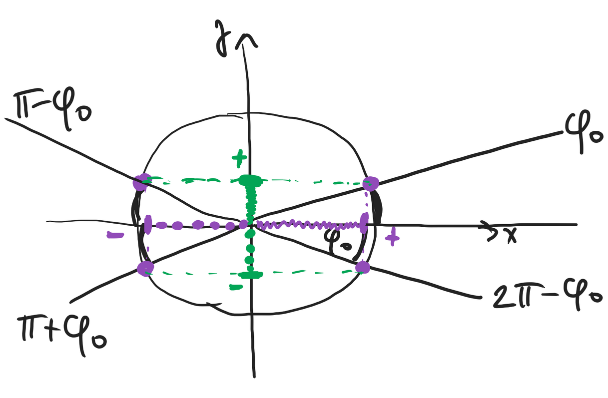


$\Rightarrow$

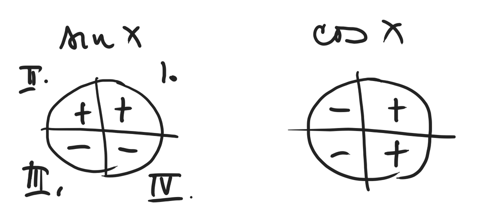


⇓

! UHOL V AKOMKOĽVEK KVADRANTE VIEME VYJADRIŤ POMOCOU NEJAKÉHO UHLA Z PRVÉHO KVADRANTU !



$\Rightarrow$



RAD x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
↗ sin x	$\frac{\sqrt{0}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{4}}{2}$
↘ cos x	$\frac{\sqrt{4}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{0}}{2}$

$\Rightarrow$

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
sin x	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos x	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0

PRÍKLAD: URČME HODNOTY F-CIÍ:

a) $\cos\frac{131}{3}\pi = \cos\frac{5}{3}\pi = +\frac{1}{2}$ LEBO $\cos\frac{\pi}{3} = \frac{1}{2}$

IV. KV.

$\frac{\frac{131}{3}\pi}{2\pi} = \frac{\frac{131}{3}\cancel{\pi}}{\frac{6}{3}\cancel{\pi}} = \frac{131}{6} \Rightarrow 131 : 6 = 21$ → KRÁT OBEHNEM CELÚ KRUŽNICU ↓ NEPODSTATNÁ INFORMÁCIA

5 ZVYŠOK PO DELENÍ ↓ DÔLEŽITÁ INFORMÁCIA

b) $\sin\frac{97}{6}\pi = \sin\frac{1}{6}\pi = +\frac{1}{2}$ LEBO $\sin\frac{\pi}{6} = \frac{1}{2}$

I. KV.

$97 : 12 = 8$

1

c) $\cos\frac{35}{4}\pi = \cos\frac{3}{4}\pi = -\frac{\sqrt{2}}{2}$ LEBO $\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

II. KV

$35 : 8 = 4$

3

UŽ VIEME, ŽE PLATÍ: AK $z = x + i \cdot y$, TAK

$\boxed{\cos\varphi = \frac{x}{|z|} \wedge \sin\varphi = \frac{y}{|z|}}$ PRE ĽUBOVOĽNÉ $\varphi \in \langle 0, 2\pi \rangle$ → ĽUBOVOĽNÝ KVADRANT

AK JE UHOL $\varphi = \varphi_0$ Z I. KVADRANTU ⇒ HODNOTY $\sin\varphi_0$ a $\cos\varphi_0$ SÚ KLADNÉ A PLATÍ:

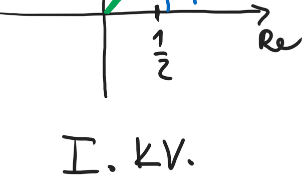
$\boxed{\cos\varphi_0 = \frac{|x|}{|z|} \wedge \sin\varphi_0 = \frac{|y|}{|z|}}$ → ABSOLÚTNA HODNOTA

MODUL

PRÍKLAD: PREPÍŠME DO GONIOM. A EXP. TVARU KČ:

a) $z_1 = \frac{1}{2} + \frac{\sqrt{3}}{2} i$

$|z| = \sqrt{x^2 + y^2} = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \sqrt{\frac{1}{4} + \frac{3}{4}} = 1$



I. KV.

STAČÍ PRACOVAŤ S JEDNOU GONIOMETRICKOU FUNKCIOU

NAPR.:

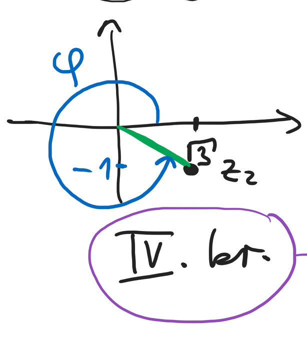
$\sin\varphi_0 = \frac{|y|}{|z|} = \frac{\left|\frac{\sqrt{3}}{2}\right|}{1} = \frac{\sqrt{3}}{2} \Rightarrow \varphi_0 = \frac{\pi}{3} = \varphi$

$z_1 = 1 . \left(\cos\frac{\pi}{3} + i \sin\frac{\pi}{3}\right) = e^{\frac{\pi}{3} . i}$

b) $z_2 = \sqrt{3} - 1i$

$|z| = \sqrt{x^2 + y^2} = \sqrt{3 + 1} = 2$

$\sin\varphi_0 = \frac{|y|}{|z|} = \frac{|-1|}{2} = \frac{1}{2} \Rightarrow \varphi_0 = \frac{\pi}{6}$



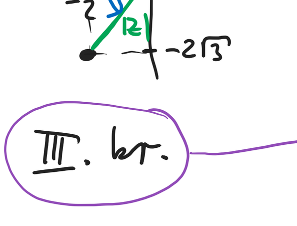
IV. kv. → $\varphi = 2\pi - \varphi_0 = \frac{11}{6}\pi$

$|z| = 2\left(\cos\frac{11}{6}\pi + i \sin\frac{11}{6}\pi\right) = 2 . e^{\frac{11}{6}\pi i}$

c) $z_3 = -2 - 2\sqrt{3} i$

$|z| = \sqrt{x^2 + y^2} = \sqrt{4 + 12} = 4$

$\cos\varphi_0 = \frac{|x|}{|z|} = \frac{|-2|}{4} = \frac{1}{2} \Rightarrow \varphi_0 = \frac{\pi}{3}$



III. kv. → $\varphi = \pi + \varphi_0 = \pi + \frac{\pi}{3} = \frac{4}{3}\pi$

$z = 4\left(\cos\frac{4}{3}\pi + i \sin\frac{4}{3}\pi\right) = 4 . e^{\frac{4}{3}\pi i}$