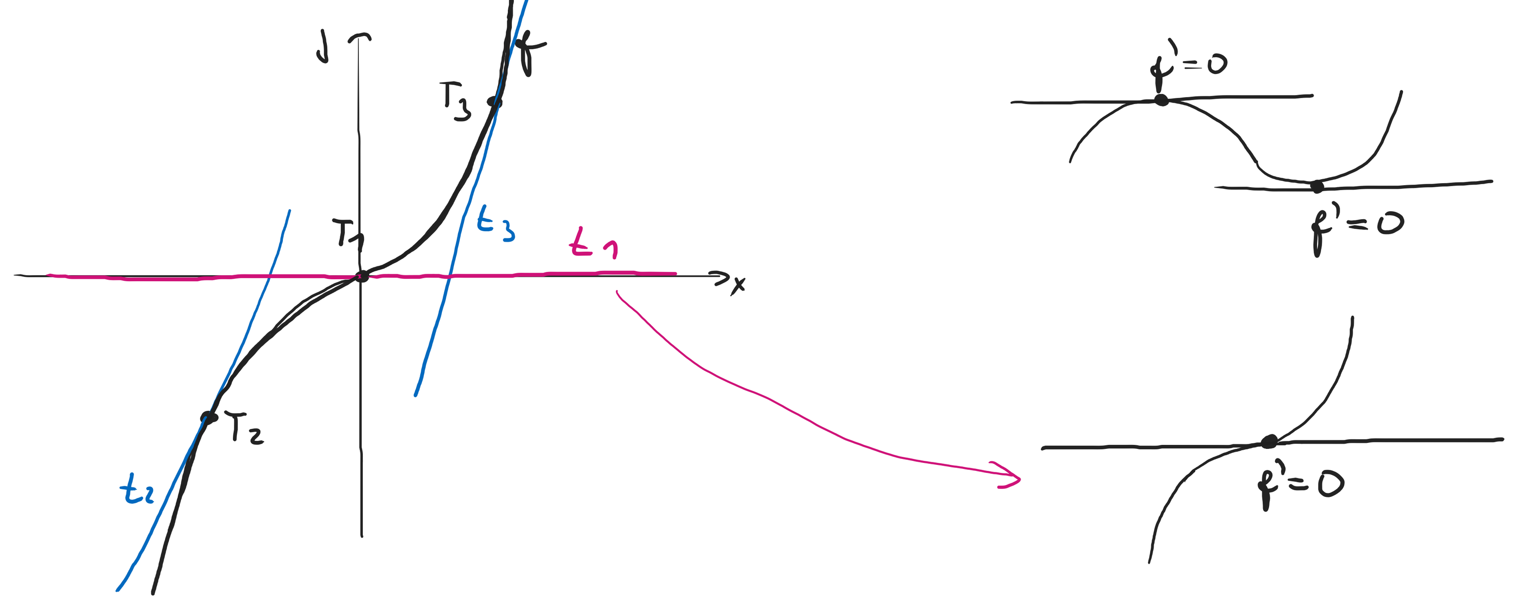
PRÍKLAD: NÁJDIME ROVNICU DOTYČNICE V BODE **T** KU GRAFU F-CIE $f(x)$

① $y = f(x) = x^3$; $T_1 = [\underset{x_0}{0} ; \underset{y_0 = (x_0)^3}{?}]$; $T_2 = [-2 ; ?]$; $T_3 = [2 ; ?]$

DOTYČNICA t : $y - f(x_0) = f'(x_0)(x - x_0)$ ALEBO $\boxed{y - y_0 = \boxed{f'(x_0)}(x - x_0)}$!

$T_1 = [\overset{x_0}{0} ; \overset{y_0}{0}]$
$T_2 = [-2 ; -8]$ LEBO $y_0 = f(x_0) = (x_0)^3$
$T_3 = [2 ; 8]$

$f'(x) = (x^3)' = 3x^2 \Rightarrow \boxed{f'(\overset{x_0}{0})} = 3 \cdot 0^2 = 0 \Rightarrow t_1 : y - 0 = 0 \cdot (x - 0) \Rightarrow y = 0$
$f'(-2) = 3 \cdot (-2)^2 = 12 \Rightarrow t_2 : y - (-8) = 12(x - (-2)) \Rightarrow y + 8 = 12(x+2)$
$f'(2) = 3 \cdot (2)^2 = 12 \Rightarrow t_3 : y - 8 = 12(x - 2)$



② $f(x) = 2x \cdot \ln x$; $T = [\underset{x_0}{e} ; ?] \Rightarrow T = [e ; 2e]$
$y_0 = 2 \cdot e \cdot \underset{1}{\ln e} = 2e$

$f'(x) = [2x \cdot \ln x]' = 2 \cdot \ln x + 2\cancel{x} \cdot \frac{1}{\cancel{x}} = 2\ln x + 2$

$f'(x_0) = f'(e) = 2\underset{1}{\ln e} + 2 = 4$

$t : y - 2e = 4(x - e)$

L'HOSPITALOVO PRAVIDLO

- POUŽÍVAME NA VÝPOČET LIMÍT SO VŠETKÝMI NEURČITOSŤAMI
- PRE NEURČITOSTI $\frac{0}{0}$ | $\frac{\pm\infty}{\pm\infty}$ SA VYUŽÍVA PRIAMO (BEZ ÚPRAV F-CIE)
- PRE OSTATNÉ NEURČITOSTI JE POTREBNÉ F-CIE NAJPRV VHODNE UPRAVIŤ NA $\frac{0}{0}$ ALEBO $\frac{\infty}{\infty}$

VETA (L'HOSPITALOVA): NECH $\lim\limits_{x \to a} f(x) = \lim\limits_{x \to a} g(x) = 0$ ALEBO

$\lim\limits_{x \to a} f(x) = \lim\limits_{x \to a} |g(x)| = \infty$ A NECH $\lim\limits_{x \to a} \frac{f'(x)}{g'(x)}$ EXISTUJE.

POTOM EXISTUJE AJ $\lim\limits_{x \to a} \frac{f(x)}{g(x)}$ A PLATÍ $\lim\limits_{x \to a} \frac{f(x)}{g(x)} = \lim\limits_{x \to a} \frac{f'(x)}{g'(x)}$.

PRÍKLAD: VYPOČÍTAJME LIMITU

① $\lim\limits_{x \to 3} \frac{x^2 - 9}{x^2 - 5x + 6} = \left\{\frac{0}{0}\right\} \overset{L'H}{=} \lim\limits_{x \to 3} \frac{2x}{2x - 5} = \frac{6}{1} = \underline{\underline{6}}$

② $\lim\limits_{x \to \infty} \frac{1x^2 + 2x - 1}{3x^2 + 4} = \left\{\frac{\infty}{\infty}\right\} \overset{L'H}{=} \lim\limits_{x \to \infty} \frac{2x + 2}{6x} = \left\{\frac{\infty}{\infty}\right\} \overset{L'H}{=} \lim\limits_{x \to \infty} \frac{2}{6} = \underline{\underline{\frac{1}{3}}}$

③ $\lim\limits_{x \to 2} \frac{\ln(x-1)}{x - 2} = \left\{\frac{0}{0}\right\} \overset{L'H}{=} \lim\limits_{x \to 2} \frac{\frac{1}{x-1}}{1} = \underline{\underline{1}}$

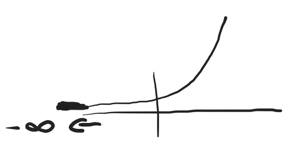
④ $\lim\limits_{x \to 0} \frac{\sin x - x}{x^2} = \left\{\frac{0}{0}\right\} \overset{L'H}{=} \lim\limits_{x \to 0} \frac{\cos x - 1}{2x} = \left\{\frac{0}{0}\right\} \overset{L'H}{=} \lim\limits_{x \to 0} \frac{-\sin x}{2} = \frac{0}{2} = \underline{\underline{0}}$

⑤ $\lim\limits_{x \to \infty} \frac{2^x}{x} = \left\{\frac{\infty}{\infty}\right\} \overset{L'H}{=} \lim\limits_{x \to \infty} \frac{2^x \cdot \ln 2}{1} = \frac{\infty}{1} = \underline{\underline{\infty}}$

⑥ $\lim\limits_{x \to \infty} \frac{2\ln x}{x^2} = \left\{\frac{\infty}{\infty}\right\} \overset{L'H}{=} \lim\limits_{x \to \infty} \frac{\frac{\cancel{2}}{x}}{\underset{1}{\cancel{2}x}} = \lim\limits_{x \to \infty} \frac{1}{x^2} = \underline{\underline{0}}$

NEURČITOSŤ $\{0 \cdot \infty\}$ VIEME UPRAVIŤ TAKTO
$0 \cdot \infty = \frac{0}{\frac{1}{\infty}} = \frac{0}{0}$
$0 \cdot \infty = \frac{\infty}{\frac{1}{0}} = \frac{\infty}{\infty}$

$a \cdot b = \frac{a}{b^{-1}} = \frac{a}{\frac{1}{b}}$



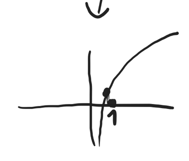
⑦ $\lim\limits_{x \to \infty} x \cdot 2^{-x} = \{\infty \cdot 0\} = \lim\limits_{x \to \infty} \frac{x}{2^x} = \left\{\frac{\infty}{\infty}\right\} \overset{L'H}{=} \lim\limits_{x \to \infty} \frac{1}{2^x \cdot \ln 2} = \frac{1}{\infty} = \underline{\underline{0}}$

⑧ $\lim\limits_{x \to 0^+} x \cdot e^{\frac{1}{x}} = \{0 \cdot \infty\} = \lim\limits_{x \to 0^+} \frac{e^{\frac{1}{x}}}{\frac{1}{x}} = \left\{\frac{\infty}{\infty}\right\} \overset{L'H}{=} \lim\limits_{x \to 0^+} \frac{e^{\frac{1}{x}} \cdot \cancel{(-1x^{-2})}}{\cancel{-1x^{-2}}} = \lim\limits_{x \to 0^+} e^{\frac{1}{x}} = \underline{\underline{\infty}}$

$\frac{1}{x} \to +\infty$, $\frac{1}{x} = x^{-1}$

NEURČITOSŤ $\{\infty - \infty\}$ UPRAVÍME POMOCOU SPOLOČNÉHO MENOVATEĽA NA $\left\{\frac{0}{0}\right\}$

⑨ $\lim\limits_{x \to 1^+} \left(\frac{1}{\underset{\downarrow 0^+}{\ln x}} + \frac{1}{\underset{\to 0^-}{1 - x}}\right) = \{\infty - \infty\} = \lim\limits_{x \to 1^+} \frac{1 - x + \ln x}{(\ln x) \cdot (1 - x)} = \left\{\frac{0}{0}\right\} \overset{L'H}{=}$



$= \lim\limits_{x \to 1^+} \frac{-1 + \frac{1}{x}}{\frac{1}{x}(1-x) + (\ln x)(-1)} = \lim\limits_{x \to 1^+} \frac{\frac{1}{x} - 1}{\frac{1}{x} - 1 - \ln x} = \left\{\frac{0}{0}\right\} \overset{L'H}{=}$

$\frac{1}{x} = x^{-1}$

$= \lim\limits_{x \to 1^+} \frac{-1x^{-2}}{-1x^{-2} - \frac{1}{x}} = \lim\limits_{x \to 1^+} \frac{-\frac{1}{x^2}}{-\frac{1}{x^2} - \frac{1}{x}} = \frac{-\frac{1}{1}}{-\frac{1}{1} - \frac{1}{1}} = \frac{-1}{-2} = \underline{\underline{\frac{1}{2}}}$