OPERÁCIE S KČ V GONIOMETRICKOM A EXPONENCIÁLNOM TVARE

- **SÚČIN KČ V GONIOM. A EXPON. TVARE**

– ROBÍME TAK, ŽE MODULY NÁSOBÍME A AMPLITÚDY SČÍTAME

NECH $\boxed{\begin{array}{l} z_1 = |z_1|(\cos\varphi_1 + i\sin\varphi_1) \\ z_2 = |z_2|(\cos\varphi_2 + i\sin\varphi_2) \end{array}}$ A $\boxed{\begin{array}{l} z_1 = |z_1| e^{i\varphi_1} \\ z_2 = |z_2| \cdot e^{i\varphi_2} \end{array}}$

SÚČIN $z_1 \cdot z_2 = |z_1| \cdot |z_2| \left(\cos(\varphi_1 + \varphi_2) + i \sin(\varphi_1 + \varphi_2)\right)$

$z_1 \cdot z_2 = |z_1| \cdot |z_2| \cdot e^{i(\varphi_1 + \varphi_2)}$

PRÍKLAD: NECH $z_1 = 2(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2})$

$z_2 = \frac{3}{2}(\cos\pi + i\sin\pi)$

VYPOČÍTAJME SÚČIN $z_1 \cdot z_2$ V GONIOM. AJ EXPONENC. TVARE

$$z_1 \cdot z_2 = \cancel{2} \cdot \frac{3}{\cancel{2}}\left(\cos\left(\frac{\pi}{2}+\pi\right) + i\sin\left(\frac{\pi}{2}+\pi\right)\right) = \underbrace{3\left(\cos\frac{3}{2}\pi + i\sin\frac{3}{2}\pi\right)}_{\text{GONIOM. TVAR VÝSLEDKU}} = \underbrace{3 \cdot e^{\frac{3}{2}\pi \cdot i}}_{\text{EXPON. TVAR}} = 3(0 + i(-1)) = \underbrace{-3i}_{\text{ALGEBR. TVAR}}$$

- **PODIEL KČ V GONIOM. A EXPON. TVARE**

– ROBÍME TAK, ŽE MODULY PREDELÍME A AMPLITÚDY ODČÍTAME

PODIEL $\frac{z_1}{z_2} = \frac{|z_1|}{|z_2|}\left(\cos(\varphi_1 - \varphi_2) + i\sin(\varphi_1 - \varphi_2)\right)$

$\frac{z_1}{z_2} = \frac{|z_1|}{|z_2|} \cdot e^{i(\varphi_1 - \varphi_2)}$

PRÍKLAD: NECH $z_1 = 2(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2})$

$z_2 = \frac{3}{2}(\cos\pi + i\sin\pi)$

VYPOČÍTAJME $\frac{z_1}{z_2}$ V GONIOM. AJ EXPON. TVARE

$$\frac{z_1}{z_2} = \frac{\frac{2}{1}}{\frac{3}{2}}\left(\cos\left(\frac{\pi}{2}-\pi\right) + i\sin\left(\frac{\pi}{2}-\pi\right)\right) = \frac{4}{3}\left(\overset{\cos\frac{\pi}{2}}{\cos\left(-\frac{\pi}{2}\right)} + i\overset{-\sin\frac{\pi}{2}}{\sin\left(-\frac{\pi}{2}\right)}\right) = \underbrace{\frac{4}{3}\left(\overset{0}{\cos\frac{\pi}{2}} - i\overset{1}{\sin\frac{\pi}{2}}\right)}_{\text{GONIOM. TV.}} = \underbrace{\frac{4}{3} \cdot e^{-\frac{\pi}{2}i}}_{\text{EXPON. TV.}} = \frac{4}{3}(0 - i \cdot 1) = \underbrace{-\frac{4}{3}i}_{\text{ALGEBR. TVAR}}$$

cos → PÁRNA F-CIA

$f(-x) = f(x)$

NEPÁRNA F-CIA

$f(-x) = -f(x)$

- **n-TÁ MOCNINA KČ V GONIOM. A EXPONENC. TVARE**

– ROBÍME JU TAK, ŽE MODUL UMOCNÍME NA $\underline{\underline{n}}$-TÚ A AMPLITÚDU VYNÁSOBÍME ČÍSLOM $\underline{\underline{n}}$

NECH $z = |z| \cdot (\cos\varphi + i\sin\varphi)$; RESP. $z = |z| \cdot e^{i\varphi}$

n-TÁ MOCNINA $z^n = |z|^n(\cos n\varphi + i\sin n\varphi)$ MOIVREOVA VETA

$z^n = |z|^n \cdot e^{in\varphi}$

PRÍKLAD: NECH $z = 2(\cos\frac{5}{6}\pi + i\sin\frac{5}{6}\pi)$

VYPOČÍTAJME z^{12} V GONIOM. AJ EXPON. TVARE

$$z^{12} = 2^{12} \cdot \left(\cos \cancel{12} \cdot \frac{5}{\cancel{6}}\pi + i\sin \cancel{12} \cdot \frac{5}{\cancel{6}}\pi\right) = \underbrace{2^{12}\left(\cos \overset{0}{10\pi} + i\sin \overset{0}{10\pi}\right)}_{\text{GONIOM. TVAR}} = 2^{12}\left(\overset{1}{\cos 0} + i\overset{0}{\sin 0}\right) = \underbrace{2^{12} \cdot e^{10\pi i}}_{\text{EXPON. TV.}} = 2^{12} \cdot \overset{1}{e^{0i}} = \underbrace{2^{12}}_{\text{ALGEBR. TVAR}}$$

- **n-TÁ ODMOCNINA KČ V GONIOM. A EXPON. TVARE**

– MÁ $\underline{\underline{n}}$ RÔZNYCH KOMPLEXNÝCH VÝSLEDKOV V TVARE:

$$z_k = \sqrt[n]{z} = \sqrt[n]{|z|} \cdot \left(\cos\frac{\varphi + 2k\pi}{n} + i\sin\frac{\varphi + 2k\pi}{n}\right); \ k = 0, 1, \dots n-1$$

$$z_k = \sqrt[n]{|z|} \cdot e^{i\frac{\varphi + 2k\pi}{n}} \ ; \ k = 0, 1, 2, \dots, n-1$$

PRÍKLAD: NECH $z = 8(\cos\pi + i\sin\pi)$

VYPOČÍTAJME $\sqrt[3]{z}$

$$z_0 = \sqrt[3]{8} \cdot \left(\cos\frac{\pi + 2 \cdot 0\pi}{3} + i\sin\frac{\pi + 2 \cdot 0\pi}{3}\right) = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right) = 2\left(\frac{1}{2} + i \cdot \frac{\sqrt{3}}{2}\right) = 1 + \sqrt{3} \cdot i$$

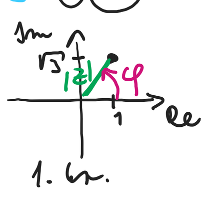
$$z_1 = \sqrt[3]{8} \cdot \left(\cos\frac{\pi + 2 \cdot 1\pi}{3} + i\sin\frac{\pi + 2 \cdot 1\pi}{3}\right) = 2(\cos\pi + i\sin\pi) = 2(-1 + i \cdot 0) = -2$$

$$z_2 = \sqrt[3]{8} \cdot \left(\cos\frac{\pi + 2 \cdot 2\pi}{3} + i\sin\frac{\pi + 2 \cdot 2\pi}{3}\right) = 2\left(\cos\underset{\text{4. kv.}}{\frac{5}{3}\pi} + i\sin\frac{5}{3}\pi\right) = 2\left(+\frac{1}{2} + i\left(-\frac{\sqrt{3}}{2}\right)\right) = 1 - \sqrt{3}i$$

PRÍKLAD: VYPOČÍTAJME $(\overset{z}{1 + \sqrt{3} \cdot i})^{17}$

A VÝSLEDOK ZAPÍŠME V ALGEBRAICKOM TVARE

$z = \overset{x}{1} + \overset{y}{\sqrt{3}}\, i$ => PREPIS DO GONIOM. TVARU



1. kv.

$|z| = \sqrt{x^2 + y^2} = \sqrt{1 + 3} = 2$

$\underline{\underline{\cos\varphi}} = \frac{x}{|z|} = \underline{\underline{\frac{1}{2}}} \Rightarrow \varphi = \frac{\pi}{3}$

$\Big\} \ z = 2(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3})$

$$z^{17} = \left[2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)\right]^{17} = 2^{17}\left(\cos\overset{\frac{5}{3}\pi}{\frac{17}{3}\pi} + i\sin\frac{17}{3}\pi\right) = 2^{17}\left(\cos\underset{\text{4. kv.}}{\frac{5}{3}\pi} + i\sin\frac{5}{3}\pi\right) =$$

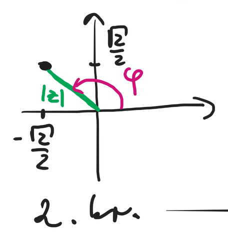
17 : 6 = 2

zv. 5

$$= 2^{17}\left(+\frac{1}{2} + i\left(-\frac{\sqrt{3}}{2}\right)\right) = \underline{\underline{2^{16}(1 - \sqrt{3}i)}}$$

PRÍKLAD: VYPOČÍTAJME $\left(\overset{z}{-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i}\right)^{99}$

$z = \overset{x}{-\frac{\sqrt{2}}{2}} + \overset{y}{\frac{\sqrt{2}}{2}}\, i$



2. kv. ⟶ $\varphi = \pi - \varphi_0 = \pi - \frac{\pi}{4} = \frac{3}{4}\pi$

$|z| = \sqrt{x^2 + y^2} = \sqrt{\frac{2}{4} + \frac{2}{4}} = \underline{\underline{1}}$

$\underline{\underline{\cos\varphi_0}} = \frac{|x|}{|z|} = \frac{\frac{\sqrt{2}}{2}}{1} = \underline{\underline{\frac{\sqrt{2}}{2}}} \Rightarrow \varphi_0 = \frac{\pi}{4}$

$\Big\} \ z = \cos\frac{3}{4}\pi + i\sin\frac{3}{4}\pi$

$$z^{99} = \left[\cos\frac{3}{4}\pi + i\sin\frac{3}{4}\pi\right]^{99} = \cos\overset{\sim\frac{1}{4}\pi}{\frac{297}{4}\pi} + i\sin\frac{297}{4}\pi =$$

297 : 8 = 37

57

zv. 1

$$= \cos\frac{\pi}{4} + i\sin\frac{\pi}{4} = \underline{\underline{\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}}}$$