Monotónnosť, lokálne extrémy,
konvexnosť/konkávnosť

streda 19. novembra 2025 7:30

MONOTÓNNOSŤ FUNKCIE

1) $f(x) = x^4 - 4x^3 \qquad D(f) = R$

$f'(x) = 4x^3 - 12x^2 = 4x^2(x-3)$

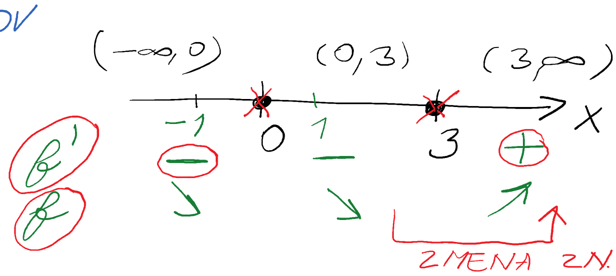
$f'(x) > 0 \Rightarrow$ RASTÚCA ↗

$f'(x) < 0 \Rightarrow$ KLESAJÚCA ↘

METÓDA NULOVÝCH BODOV

$4x^2(x-3)^1 = 0$

$x_1 = 0 \qquad x_2 = 3$



x	$(-\infty,0)$	0	$(0,3)$	3	$(3,\infty)$
f'	$-$	0	$-$	0	$+$
f	↘		↘		↗

FUNKCIA JE KLESAJÚCA NA: $(-\infty,0) \cup (0,3)$

FUNKCIA JE RASTÚCA NA: $(3,\infty)$

2) $f(x) = x \cdot \ln x \qquad x > 0 \qquad D(f) = (0,\infty) = R^+$

↑ SÚČIN

$f'(x) = 1 \cdot \ln x + \cancel{x} \cdot \frac{1}{\cancel{x}} = \ln x + 1$

$\ln x^n = n \ln x$ ✓

$\boxed{f \circ f^{-1} = id}$

$f'(x) = 0$

$\ln x + 1 = 0$

$\ln x = -1 \quad / e^{(\ )}$

$f \circ f^{-1} = e^{\ln x} = e^{-1}$

$x = e^{-1} = \frac{1}{e}$

inak: $\ln e = 1$

$\ln x = -1 \cdot \ln e$

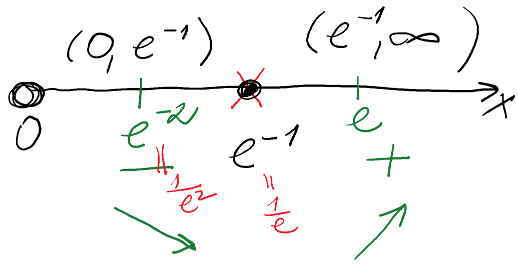
$\ln x = \ln e^{-1}$

$\ln e = 1$ ✓

$x = e^{-1}$

$f'(e^{-2}) = \ln e^{-2} + 1 = -2 \cdot \ln e + 1 = -2 + 1 = -1$

$f'(e) = \ln e + 1 = 1 + 1 = 2$



x	$(0,e^{-1})$	e^{-1}	(e^{-1},∞)
f'	$-$	0	$+$
f	↘		↗

FUNKCIA JE KLESAJÚCA NA: $(0,e^{-1})$

FUNKCIA JE RASTÚCA NA: (e^{-1},∞)

STACIONÁRNE BODY

V: $P \Rightarrow Z$

obmena V: $Z' \Rightarrow P'$

STACIONÁRNE BODY

V: $P \Rightarrow Z$,
obmenená V: $Z' \Rightarrow P'$

1) $f(x) = x^4 - 4x^3 \quad D(f) = \mathbb{R}$

$f'(x) = 4x^3 - 12x^2 = 4x^2(x-3)$

S.B. $f'(x) = 0$ ✓ STACIONÁRNE BODY

$4x^2(x-3)$

$x_1 = 0 \quad x_2 = 3$

2) $f(x) = x \cdot \ln x \quad D(f) = \mathbb{R}^+$

$f'(x) = \ln x + 1$

S.B. $f'(x) = 0$

$\ln x + 1 = 0$

$x = e^{-1}$ STACIONÁRNY BOD

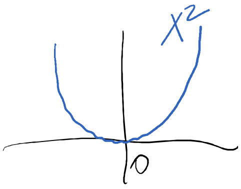
3) $f(x) = x^2 \quad D(f) = \mathbb{R}$

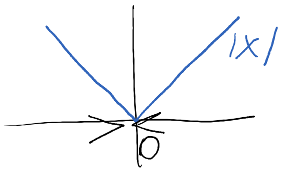
$f'(x) = 2x$

S.B. $f'(x) = 0$

$2x = 0$

$x = 0$ STAC. BOD





4) $f(x) = |x| \quad D(f) = \mathbb{R}$

$\left.\begin{array}{l} f'_+(0) = 1 \\ f'_-(0) = -1 \end{array}\right\} \nexists f'(0)$

$|x| = \begin{cases} x & x \geq 0 \ ✓ \\ -x & x < 0 \ ✓ \end{cases}$

$f'_+(0) = x' = 1$

$f'_-(0) = -x' = -1$

LOKÁLNE EXTRÉMY POMOCOU MONOTÓNNOSTI

1) $f(x) = x^4 - 4x^3 \quad D(f) = R$

$f'(x) = 4x^3 - 12x^2 = 4x^2(x-3)$

S.B. $f'(x) = 0$

$4x^2(x-3) = 0$

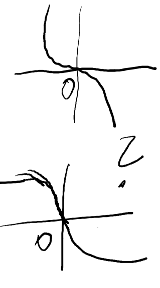
$x_1 = 0 \quad x_2 = 3$

$f'(x) > 0 \quad \nearrow$

$f'(x) < 0 \quad \searrow$

	$(-\infty,0)$	0	$(0,3)$	3	$(3,\infty)$
f'	$-$	0	$-$	0	$+$
f	$\searrow$	∄ extrém	$\searrow$	lok. min	$\nearrow$

f' NEMENÍ ZNAMIENKO   MENÍ ZNAM.



FUNKCIA NEMÁ V BODE $X=0$ LOKÁLNY EXTRÉM

FUNKCIA MÁ V BODE $X=3$ LOKÁLNE MINIMUM

$f(3)=3^4-4\cdot 3^3=3^3(3-4)=-27$

2) $f(x)=x\cdot \ln x \quad D(f)=\mathbb{R}^+$

	$(0,e^{-1})$	e^{-1}	(e^{-1},∞)
$f'(x)$	$-$	0	$+$
$f(x)$	$\searrow$	lok. min.	$\nearrow$

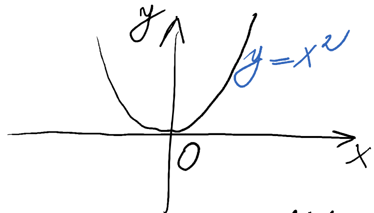
FUNKCIA NADOBÚDA V BODE $X=e^{-1}$ LOKÁLNE MINIMUM

$f(e^{-1})=e^{-1}\cdot \ln e^{-1}=-e^{-1}=-\frac{1}{e}$

3) $f(x)=x^2 \qquad D(f)=\mathbb{R}$

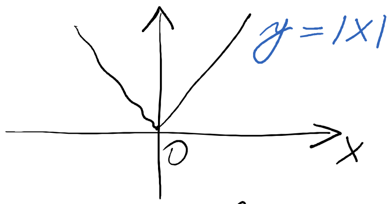
$f'(x)=2x$

$\underset{\searrow}{-} \quad 0 \quad \underset{\nearrow}{+}$



FUNKCIA NADOBÚDA V BODE $X=0$ LOKÁLNE MINIMUM $f(0)=0$

4) $f(x)=|x|$



$X=0 \quad \nexists$ DERIVÁCIA (KRITICKÝ BOD)

V $X=0$ MÁ FUNKCIA LOKÁLNE MINIMUM $f(0)=0$

URČENIE LOK. EXTRÉMU POMOCOU VYŠŠÍCH DERIVÁCIÍ

1) $f(x)=x^4-4x^3$

$f'(x)=4x^3-12x^2=4x^2(x-3)$

S.B. $f'(x)=0$

$4x^2(x-3)=0$

$x_1=0 \quad x_2=3$

$f''(x)=12x^2-24x$

$f''(3)=12\cdot 9-24\cdot 3=36>0 \Rightarrow$ LOKÁLNE MINIMUM

PÁRNA $\Rightarrow$ V BODE $X=3$ MÁ EXTRÉM

$f''(0)=0$

$f'''(x)=24x-24$

$f'''(0)=-24\neq 0$

$$f'''(x) = 24x - 24$$
$$f'''(0) = -24 \neq 0$$

NEPÁRNA $\Longrightarrow$ $\nexists$ EXTRÉM V BODE $x=0$

2) $f(x) = x^5 - 5x^4 \qquad D(f) = \mathbb{R}$

$$f'(x) = 5x^4 - 20x^3$$

S.B. $f'(x) = 0$

$$5x^3(x-4) = 0$$
$$x_1 = 0 \quad x_2 = 4$$

PÁRNA $\Longrightarrow$ $\exists$ EXTRÉM

$$f''(4) = 320 > 0$$

V BODE $x=4$ MÁ LOKÁLNE MINIMUM

$$f''(x) = 20x^3 - 60x^2$$
$$f''(0) = 0$$
$$f'''(x) = 60x^2 - 120x$$
$$f'''(0) = 0$$
$$f^{IV}(x) = 120x - 120$$
$$f^{IV}(0) = -120 < 0 \Longrightarrow \text{LOKÁLNE MAXIMUM}$$

PÁRNA $\Longrightarrow$ V BODE $x=0$ LOKÁLNY EXTRÉM

3) $f(x) = x \cdot \ln x \qquad D(f) = \mathbb{R}^+$

$$f'(x) = \ln x + 1$$

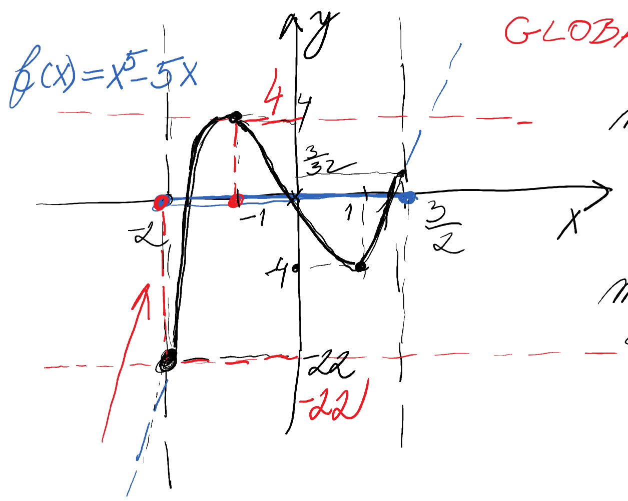
S.B. $x = e^{-1} = \frac{1}{e}$

$$f''(x) = \frac{1}{x}$$
$$f''\left(\frac{1}{e}\right) = e > 0 \Longrightarrow \text{L. MIN}$$

PÁRNA

MÁ V BODE $x = e^{-1}$ LOKÁLNE MINIMUM $f(e^{-1}) = e^{-1} \ln e^{-1} = -e^{-1}$

GLOBÁLNE EXTRÉMY



$$\min_{\langle -2, \frac{3}{2}\rangle} f(x) = \min\left\{f(-1), f(1), f(-2), f\left(\tfrac{3}{2}\right)\right\} = \min\left\{4, -4, -22, \tfrac{3}{32}\right\} = -22$$

$$\max_{\langle -2, \frac{3}{2}\rangle} f(x) = \max\left\{f(-1), f(1), f(-2), f\left(\tfrac{3}{2}\right)\right\} = \max\left\{4, -4, -22, \tfrac{3}{32}\right\} = 4$$

KONVEXNOSŤ / KONKÁVNOSŤ

1) $f(x) = x^4 - 4x^3$ $\quad D(f) = \mathbb{R}$

$f''(x) > 0 \Rightarrow$ KONVEXNÁ $\cup$

$f''(x) < 0 \Rightarrow$ KONKÁVNA $\cap$

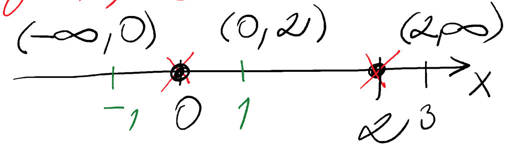
$f'(x) = 4x^3 - 12x^2$

$f''(x) = 12x^2 - 24x = 12x(x-2)$

NULOVÉ BODY: $12x^2 - 24x = 0$

$12x^1(x-2)^1 = 0$

$x_1 = 0 \quad x_2 = 2$



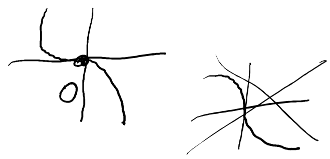
$f''(x)$: $+$ $\quad -$ $\quad +$

$f(x)$: $\cup$ $\quad \cap$ $\quad \cup$

ZMENA ZN. $\quad$ ZMENA ZNAM.

$\Rightarrow x=0$ je I.B. $\quad x=2$ je IB

x	$(-\infty,0)$	0	$(0,2)$	2	$(2,\infty)$
f''	$+$	0	$-$	0	$+$
f	$\cup$	I.B.	$\cap$	I.B.	$\cup$



$x = 0$, $x = 2$ SÚ INFLEXNÉ BODY

FUNKCIA JE R. KONVEXNÁ NA $(-\infty, 0) \cup (2,\infty)$

FUNKCIA JE R. KONKÁVNA NA $(0,2)$

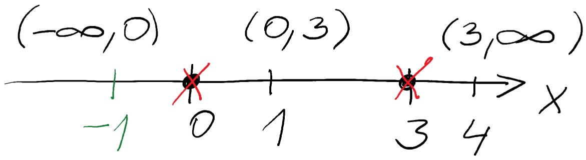
2) $f(x) = x^5 - 5x^4$ $\quad D(f) = \mathbb{R}$

$f'(x) = 5x^4 - 20x^3$

$f''(x) = 20x^3 - 60x^2$

N.B. $\quad 20x^2(x-3)^1 = 0$

$x_1 = 0 \quad x_2 = 3$



$f''(x)$: $-$ NIE JE ZMENA $-$ ZMENA $+$

$f(x)$: $\cap$ $\quad \cap$ $\quad \cup$

	$(-\infty,0)$	0	$(0,3)$	3	$(3,\infty)$
f''	$-$	0	$-$	0	$+$
f	$\cap$	X	$\cap$	IB	$\cup$

$x=0$ NIE JE INFLEXNÝ BOD

$x=3$ JE INFLEXNÝ BOD

FUNKCIA JE RÝDZO KONKÁVNA NA $(-\infty, 0) \cup (0,3)$

FUNKCIA JE RÝDZO KONVEXNÁ NA $(3,\infty)$

3) $f(x) = x \cdot \ln x$ $\quad D(f) = \mathbb{R}^+ = \boxed{(0,\infty)}$

$f'(x) = \ln x + 1$

$f''(x) = \frac{1}{x} > 0 \quad \forall x \in D(f)$

$f''(x) = \frac{1}{x} \neq 0$

FUNKCIA JE KONVEXNÁ NA CELOM $D(f)$
FUNKCIA NEMÁ INFLEXNÉ BODY