Priebeh funkcie

streda 3. decembra 2025 7:30

VYŠETRITE PRIEBEH A NAKRESLITE GRAF FUNKCIE

PRÍKLAD 2)

$f(x) = x^5 - 5x^4 \qquad (= x^4(x-5))$

1) $D(f) = \mathbb{R}$

2) $\sigma_y: \ x = 0 \Rightarrow y = f(0) = 0 \qquad P_1[0,0]$ ✓

$\sigma_x: \ y = 0 \Rightarrow x^5 - 5x^4 = 0$

$x^4 \cdot (x-5) = 0 \qquad P_2[5,0]$ ✓

$x_1 = 0 \quad x_2 = 5$

3) $\forall x \in D(f) \ -x \in D(f)$ ✓

$f(-x) = f(x)$ P

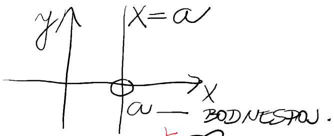
$f(-x) = -f(x)$ N

$f(-x) = (-x)^5 - 5(-x)^4 = -x^5 - 5x^4 \neq f(x) \Rightarrow$ NIE JE PÁRNA

$-f(x) = -(x^5 - 5x^4) = -x^5 + 5x^4 \neq f(-x) \Rightarrow$ NIE JE NEPÁRNA

4) NEPERIODICKÁ

5) BODY NESPOJITOSTI (BN) NEMÁ $D(f) = \mathbb{R}$



6) AS:

ABS NEMÁ, PRETOŽE NEMÁ BN

ASS: $y = kx + q \qquad x \to \pm\infty$

$k = \lim_{x\to\infty} \frac{f(x)}{x} = \lim_{x\to\pm\infty} \frac{x^5 - 5x^4}{x} = \lim_{x\to\pm\infty} \frac{x^{\cancel{4}3}(x-5)}{\cancel{x}} = \infty$

$\boxed{\text{NIE JE KONEČNÉ}}$

$\Rightarrow \nexists$ ASS $x \to \pm\infty$

7) $f'(x) = 5x^4 - 20x^3$

S.B.: $f'(x) = 0$

$5x^4 - 20x^3 = 0$

$5x^3(x-4)^1 = 0$

$x_1 = 0 \quad x_2 = 4$

f': $+$ (0) $-$ (4) $+$

	$(-\infty,0)$	0	$(0,4)$	4	$(4,\infty)$
f'	$+$	0	$-$	0	$+$
f	↗	LOK MAX	↘	LOK MIN	↗

$f(4)$

$P_3 = [4, -256]$

8) FUNKCIA MÁ V BODE $x = 0$ LOK. MAX. $f(0) = 0$

$x = 4$ LOK. MIN. $f(4) = -256$

9) $f''(x) = 20x^3 - 60x^2$

N.B. $f''(x) = 0$

$20x^3 - 60x^2 = 0$

$20x^2(x-3)^1 = 0$

$x_1 = 0 \quad x_2 = 3$

f'': $-$ (0) $-$ (3) $+$

	$(-\infty,0)$	0	$(0,3)$	3	$(3,\infty)$
f''	$-$	0	$-$	0	$+$
f	∩		∩	I.B.	∪

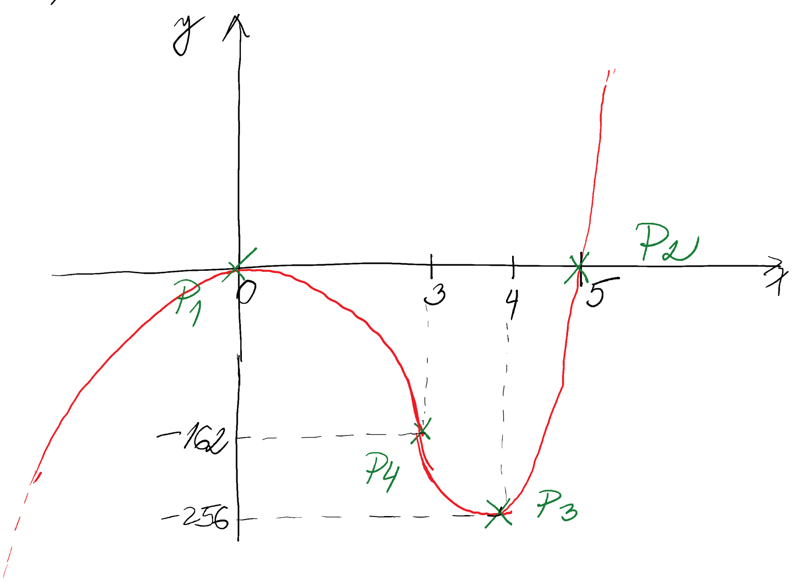
nie je I.B.

$f(3)$

$P_4 = [3, -162]$

10) $x = 3$ JE INFLEXNÝ BOD

y



$H(f) = \mathbb{R}$

PRÍKLAD 3

$$f(x) = \frac{-x^3+x^2+4}{x^2}$$

1) $D(f): x^2 \neq 0 \qquad D(f) = \mathbb{R} - \{0\} = (-\infty, 0) \cup (0, \infty)$

2) $o_y: \; x = 0 \notin D(f)$

$o_x: \; y = 0 \implies \dfrac{-x^3+x^2+4}{x^2} = 0$

$$-x^3+x^2+4 = 0 \quad /\cdot(-1)$$

$$x^3-x^2-4 = 0$$

$\alpha \in \{\pm 1, \pm 2, \pm 4\}$

DELITELE (-4)

$$\begin{array}{c|cccc} & 1 & -1 & 0 & -4 \\ 2 & & 2 & 2 & 4 \\ \hline & 1 & 1 & 2 & \boxed{0} \end{array}$$

$x^3-x^2-4 = (x-2)(x^2+x+2)$ $\quad D = -7$

✓ $x^3 \underbrace{-8\;-x^2+4}_{-4} = 0$

$x^3-2^3 =$

$$(x^3-8) - (x^2-4) = 0$$

$$(x-2)(x^2+2x+4) - (x-2)(x+2) = 0$$

$$(x-2)\left[x^2+2x+4-(x+2)\right] = 0$$

$$(x-2)\underbrace{(x^2+x+2)}_{\neq 0} = 0 \qquad x = 2 \qquad P_1[2,0]$$

$a^3-b^3 = (a-b)(a^2+ab+b^2)$ ✓

$a^2-b^2 = (a-b)(a+b)$

3) $\forall x \in D(f) \; -x \in D(f)$

$$f(-x) = \frac{-(-x)^3+(-x)^2+4}{(-x)^2} = \frac{x^3+x^2+4}{x^2} \neq f(x) \quad \times$$

$$-f(x) = - \frac{-x^3+x^2+4}{x^2} = \frac{+x^3-x^2-4}{x^2} \neq f(-x) \quad \times$$

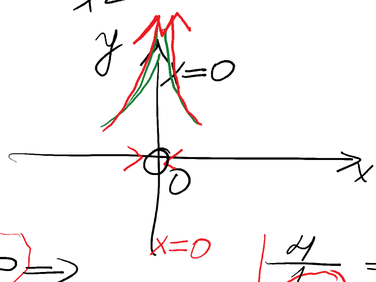
4) NEPERIODICKÁ

5) BN: $x = 0$ $(\notin D(f))$

6) ABS: $x = 0$ (PRIAMKA $\parallel o_y$)

$$\lim_{x\to 0} \frac{\overset{\to 0}{-x^3}\overset{\to 0}{+x^2}+4 \to 4}{x^2 \to 0^+} = \infty \implies \boxed{x = 0 \text{ JE ABS}}$$

$$\frac{4}{\frac{1}{10}} = 40 \qquad \frac{4}{\frac{1}{100}} = 400$$



$x \to 0$ ... (0)

$\boxed{x=0 \text{ JE ASS}}$

$\dfrac{10}{\frac{1}{100}} = 400$!

ASS: $y = kx+q$

$$k = \lim_{x\to\infty} \frac{f(x)}{x} = \lim_{x\to\infty} \frac{\frac{-x^3+x^2+4}{x^2}}{\frac{x}{1}} =$$

$$= \lim_{x\to\infty} \frac{-x^3+x^2+4}{1\cdot x^3} \overset{\frac{-\infty}{\infty}}{\underset{L'H}{=}} \lim_{x\to\infty} \frac{-3x^2+2x}{3x^2} \overset{\frac{-\infty}{\infty}}{\underset{L'H}{=}}$$

$$= \lim_{x\to\infty} \frac{-6x+2}{6x} \overset{\frac{-\infty}{\infty}}{\underset{L'H}{=}} \lim_{x\to\infty} \frac{-6}{6} = -1 < \infty$$

$$q = \lim_{x\to\infty} (f(x) - k\cdot x) = \lim_{x\to\infty} \left(\frac{-x^3+x^2+4}{x^2} - (-1)\cdot x\right) = \quad -\infty+\infty$$

$$= \lim_{x\to\infty} \frac{\cancel{-x^3}+x^2+4 \cancel{+x\cdot x^2}}{x^2} = \lim_{x\to\infty} \frac{x^2+4}{x^2} \overset{\frac{\infty}{\infty}}{\underset{L'H}{=}} \lim_{x\to\infty} \frac{2x}{2x} = \boxed{1}$$

$y = -1\cdot x + 1$ ASS $x \to \pm\infty$

7) $$f'(x) = \frac{(-3x^2+2x)\cdot x^2 - (-x^3+x^2+4)\cdot 2x}{x^4} =$$

$$= \frac{x\left[-3x^3 \cancel{+2x^2} + 2x^3 \cancel{-2x^2} - 8\right]}{x^{\cancel{4}\,3}} = \qquad (x-1)^2+3 > 0$$

$$= \frac{-x^3-8}{x^3} = \frac{-(x^3+\underset{2^3}{8})}{x^3} = -\frac{(x+2)\overbrace{(x^2-2x+4)}^{>0}}{x^3}$$

S.B. $f'(x) = 0$

$-\dfrac{(x^3+8)}{x^3} = 0$ $\qquad a^3+b^3 = (a+b)(a^2-ab+b^2)$

$x^3+8 = 0$ $\quad D=-12$ $\qquad$ delitele 8: $\pm1, \pm2, \pm4, \pm8$

$(x+2)\underbrace{(x^2-2x+4)}_{\neq 0} = 0$

$x_1 = -2$

	$(-\infty,-2)$	-2	$(-2,0)$	$(0,\infty)$
f'	$-$	0	$+$	$-$
f	↘	LOK. MIN	↗	↘

f': $-$ $\quad -2 \quad$ $+$ $\quad 0 \text{ BN} \quad$ $-$

$P_2[-2, 4] = f(-2)$

8) FUNKCIA MÁ V BODE $x=-2$ LOK. MIN. $f(-2) = 4$

9) $$f''(x) = \left[\frac{-(x^3+8)}{x^3}\right]' = -1\left[\frac{x^3+8}{x^3}\right]' = -\left(1+\frac{8}{x^3}\right)' = -(1+8\cdot x^{-3})'$$

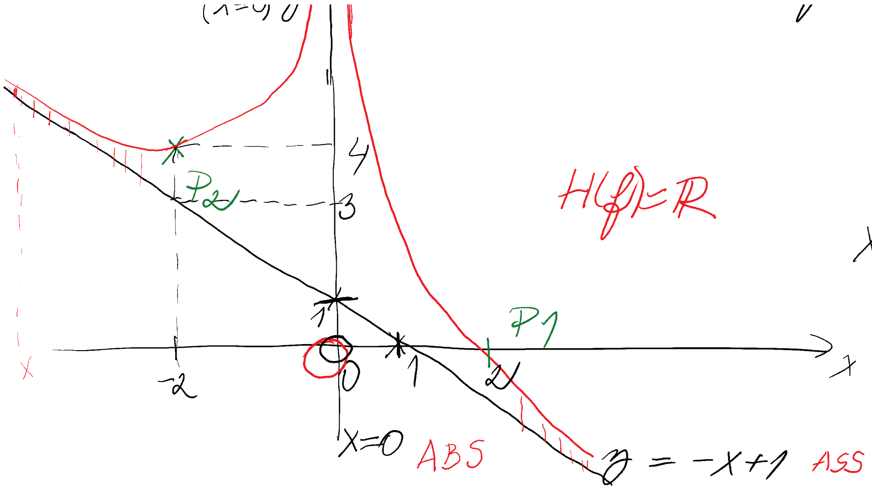
$$= -\left(8\cdot(-3)\cdot x^{-4}\right) = \frac{24}{x^4} > 0$$

	$(-\infty,0)$	$(0,\infty)$
f''	$+$	$+$
f	∪	∪

10) NEMÁ INFLEXNÉ BODY

$(x=0)$ y

$y = -x+1$



$H(f) = R$

$y = -x+1$

$f(x) = \frac{-x^3+x^2+4}{x^2}$

$x=-2: \quad y = -(-2)+1 = 3$

$f(-2) = 4$

PRÍKLAD 4

$f(x) = x \cdot \ln x$

1) $D(f): \ x>0 \qquad D(f) = (0,\infty) = R^+$

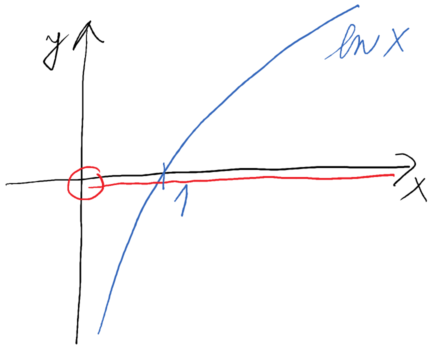
2) $O_y: \ x=0 \notin D(f)$

$O_x: \ y=0 \qquad x \cdot \ln x = 0$

$x=0 \ \vee \ \ln x = 0$

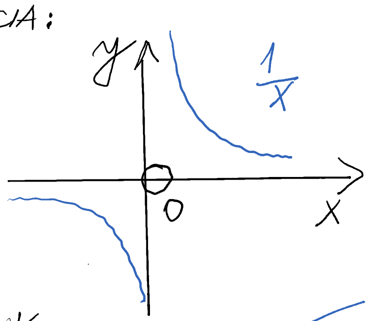
$\notin D(f) \qquad x=1$

$P_1 = [1,0]$



3) $\forall x \in D(f) \ -x \in D(f)$ NEPLATÍ, ALE PLATÍ NEGÁCIA:

$\exists x \in D(f) \ -x \notin D(f) \implies$ ANI $\cancel{P}$ ANI $\cancel{N}$



4) NEPERIODICKÁ

5) BODY NESPOJITOSTI NEMÁ

6) $\boxed{\text{ABS}}$ $\quad x=0$

$\cancel{x \to 0^-}$

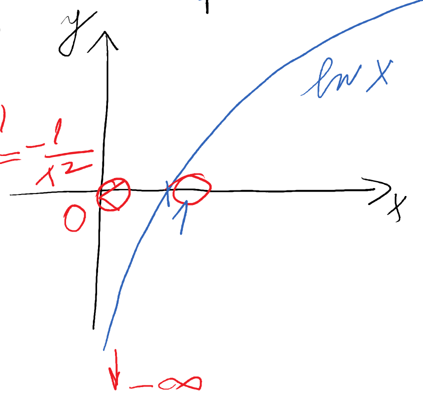
$D(f) = (0,\infty)$

$\lim_{x\to 0^+} f(x) = 0 \cdot (-\infty)$

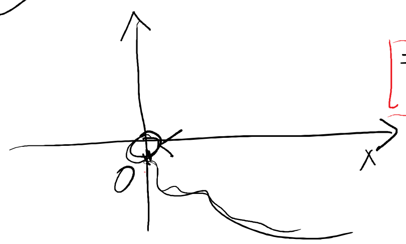
$= \lim_{x\to 0^+} x \cdot \ln x = \lim_{x\to 0^+} \frac{\ln x}{\frac{1}{x}} \quad \frac{-\infty}{\infty}$ ✓

$\underset{L'H}{=} \lim_{x\to 0^+} \frac{\frac{1}{x}}{-\frac{1}{x^2}} = \lim_{x\to 0^+} \frac{-x^2}{x} = 0$ KONEČNÉ

$(x^{-1})' = -\frac{1}{x^2}$



$\boxed{\implies x=0 \text{ NIE JE ABS}}$



ASS $\quad y = kx+q \qquad x \to \infty$

$x \to -\infty$ NEMÔŽE $D(f) = (0,\infty)$

$k = \lim_{x\to\infty} \frac{f(x)}{x} = \lim_{x\to\infty} \frac{\cancel{x} \ln x}{\cancel{x}} = \infty$ NIE JE KONEČNÉ

$\implies$ ASS $\nexists$

7) $f'(x) = 1\ln x + x\cdot\frac{1}{x} = \ln x + 1$

S.B.: $f'(x) = 0$

$\ln x + 1 = 0$

$\ln x = -1 \;/e^{()}$

$x = e^{-1} = \frac{1}{e}$

	$(0, e^{-1})$	e^{-1}	(e^{-1}, ∞)
f'	$-$	0	$+$
f	$\searrow$	LOK MIN.	$\nearrow$

$P_2[e^{-1}; -e^{-1}]$

$\ln x = -1 \;/e^{()}$

$e^{\ln x} = e^{-1}$

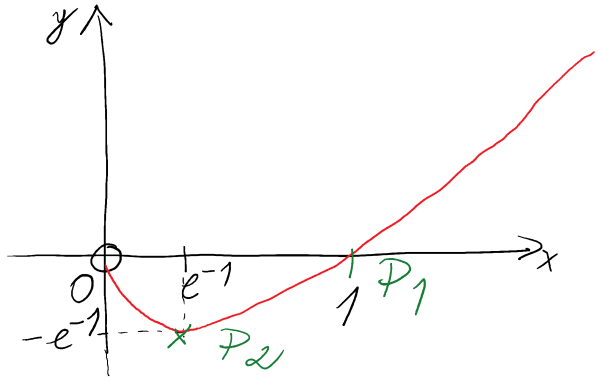
$x = e^{-1}$

$\ln e^{-2} + 1 = -2\ln e + 1$
$= -2 + 1 = -1$

8) FUNKCIA MÁ V BODE $x = e^{-1}$ LOK. MIN.. $f(e^{-1}) = -e^{-1}$

9) $f''(x) = (\ln x + 1)' = \frac{1}{x} > 0 \quad \forall x \in D(f) = (0, \infty)$ $\cup$

10) NEMÁ INFLEXNÝ BOD



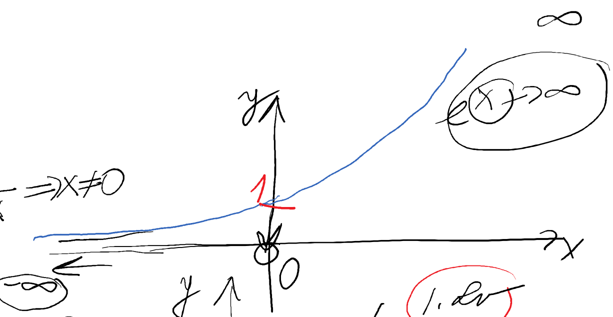
$H(f) = \langle -e^{-1}; \infty)$

PRÍKLAD 5

$f(x) = x\cdot e^{\frac{1}{x}}$

$\frac{1}{x} \Rightarrow x \neq 0$

$H(f) = \mathbb{R}^+$



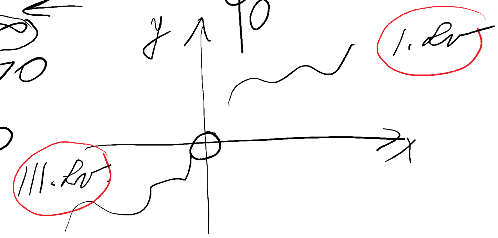
1) $D(f) = \mathbb{R} - \{0\}$

2) σ_y: $x = 0 \notin D(f)$

σ_x: $y = 0 \quad x\cdot e^{\frac{1}{x}} = 0$

$x = 0 \notin D(f) \vee e^{\frac{1}{x}} = 0$

$\emptyset \qquad \emptyset$

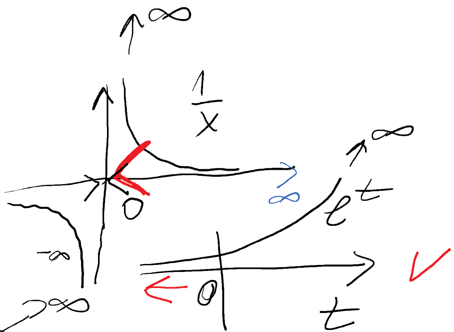


NEMÁ PRIESEKY S OSAMI

3) $\forall x \in D(f) \;\exists -x \in D(f)$

$f(-x) = -x\, e^{-\frac{1}{x}} \neq f(x)$ ANI ~~P~~ ANI ~~N~~

$-f(x) = -x e^{\frac{1}{x}} \neq f(-x)$



4) NEPERIODICKÁ

5) BN $x = 0$

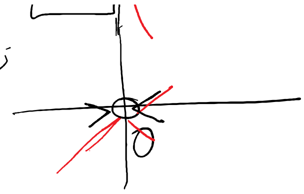
6) ABS $x = 0$

$\lim_{x\to 0^+} x\cdot e^{\frac{1}{x}} = 0\cdot\infty \quad \lim_{x\to 0^+} \frac{e^{\frac{1}{x}}}{\frac{1}{x}} \overset{L'H}{=} \frac{\infty}{\infty} = \lim_{x\to 0^+} \frac{e^{\frac{1}{x}}\cdot\left(\frac{1}{x}\right)'}{\left(\frac{1}{x}\right)'} = \lim_{x\to 0^+} e^{\frac{1}{x}} = \boxed{\infty}$

$\Longrightarrow$ $x = 0$ JE ABS

$\lim \left(\frac{1}{x}\right) \to -\infty \to 0$

$\Rightarrow x=0$ JE ABS



$$\lim_{x\to 0^-} x\, e^{\frac{1}{x}} = 0\cdot 0 = \underline{\underline{0}}$$

ASS $y = kx+q$

$$k = \lim_{x\to\infty} \frac{\cancel{x} e^{\frac{1}{x}}}{\cancel{x}} = e^0 = \boxed{1} < \infty$$

$$k = \lim_{x\to\infty} \frac{f(x)}{x}$$

$$q = \lim_{x\to\infty}\left(xe^{\frac{1}{x}} - 1\cdot x\right) = \lim_{x\to\infty} x\left(e^{\frac{1}{x}}-1\right) \overset{\infty\cdot 0}{=}$$

$$q = \lim_{x\to\infty}\left(f(x)-k\cdot x\right)$$

$$= \lim_{x\to\infty} \frac{e^{\frac{1}{x}}-1}{\frac{1}{x}} \overset{\frac{0}{0}}{\underset{L'H}{=}} \lim_{x\to\infty} \frac{e^{\frac{1}{x}}\cdot\cancel{\left(\frac{1}{x}\right)'}}{\cancel{\left(\frac{1}{x}\right)'}} = \boxed{1}$$

$$y = kx+q$$

$$y = 1\cdot x + 1 \qquad x\to\pm\infty$$

7) $f'(x) = e^{\frac{1}{x}} + \cancel{x}\cdot e^{\frac{1}{x}}\cdot\left(-\frac{1}{x^{\cancel{2}}}\right) = e^{\frac{1}{x}} - \frac{e^{\frac{1}{x}}}{x} =$ $\qquad \left(\frac{1}{x}\right)' = \left(x^{-1}\right)'$

$$= e^{\frac{1}{x}}\left(1-\frac{1}{x}\right) = \underbrace{e^{\frac{1}{x}}}_{>0}\left(\frac{x-1}{x}\right)$$

$\neq 0$

S.B. $f'(x)=0$

$\frac{x-1}{x}=0$

$x-1=0$

$x=1$

	$(-\infty,0)$	$(0,1)$	1	$(1,\infty)$
f'	+	−	0	+
f	↗	↘	LOK MIN	↗

8) FUNKCIA MÁ V BODE $x=1$ LOK. MIN. $f(1)=1\cdot e^{\frac{1}{1}}=e$

$P_1=[1,e]$

// $\left(\frac{1}{x}\right)'$

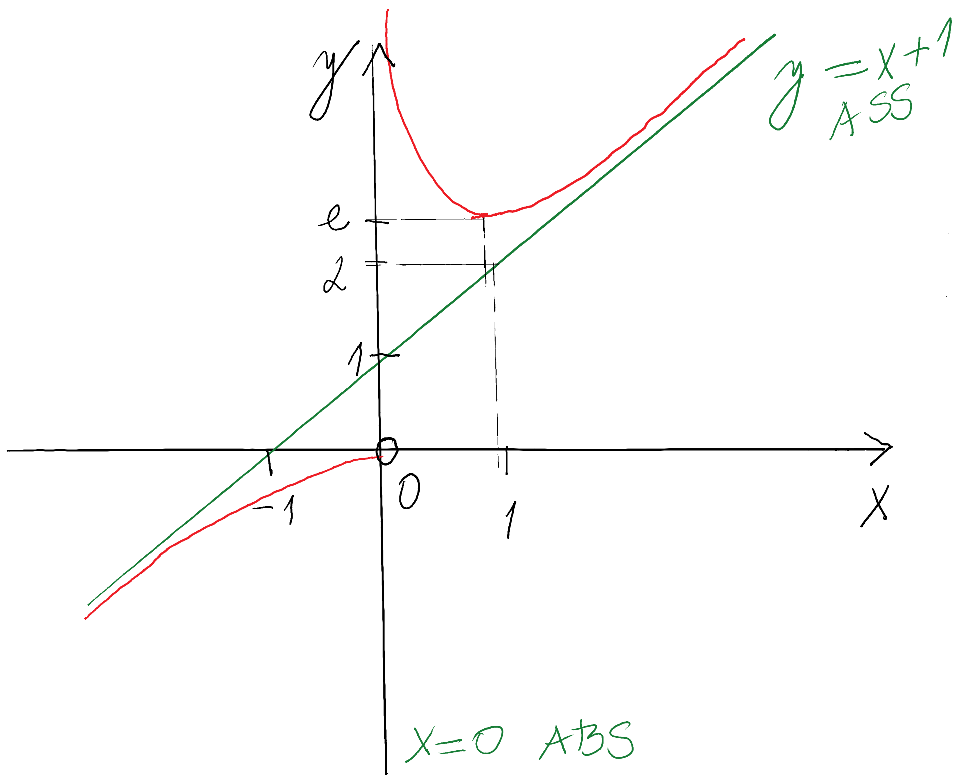
$(1-x^{-1})' = -1(-1x^{-2})$

9) $f''(x) = \left[\left(1-\frac{1}{x}\right)e^{\frac{1}{x}}\right]' = \boxed{\frac{1}{x^2}}\boxed{e^{\frac{1}{x}}} + \left(1-\frac{1}{x}\right)\boxed{e^{\frac{1}{x}}}\left(-\boxed{\frac{1}{x^2}}\right) =$ $\qquad -\left(1-\frac{1}{x}\right) = \left(-1+\frac{1}{x}\right)$

$$= \boxed{\frac{1}{x^2}}\boxed{e^{\frac{1}{x}}}\left(1-1+\frac{1}{x}\right) = \frac{\overset{>0}{e^{\frac{1}{x}}}}{x^3} \neq 0 \quad \nexists \text{ NB}$$

	$(-\infty,0)$	$(0,\infty)$
f''	−	+
f	∩	∪

10) NEMÁ INFLEXNÉ BODY


y
e
2
1
0
-1
0
1
x
y = x+1
ASS
x=0 ABS