PRÍKLAD: VYŠETRIME MONOTÓNNOSŤ A EXTRÉMY F-CIE

a) $\boxed{f(x) = 3x^4 - 8x^3 + 6x^2 + 2}$

1) $D(f) = (-\infty, \infty)$ $\qquad \longrightarrow x$

2) $f'(x) = 12x^3 - 24x^2 + 12x = 12x(x^2 - 2x + 1) = 12x(x-1)^2$ $\quad +\ \ +$

3) $f'(x) = 0 \iff x = 0 ;\ x = 1$
$f'(x)$ – NEEX $\iff$ NIKDY

4) $0 \quad 1 \quad \longrightarrow x$

5,6)

	$(-\infty, 0)$	0	$(0,1)$	1	$(1,\infty)$
f'	$-$		$+$		$+$
f	$\searrow$	MIN	$\nearrow$		$\nearrow$

b) $\boxed{f(x) = (x^2-1)^4}$

1) $D(f) = (-\infty, \infty)$ $\qquad \longrightarrow x$

2) $f'(x) = 4(x^2-1)^3 \cdot 2x = 8x(x-1)^3(x+1)^3$

3) $f'(x) = 0 \iff x = 0 ;\ x = \pm 1$

4) $-1 \quad 0 \quad 1 \quad \longrightarrow x$

5,6)

	$(-\infty,-1)$	-1	$(-1,0)$	0	$(0,1)$	1	$(1,\infty)$
f'	$- - -$ $\ -$		$- - +$ $\ +$		$+ - +$ $\ -$		$+ + +$ $\ +$
f	$\searrow$	MIN	$\nearrow$	MAX	$\searrow$	MIN	$\nearrow$

c) $\boxed{f(x) = \dfrac{x^3}{2(x+1)^2}}$

1) $D(f) = (-\infty, -1) \cup (-1, \infty)$ $\qquad \oplus\ -1$

2) $f'(x) = \dfrac{3x^2 \cdot 2(x+1)^2 - x^3 \cdot 4(x+1) \cdot 1}{4(x+1)^4} = \dfrac{\cancel{(x+1)}[6x^2(x+1) - 4x^3]}{4(x+1)^{\cancel{4}}} = \dfrac{6x^3 + 6x^2 - 4x^3}{4(x+1)^3} = \dfrac{2x^3 + 6x^2}{4(x+1)^3} = \dfrac{2x^2(x+3)}{4(x+1)^3}$ $\quad +$

3) $f'(x) = 0 \Leftarrow x = 0 ;\ x = -3$
$f'(x)$ – NEEX $\iff x = -1$

4) $-3 \quad \oplus\,{-1} \quad 0 \quad \longrightarrow x$

5,6)

	$(-\infty,-3)$	-3	$(-3,-1)$	-1	$(-1,0)$	0	$(0,\infty)$
f'	$+$		$-$	$*$	$+$		$+$
f	$\nearrow$	MAX	$\searrow$	$*$	$\nearrow$		$\nearrow$

d) $\boxed{f(x) = \dfrac{x^2}{x^2-1}}$

1) $D(f) = (-\infty, -1) \cup (-1, 1) \cup (1, \infty)$ $\qquad \oplus\,{-1} \quad \oplus\,1$

2) $f'(x) = \dfrac{2x(x^2-1) - x^2 \cdot 2x}{(x^2-1)^2} = \dfrac{\cancel{2x^3} - 2x - \cancel{2x^3}}{(x^2-1)^2} = \dfrac{-2x}{(x^2-1)^2}$ $\ +$

3) $f'(x) = 0 \iff x = 0$
$f'(x)$ – NEEX $\iff x = \pm 1$

4) $\oplus\,{-1} \quad 0 \quad \oplus\,1 \quad \longrightarrow x$

5,6)

	$(-\infty,-1)$	-1	$(-1,0)$	0	$(0,1)$	1	$(1,\infty)$
f'	$+$	$*$	$+$		$-$	$*$	$-$
f	$\nearrow$	$*$	$\nearrow$	MAX	$\searrow$	$*$	$\searrow$

PRÍKLAD: VYŠETRIME KONVEXNOSŤ, KONKÁVNOSŤ A INFL. BODY F-CIE

a) $\boxed{f(x) = 2x^3 + 6x^2 + 6x + 5}$

1) $D(f) = (-\infty, \infty)$ $\qquad \longrightarrow x$

2) $f'(x) = 6x^2 + 12x + 6$
$f''(x) = 12x + 12 = 12(x+1)$

3) $f''(x) = 0 \iff x = -1$

4) $-1 \quad \longrightarrow x$

5,6)

	$(-\infty,-1)$	-1	$(-1,\infty)$
f''	$-$		$+$
f	$\cap$	IB	$\cup$

b) $\boxed{f(x) = \dfrac{10x}{(x+2)^2}}$

1) $D(f) = (-\infty, -2) \cup (-2, \infty)$ $\qquad \oplus\,{-2}$

2) $f'(x) = \dfrac{10(x+2)^2 - 10x \cdot 2(x+2) \cdot 1}{(x+2)^4} = \dfrac{\cancel{(x+2)}[10(x+2) - 20x]}{(x+2)^{\cancel{4}}} = \dfrac{10x + 20 - 20x}{(x+2)^3} = \dfrac{20 - 10x}{(x+2)^3}$

$f''(x) = \dfrac{-10(x+2)^3 - (20-10x) \cdot 3(x+2)^2 \cdot 1}{(x+2)^6} = \dfrac{\cancel{(x+2)^2}[-10(x+2) - 3(20-10x)]}{(x+2)^{\cancel{6}}} = \dfrac{-10x - 20 - 60 + 30x}{(x+2)^4} = \dfrac{20x - 80}{(x+2)^4} = \dfrac{20(x-4)}{(x+2)^4}$ $\ +$

3) $f''(x) = 0 \iff x = 4$
$f''(x)$ – NEEX $\iff x = -2$

4) $\oplus\,{-2} \quad 4$

5,6)

	$(-\infty,-2)$	-2	$(-2,4)$	4	$(4,\infty)$
f''	$-$	$*$	$-$		$+$
f	$\cap$	$*$	$\cap$	IB	$\cup$

c) $\boxed{f(x) = x^2 + \dfrac{1}{x}} = \dfrac{x^3+1}{x}$

1) $D(f) = (-\infty, 0) \cup (0, \infty)$ $\qquad \oplus\,0$

2) $f'(x) = \dfrac{3x^2 \cdot x - (x^3+1) \cdot 1}{x^2} = \dfrac{3x^3 - x^3 - 1}{x^2} = \dfrac{2x^3 - 1}{x^2}$

$A^3 + B^3 = (A+B)(A^2 - AB + B^2)$

$f''(x) = \dfrac{6x^2 \cdot x^2 - (2x^3 - 1) \cdot 2x}{x^4} = \dfrac{6x^4 - 4x^4 + 2x}{x^4} = \dfrac{2x^4 + 2x}{x^4} = \dfrac{2\cancel{x}(x^3+1)}{x^{\cancel{4}}} = \dfrac{2(x+1)(x^2 - x + 1)}{x^3}$ $\ +$

3) $f''(x) = 0 \iff x = -1$
$f''(x)$ – NEEX $\iff x = 0$

4) $-1 \quad \oplus\,0$

5,6)

	$(-\infty,-1)$	-1	$(-1,0)$	0	$(0,\infty)$
f''	$+$		$-$	$*$	$+$
f	$\cup$	IB	$\cap$	$*$	$\cup$