Marginálna analýza, elasticita

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MARGINÁLNA ANALÝZA

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$f'(x) \doteq \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$\Delta x = 1$

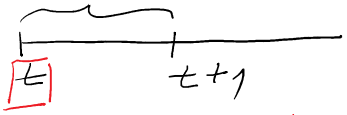
$$f'(x) \doteq f(x+1) - f(x)$$

$Mf(x) = f'(x)$

PR. $C(t) = 50t^2 + 100t + 10\,000$

1) tempo rastu po t rokoch
2) prírastok počas 1. roka
3) prírastok počas 6. roka

$\Delta t = 1 \Rightarrow$ MARGINÁLNA VEĽČINA



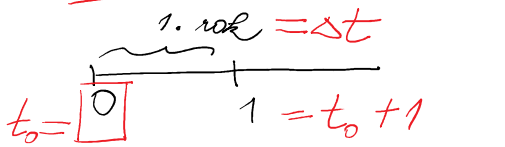
1) $MC(t) = C'(t)$ ✓

$C'(t) = 100t + 100$

2) $t_0 = 0$

$MC(t_0) = C'(t_0)$

$C'(0) = 100 \cdot 0 + 100 = 100$

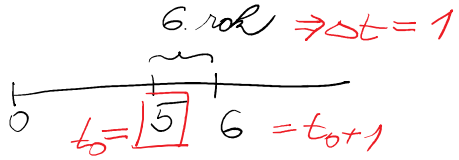


$t_0 =$ 0

3) $t_0 = 5$

$MC(t_0) = C'(t_0)$

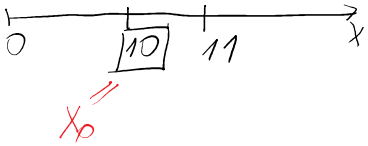
$C'(5) = 100 \cdot 5 + 100 = 600$



PR. $C(x) = x^2 + 15x + 5000$ ✓

1) 11. výrobok odlad
2) presne

$x_0 = 10$ ✓



1) $MC(x) = C'(x)$

$MC(10) = ?$

$C'(x) = 2x + 15$

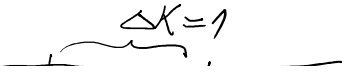
$MC(10) = C'(10) = 2 \cdot 10 + 15 = 35$ približne

2) $C(11) - C(10) = 5286 - 5250 = 36$ presne

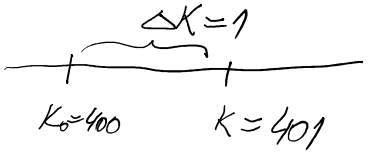
PR. $Q(K) = 1200 \cdot \sqrt{K} = 1200 \cdot K^{\frac{1}{2}}$

$K_0 = 400$ tisíc

$\Delta K = 1$



$K_0 = 400$ tisíc
$\Delta K = 1$



$MQ(K_0) = ?$

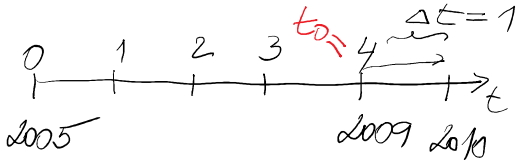
$MQ(K_0) = Q'(K_0)$

$Q'(K) = 1200 \cdot \frac{1}{2} K^{-\frac{1}{2}} = \frac{600}{\sqrt{K}}$

$MQ(400) = Q'(400) = \frac{600}{\sqrt{400}} = \frac{600}{20} = 30$

DENNÁ PRODUKCIA SA ZVÝŠI PRIBLIŽNE O 30 KS.

PR. $R(t) = 20t^2 + 1000t + 2000$



1) ročné tempo rastu na začiatku 2009

2) percentuálne ...

1) $MR(4) = ?$ $t_0 = 4$ ✓

$MR(t) = R'(t)$

$R'(t) = 40t + 1000$

$MR(4) = R'(4) = 40 \cdot 4 + 1000 = 1160$ tisíc eur

2) $\frac{R'(4)}{R(4)} \cdot 100\% = \frac{1160}{16\,336} \cdot 100\% = 18{,}3\%$

ELASTICITA FUNKCIE

$\mathcal{E}(f(x)) = \frac{f'(x)}{f(x)} \cdot x$

PR. $Q(K) = 1200\sqrt{K}$

$\Delta K = \frac{K}{100}$

$\mathcal{E}(Q(K)) = ?$

$Q'(K) = \frac{600}{\sqrt{K}}$

$\mathcal{E}(Q(K)) = \frac{Q'(K)}{Q(K)} \cdot K = \frac{\frac{600}{\sqrt{K}}}{1200\sqrt{K}} \cdot K =$

$\sqrt{K}\sqrt{K} = K^{\frac{1}{2}} K^{\frac{1}{2}} = K^{\frac{1}{2}+\frac{1}{2}} = 1$

$= \frac{600}{2 \cdot 1200 \underbrace{\sqrt{K}\sqrt{K}}_{K}} \cdot K = \frac{K}{2K} = \frac{1}{2}\ \%$

ELASTICITA DOPYTU

$q = D(p)$ $\quad p > 0,\ D(p) > 0$

$E_D(p_0) = -\frac{D'(p_0)}{D(p_0)} \cdot p_0$ $\quad p > 0 \quad q = D(p) > 0$

PR $q = 240 - 2p \ (= D(p))$
$0 < p < 120$

$q > 0$
$240 - 2p > 0$
$240 > 2p$
$120 > p$

1) $E_D = ?$
2) $E_D(100) = ?$
3) $E_D(30) = ?$
4) $E_D = 1$

1) $E_D = -\frac{D'(p)}{D(p)} \cdot p = \frac{-(-2)}{240 - 2p} \cdot p = \frac{2p}{240 - 2p}$ ✓ $\quad D'(p) < 0$

$= \frac{2p}{2(120 - p)} = \frac{p}{120 - p}$ ✓ $\quad p = p_0 + \frac{p_0}{100} = 100 + 1 = 101$ ✓

2) $p_0 = 100$ $\quad E_D(100) = \frac{100}{120 - 100} = \frac{100}{20} = 5 > 1$ ELASTICKÁ

ZVÝŠENIE CENY O 1%, ZNÍŽI POČET PREDANÝCH LS O 5%

3) $p_0 = 50$ $\quad E_D(30) = \frac{50}{120 - 50} = \frac{50}{70} \doteq 0{,}71 < 1$ NEELAST.

4) $E_D = 1$

$\frac{p}{120 - p} = 1$

$p = 120 - p$

$2p = 120$

$p = 60$ JEDNOTKOVÁ ELASTICITA

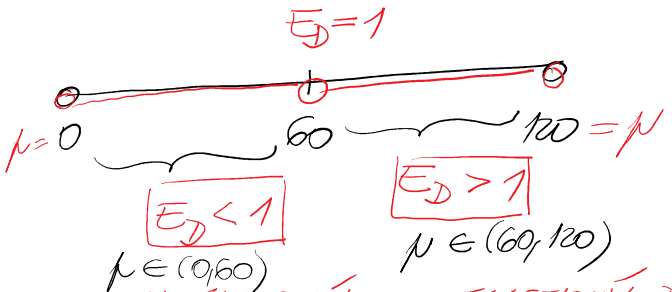
VPLYV ELASTICITY DOPYTU NA CELKOVÉ PRÍJMY

PR $q = 240 - 2p$
$0 < p < 120 \Rightarrow p \in (0, 120)$

1) $E_D > 1$
$E_D < 1$
$E_D = 1$

$E_D = \frac{p}{120 - p} = 1$

$p = 60$



$E_D > 1 \ / \ \frac{p}{120 - p} > 1 \ / \cdot (120 - p)$

$\mu = 60$ $\quad \mu \in (0;60)$ NEELASTICKÝ $\quad$ ELASTICKÝ DOPYT

$E_D > 1$ ✓ $\quad \frac{\mu}{120-\mu} > 1 \quad / \cdot (120-\mu)$

$\mu > 120 - \mu$

$2\mu > 120$

$\mu > 60$

$= q$

$240 - 2\mu > 0$

$120 - \mu > 0$

$E_D < 1 \quad \frac{\mu}{120-\mu} < 1 \quad / \cdot (120-\mu)$

$\mu < 120 - \mu$

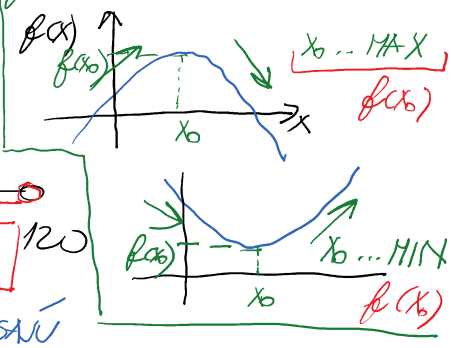
$2\mu < 120$

$\mu < 60$

$f'(x) > 0$ $f(x)$ RASTIE ↗

$f'(x) < 0$ $f(x)$ KLESÁ ↘

$f'(x_0) = 0$ NEMENÍ SA



2) intervaly rastu a klesania $R(\mu)$ pomocou E_D

$R'(\mu) = D(\mu)\,[1 - E_D]$

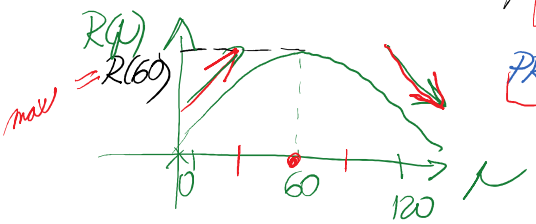
$E_D < 1$ $\quad E_D = 1$ $\quad E_D > 1$

0 $\quad$ 60 $\quad$ 120

$R'(\mu) > 0$ $\quad$ $R'(\mu) < 0$

PRÍJMY RASTÚ $\quad$ PRÍJMY KLESAJÚ

NEMENIA SA



$\mu \in (0;60)$ PRÍJMY RASTÚ

$\mu \in (60;120)$ KLESAJÚ

$\mu = 60$ PRÍJEM JE MAXIMÁLNY

3) $R(\mu) = \mu \cdot q = \mu \cdot D(\mu) =$

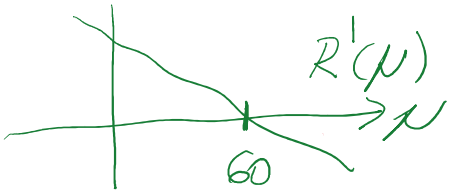
$= \mu\,(240 - 2\mu) = 240\mu - 2\mu^2$

$R'(\mu) = 240 - 4\mu$

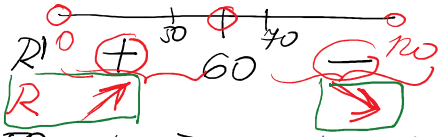
$R'(\mu) = 0 \quad 240 - 4\mu = 0$

$4\mu = 240$

$\mu = 60$



METÓDA NULOVÝCH BODOV



$R'(50) = 240 - 4 \cdot 50 = 40 > 0$ $\quad \mu \in (0;60)$ $R' > 0$ ✓

$R'(70) = 240 - 4 \cdot 70 = -40 < 0$ $\quad \mu \in (60;120)$ $R' < 0$

KLASICKY

$R'(\mu) > 0 \quad 240 - 4\mu > 0$

$240 > 4\mu$

$60 > \mu \; (> 0)$

$(R(\mu)$ RASTÚCA$)$

$R'(\mu) < 0 \quad 240 - 4\mu < 0$

$240 < 4\mu$

$60 < \mu \; (< 120)$

$(R(\mu)$ KLESAJÚCA$)$

PR

$D(p) = 4000 - p^3$

$0 < p < \sqrt[3]{4000}$

$p_0 = 11$

$E_D(p_0) = ?$

$q = 4000 - p^3 > 0$

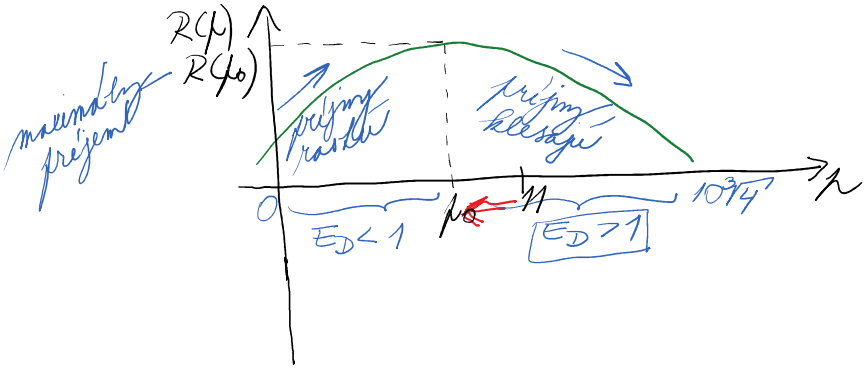
$p^3 < 4000$

$p < \sqrt[3]{4000} = \sqrt[3]{4 \cdot 1000} = 10\sqrt[3]{4}$

$$E_D(p_0) = -\frac{D'(p_0)}{D(p_0)} \cdot p = -\frac{-3p^2}{4000 - p^3} \cdot p = \frac{3p^3}{4000 - p^3}$$

$$E_D(11) = \frac{3 \cdot 11^3}{4000 - 11^3} = 1{,}49 > 1$$ DOPYT JE ELASTICKY

ODPORÚČAME ZNÍŽIŤ CENU (NA HODNOTU, KEĎ $E_D = 1$)



$E_D = 1$:

$$\frac{3p^3}{4000 - p^3} = 1 \quad / \cdot (4000 - p^3)$$

$$3p^3 = 4000 - p^3 \quad / + p^3$$

$$4p^3 = 4000 \quad / : 4$$

$$p^3 = 1000$$

$$p_0 = 10$$

JEDNOTKOVÁ ELASTICITA

$\Rightarrow$ PRÍJMY MAXIMÁLNE

$d f(x_0) = f'(x_0) \cdot \Delta x$

$\boxed{M f(x_0) = f'(x_0)}$ $\leftarrow$ $\boxed{\Delta x = 1}$

$Q(K) = 1200\sqrt{K}$

$K_0 = 400$

1) $\Delta K = 0{,}8$

2) $\Delta K = 1$