

INVERZNÁ MATICA

NECH A JE REGULÁRNA MATICA ($|A| \neq 0$). POTOM K MATICI A EXISTUJE INVERZNÁ MATICA $\tilde{A}^{-1}$, PRÍČOM PLATÍ:

$$A \cdot \tilde{A}^{-1} = \tilde{A}^{-1} \cdot A = E \quad ; \quad E - \text{JEDNOTKOVÁ MATICA}$$

SPÔSOB HĽADANIA INVERZNEJ MATICE - POMOCOU **ADJUNGOVANEJ MATICE** $\text{adj } A$

$$\tilde{A}^{-1} = \frac{1}{|A|} \cdot \text{adj } A = \frac{1}{|A|} \cdot \begin{pmatrix} a_{11} & a_{21} & \dots & a_{n1} \\ a_{12} & a_{22} & \dots & a_{n2} \\ \vdots & \vdots & \ddots & \vdots \\ a_{1n} & a_{2n} & \dots & a_{nn} \end{pmatrix} ; \quad \begin{array}{l} a_{ij} - \text{ALGEBRAICKÝ} \\ \text{DOPLNOK PRVKU } a_{ij} \\ \text{PÔVODNEJ MATICE } A \end{array}$$

$$a_{ij} = (-1)^{i+j} \cdot D_{ij}$$

D_{ij} - SUBDETERMINANT VZHLADOM NA PRVOK a_{ij}

PRÍKLAD: K MATICI $A = \begin{pmatrix} 1 & 1 & 1 \\ 6 & 5 & 4 \\ 13 & 10 & 9 \end{pmatrix}$ NÁJDI ME INVERZNÚ MATICU $\tilde{A}^{-1}$

1) OVERÍME, ČI $\tilde{A}^{-1}$ EXISTUJE

$$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 6 & 5 & 4 \\ 13 & 10 & 9 \end{vmatrix} = 1 \cdot 45 - 1 \cdot 54 + 1 \cdot 18 = -1 \neq 0 \quad A - \text{REGULÁRNA}$$

2) HĽADANIE PRVKOV ADJUNG. MATICE

$$\begin{array}{lll} a_{11} = (-1)^{1+1} \cdot \begin{vmatrix} 5 & 4 \\ 10 & 9 \end{vmatrix} = + (45 - 40) = 5 & a_{21} = - \begin{vmatrix} 1 & 1 \\ 10 & 9 \end{vmatrix} = 2 & a_{31} = + \begin{vmatrix} 1 & 1 \\ 5 & 4 \end{vmatrix} = -1 \\ a_{12} = (-1)^{1+2} \cdot \begin{vmatrix} 6 & 4 \\ 13 & 9 \end{vmatrix} = - (54 - 52) = -2 & a_{22} = + \begin{vmatrix} 1 & 1 \\ 13 & 9 \end{vmatrix} = -5 & a_{32} = - \begin{vmatrix} 1 & 1 \\ 6 & 4 \end{vmatrix} = 2 \\ a_{13} = (-1)^{1+3} \cdot \begin{vmatrix} 6 & 5 \\ 13 & 10 \end{vmatrix} = + (60 - 65) = -5 & a_{23} = - \begin{vmatrix} 1 & 1 \\ 13 & 10 \end{vmatrix} = 3 & a_{33} = + \begin{vmatrix} 1 & 1 \\ 6 & 5 \end{vmatrix} = -1 \end{array}$$

3) VYTVORENIE INVERZNEJ MATICE

$$\tilde{A}^{-1} = \frac{1}{-1} \cdot \begin{pmatrix} 5 & 2 & -1 \\ -2 & -5 & 2 \\ -5 & 3 & -1 \end{pmatrix} = \begin{pmatrix} -5 & -2 & 1 \\ 2 & 5 & -2 \\ 5 & -3 & 1 \end{pmatrix}$$

OVERENIE: $A \cdot \tilde{A}^{-1} = \begin{pmatrix} 1 & 1 & 1 \\ 6 & 5 & 4 \\ 13 & 10 & 9 \end{pmatrix} \cdot \begin{pmatrix} -5 & -2 & 1 \\ 2 & 5 & -2 \\ 5 & -3 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = E$

(DOBRÝ OČNÉ)

$1 \cdot 0 + 1 \cdot (-2) + 1 \cdot 5 = 1$

$13 \cdot (-2) + 10 \cdot 5 + 9 \cdot (-3) = -26 + 50 - 27 = -3$

PRÍKLAD: K MATICI A NÁJDI ME INVERZNÚ MATICU $\tilde{A}^{-1}$

$$A = \begin{pmatrix} 3 & 2 \\ 6 & 5 \end{pmatrix}$$

1) $|A| = \begin{vmatrix} 3 & 2 \\ 6 & 5 \end{vmatrix} = 15 - 12 = 3 \neq 0 \Rightarrow A - \text{REGULÁRNA}$

2) $a_{11} = +|5| = 5$ $a_{21} = -|2| = -2$
 $a_{12} = -|6| = -6$ $a_{22} = +|3| = 3$

3) $\tilde{A}^{-1} = \frac{1}{3} \begin{pmatrix} 5 & -2 \\ -6 & 3 \end{pmatrix} = \begin{pmatrix} \frac{5}{3} & -\frac{2}{3} \\ -2 & 1 \end{pmatrix}$