Renta odložená, prerušená, nekonečná, so spojitým úrokovaním

streda 25. marca 2026 10:00

ODLOŽENÁ RENTA

PR1

$${}_5A_{10} = 10\,000$$
$$i = 0{,}06$$
$$n = 10$$
$$t = 5$$
$$R = ?$$

$${}_tA_n = (1+i)^{-t} \cdot R \cdot \frac{1-(1+i)^{-n}}{i}$$

$$10\,000 = (1+0{,}06)^{-5} \cdot R \cdot \frac{1-(1+0{,}06)^{-10}}{0{,}06}$$

$$R = 10\,000 \cdot 1{,}06^{5} \cdot \frac{0{,}06}{1-1{,}06^{-10}}$$

$$\underline{R = 1818{,}21977};$$

OTÁZKA: AKÁ BY BOLA VÝŠKA SPLÁTKY, AK BY SME SPLÁCANIE NEODLOŽILI?

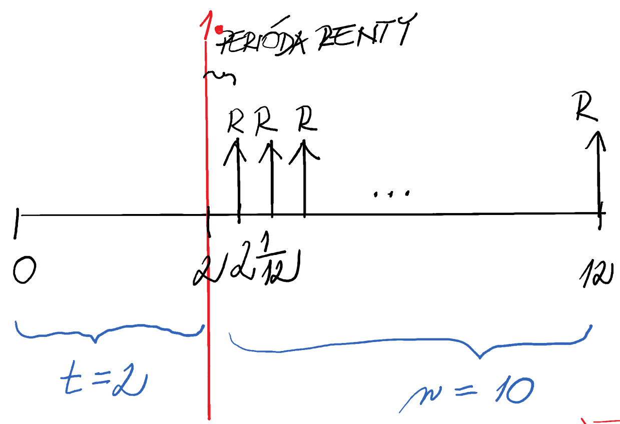
$$\underline{R^* = R \cdot 1{,}06^{-5} = \frac{R}{1{,}06^5} = 1358{,}67958};$$

VÝŠKA SPLÁTKY (R^*)

VÝŠKA SPLÁTKY PRI ODLOŽENEJ RENTE (R)

PR2

$${}_2A_{10} = 3058{,}25217$$
$$j = 0{,}1$$
$$n = 10$$
$$t = 2$$
$$p = 12$$
$$m = 4$$
$$R = ?$$



$${}_tA_n = R\left(1+\frac{j}{m}\right)^{-m \cdot t} \cdot \frac{1-\left(1+\frac{j}{m}\right)^{-mn}}{\left(1+\frac{j}{m}\right)^{\frac{m}{p}}-1}$$

$$3058{,}25217 = R\left(1+\frac{0{,}1}{4}\right)^{-4\cdot 2} \cdot \frac{1-\left(1+\frac{0{,}1}{4}\right)^{-4\cdot 10}}{\left(1+\frac{0{,}1}{4}\right)^{\frac{4}{12}=\frac{1}{3}}-1} \qquad \frac{0{,}1}{4} = 0{,}025$$

$$R = 3058{,}25217 \cdot 1{,}025^{8} \cdot \frac{1{,}025^{\frac{1}{3}}-1}{1-1{,}025^{-40}}$$

$$\underline{R = 49{,}07234}; \qquad \left(R^* = \frac{R}{1{,}025^2}\right)$$

$i \in (0,1)$

VEČNÁ RENTA

$$A_\infty = \lim_{n \to \infty} R \frac{1-(1+i)^{-n}}{i}$$

$$(1+i)^{-n} = \frac{1}{(1+i)^n} \to 0$$

$(1+i) > 1$, $(1+i)^n \to \infty$

$i = 0{,}1$ $\quad \frac{1}{1{,}1^n} \to 0$, $1{,}1^n \to \infty$ $\quad \frac{1}{2^n}$

$$A_\infty = \frac{R}{i}$$

$p > 1$

$$A_\infty = \lim_{n\to\infty} R \frac{1-\left(1+\frac{j}{m}\right)^{-m \cdot n}}{\left(1+\frac{j}{m}\right)^{\frac{m}{p}}-1}$$

$$\frac{1}{\left(1+\frac{j}{m}\right)^{m \cdot \boxed{n}}} \to 0, \quad \left(1+\frac{j}{m}\right)^{m \cdot n} \to \infty$$

$$A_\infty = \frac{R}{\left(1+\frac{j}{m}\right)^{\frac{m}{p}}-1}$$

$$S_\infty = \lim_{n\to\infty} S_n = \lim_{n\to\infty} R \frac{\left(1+\frac{j}{m}\right)^{m \cdot n}-1}{\left(1+\frac{j}{m}\right)^{\frac{m}{p}}-1} = \infty$$

$$S_\infty = \infty$$

RENTA SO SPOJITÝM ÚROKOVANÍM

$$A_n = \lim_{m\to\infty} R \frac{1-\left(1+\frac{j}{m}\right)^{-m \cdot n}}{\left(1+\frac{j}{m}\right)^{\frac{m}{p}}-1} =$$

1^∞

$$\lim_{x\to\infty}\left(1+\frac{k}{x}\right)^x = e^k \quad \checkmark$$

$k = j$
$x = m$

$(a^b)^c = a^{b \cdot c}$ ✓

$$= \lim_{m\to\infty} R \frac{1-\left[\underbrace{\left(1+\frac{j}{m}\right)^m}_{\to e^j}\right]^{-n}}{\left[\underbrace{\left(1+\frac{j}{m}\right)^m}_{\to e^j}\right]^{\frac{1}{p}}-1} =$$

$(e^j)^{-n} = e^{-jn}$

$(e^j)^{\frac{1}{p}} = e^{\frac{j}{p}}$

$$= R \frac{1-(e^j)^{-n}}{(e^j)^{\frac{1}{p}}-1} = R \frac{1-e^{-jn}}{e^{\frac{j}{p}}-1}$$

e^j $\quad \left(1+\frac{j}{m}\right)^{m \cdot n} \quad 1$

$$S_n = \lim_{m\to\infty} R \frac{\left(1+\frac{\delta}{m}\right)^{mn} - 1}{\left(1+\frac{\delta}{m}\right)^{\frac{m}{p}} - 1} =$$

$$= \lim_{m\to\infty} R \frac{\left[\left(1+\frac{\delta}{m}\right)^{m}\right]^{n} - 1}{\left[\left(1+\frac{\delta}{m}\right)^{m}\right]^{\frac{1}{p}} - 1} =$$

$$\left(1+\frac{\delta}{m}\right)^{m} \to e^{\delta}$$

$$= R \frac{\left(e^{\delta}\right)^{n} - 1}{\left(e^{\delta}\right)^{\frac{1}{p}} - 1} = R \frac{e^{\delta n} - 1}{e^{\frac{\delta}{p}} - 1}$$

VEČNÁ RENTA SO SPOJITÝM ÚROKOVANÍM

$$A_\infty = \lim_{n\to\infty} R \frac{1 - e^{-\delta n}}{e^{\frac{\delta}{p}} - 1}, \qquad e^{-\delta n} \to 0$$

$$e^{-\delta n} = \frac{1}{e^{\delta n}} \to 0, \qquad e^{\delta n} \to \infty$$

$e = 2{,}7\ldots$

$e > 1$

$$A_\infty = \frac{R}{e^{\frac{\delta}{p}} - 1}$$

$$S_\infty = \lim_{n\to\infty} R \frac{e^{\delta n} - 1}{e^{\frac{\delta}{p}} - 1} = \infty, \qquad e^{\delta n} \to \infty$$