PRÍKLAD: VYPOČÍTAJME 1. DERIVÁCIU F-CIÍ

$[f \cdot g]' = f' \cdot g + f \cdot g'$

(9) $y = (x^{10} - 4x^3 + 2x) \bullet (x^{-2} + 3 - 3x^2 + \frac{4}{x})$

$y' = (x^{10} - 4x^3 + 2x)' \cdot (x^{-2} + 3 - 3x^2 + 4x^{-1}) + (x^{10} - 4x^3 + 2x) \cdot (x^{-2} + 3 - 3x^2 + 4x^{-1})' = (10x^9 - 12x^2 + 2) \cdot (x^{-2} + 3 - 3x^2 + 4x^{-1}) + (x^{10} - 4x^3 + 2x) \cdot (2x + 0 - 6x - 4x^{-2})$

(10) $y = (3^x - e^x) \bullet (\sin x - \mathrm{tg}\, x)$

$y' = (3^x \cdot \ln 3 - e^x) \cdot (\sin x - \mathrm{tg}\, x) + (3^x - e^x) \cdot (\cos x - \frac{1}{\cos^2 x})$

(11) $y = (-\cos x + \ln x) \bullet (\log_2 x + 2\,\mathrm{cotg}\, x)$

$y' = (\sin x + \frac{1}{x}) \cdot (\log_2 x + 2\,\mathrm{cotg}\, x) + (-\cos x + \ln x) \cdot (\frac{1}{x \cdot \ln 2} - 2 \cdot \frac{1}{\sin^2 x})$

(12) $y = \dfrac{x^2 + 3x - 3}{4x + 5}$

$$y' = \frac{(x^2+3x-3)' \cdot (4x+5) - (x^2+3x-3) \cdot (4x+5)'}{(4x+5)^2} = \frac{(2x+3-0) \cdot (4x+5) - (x^2+3x-3) \cdot (4+0)}{(4x+5)^2} = \frac{(2x+3)(4x+5) - (x^2+3x-3) \cdot 4}{(4x+5)^2}$$

$$\left[\frac{f}{g}\right]' = \frac{f' \cdot g - f \cdot g'}{g^2}$$

(13) $y = \dfrac{e^x + \log_{10} x}{\ln x}$

$$y' = \frac{(e^x + \frac{1}{x \cdot \ln 10}) \cdot \ln x - (e^x + \log x) \cdot \frac{1}{x}}{(\ln x)^2}$$

$(\ln x)^2 \to \ln^2 x$

(14) $y = \dfrac{3\cos x - 2\sin x}{x^2}$

$$y' = \frac{(-3\sin x - 2\cos x) \cdot x^2 - (3\cos x - 2\sin x) \cdot 2x}{x^4}$$

IDEME OBLIEKAŤ PANÁČIKA X => VYTVÁRANIE ZLOŽENEJ F-CIE

$x \to 4x + 5 \to \ln(4x+5) \to \sin\ \ln(4x+5))$

VYTVORILI SME ZLOŽENÚ F-CIU $y = \sin \ln (4x + 5)$

F-CIU IDEME DERIVOVAŤ (PANÁČIKA X VYZLIEKAME POSTUPNE)

$$y' = [\cos \ln(4x+5)] \bullet \left[\frac{1}{4x+5}\right] \bullet [4] = \frac{4 \cos \ln(4x+5)}{4x+5}$$

PRAVIDLO NA DERIVOVANIE ZLOŽENEJ F-CIE : $[f(g(x))]' = f'(g(x)) \bullet g'(x)$

(15) $y = \ln \sin(4x+5)$

$$y' = \left[\frac{1}{\sin(4x+5)}\right] \bullet [\cos(4x+5)] \bullet [4]$$

$[\ln x]' = \frac{1}{x}$

$[\ln \heartsuit]' = \frac{1}{\heartsuit}$

(16) $y = \sqrt{\cos x^2} = (\cos x^2)^{\frac{1}{2}}$

$$y' = \left[\frac{1}{2} (\cos x^2)^{-\frac{1}{2}}\right] \bullet [-\sin x^2] \bullet [2x]$$

$\sqrt{x} = x^{\frac{1}{2}}$ $\quad \cos(x^2)$

$\sqrt{\heartsuit} = \heartsuit^{\frac{1}{2}}$ $\quad (\cos x)^2 = \cos^2 x$

(17) $y = \overset{1}{\mathrm{tg}}\ \overset{2}{\log}\ \overset{3}{(2x^2 - 1)}$

$$y' = \underbrace{\left[\frac{1}{\cos^2 \log(2x^2-1)}\right]}_{1} \bullet \underbrace{\left[\frac{1}{(2x^2-1) \cdot \ln 10}\right]}_{2} \cdot \underbrace{[4x]}_{3}$$

$[\mathrm{tg}\, x]' = \frac{1}{\cos^2 x}$

$[\mathrm{tg}\, \heartsuit] = \frac{1}{\cos^2 \heartsuit}$

(18) $y = (2 + 3e^x)^{11}$

$$y' = [11(2+3e^x)^{10}] \bullet [3e^x]$$

(19) $y = \mathrm{cotg}^4 (x^3 + x) = (\mathrm{cotg}(x^3 + x))^4$

$$y' = [4\,\mathrm{cotg}^3 (x^3 + x)] \bullet \left[\frac{-1}{\sin^2(x^3+x)}\right] \bullet [3x^2 + 1]$$

$[\heartsuit^4]' = 4 \cdot \heartsuit^3$

(20) $y = \mathrm{cotg}\, (x^3 + x)^4$

$$y' = \left[\frac{-1}{\sin^2 (x^3+x)^4}\right] \bullet [4(x^3+x)^3] \bullet [3x^2 + 1]$$

(21) $y = \ln \dfrac{x^2+1}{x^2-1}$

$$y' = \left[\frac{x^2-1}{x^2+1}\right] \bullet \left[\frac{(2x)(x^2-1) - (x^2+1)(2x)}{(x^2-1)^2}\right]$$