$2\cos(x\cdot y) - \sin x = 0$
$2x \sin y - 3y \sin x = \boxed{-1}$

$[x_0, y_0] = [1, 1]$
$\varepsilon = 0{,}15$

$2\cos(xy) - \sin x = 0$
$2x \sin y - 3y \sin x + 1 = 0$

$F(x,y) = 0$
$G(x,y) = 0$

$F(x,y) = 2\cos(xy) - \sin x$
$G(x,y) = 2x\cdot\sin y - 3y \sin x + 1$

$F'_x = -2\sin(xy)\cdot y - \cos x$ $\quad F'_y = -2\sin(xy)\cdot x$
$G'_x = 2\sin y - 3y\cdot\cos x$ $\quad G'_y = 2x\cos y - 3\sin x$

$$\left(\begin{array}{cc|c} F'_x & F'_y & -F \\ G'_x & G'_y & -G \end{array}\right)_{(x_k, y_k)}$$

V KALKULAČKE- RADIÁNY!

$$\left(\begin{array}{cc|c} -2\sin(xy)\cdot y - \cos x & -2\sin(xy)\cdot x & -[2\cos(xy) - \sin x] \\ 2\sin y - 3y\cdot\cos x & 2x\cos y - 3\sin x & -[2x\sin y - 3y\sin x + 1] \end{array}\right)_{(x_k, y_k)}$$

$\boxed{x_0 = 1 \quad y_0 = 1}$

$$\left(\begin{array}{cc|c} -2{,}22324 & -1{,}68294 & -0{,}23913 \\ 0{,}062035 & -1{,}44381 & -0{,}158529 \end{array}\right) \qquad \left|\overset{+\quad -}{\times}\right|$$

$$D = \begin{vmatrix} -2{,}22324 & -1{,}68294 \\ 0{,}062035 & -1{,}44381 \end{vmatrix} = 3{,}31434$$

$$D_1 = \begin{vmatrix} -0{,}23913 & -1{,}68294 \\ -0{,}158529 & -1{,}44381 \end{vmatrix} = 0{,}07846$$

$$D_2 = \begin{vmatrix} -2{,}22324 & -0{,}23913 \\ 0{,}062035 & -0{,}158529 \end{vmatrix} = 0{,}36728$$

$$x_1 - x_0 = \frac{D_1}{D} \Rightarrow x_1 = x_0 + \frac{D_1}{D} = 1{,}02367$$

$$y_1 - y_0 = \frac{D_2}{D} \Rightarrow y_1 = y_0 + \frac{D_2}{D} = 1{,}11082$$

STOPTEST: $\underline{|x_{k+1} - x_k| \leq \varepsilon \ \& \ |y_{k+1} - y_k| \leq \varepsilon}$

$|x_1 - x_0| = 0{,}02367 < \varepsilon$ STOP $\quad \underline{\varepsilon = 0{,}15}$
$|y_1 - y_0| = 0{,}11082 < \varepsilon$

približné riešenie je $[x_1, y_1] = [1{,}02368; 1{,}11082]$

2. krok (teraz nie je potrebný) - napr. až $\varepsilon = 0{,}001$
$\varepsilon = 0{,}1$

$$\underline{\left(\begin{array}{c|c} \quad & \quad \end{array}\right)}_{(x_1, y_1)}$$

$x_2 - x_1 = \frac{D_1}{D} \qquad y_2 - y_1 = \frac{D_2}{D}$

NUMERICKÉ RIEŠENIE SÚSTAVY LINEÁRNYCH ROVNÍC

A - štvorcová matica $n \times n$
$\vec{x}$ - stĺpcový vektor neznámych $n \times 1$
$\vec{b}$ - stĺpcový vektor pravých strán $n \times 1$

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix} \quad \vec{x} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} \quad \vec{b} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix}$$

$$A\cdot\vec{x}=\vec{b} \iff \left.\begin{array}{l} a_{11}x_1+a_{12}x_2+\dots+a_{1n}x_n=b_1\\ a_{21}x_1+a_{22}x_2+\dots+a_{2n}x_n=b_2\\ \quad\vdots\\ a_{n1}x_1+a_{n2}x_2+\dots+a_{nn}x_n=b_n\end{array}\right\}(*)$$

sústava n lineárnych rovníc

JACOBIHO ITERAČNÁ METÓDA

Nech matica A je regulárna ($|A|\neq 0$).
Teda sústava (*) má práve 1 riešenie.

Definícia Nech B je matica $n\times m$, $B=(b_{ij})$

Číslo $\|B\|_r=\max\limits_{i}\sum\limits_{j=1}^{m}|b_{ij}|$ nazývame **riadková norma matice B.**

Číslo $\|B\|_s=\max\limits_{j}\sum\limits_{i=1}^{n}|b_{ij}|$ nazývame **stĺpcová norma matice B.**

V ĎALŠOM – uvažujeme riadkové normy

PRÍKLAD a) $B=\begin{pmatrix}1&2&-3\\0&2&-1\\-1&0&4\end{pmatrix}$

$1+2+3=\boxed{6}=\|B\|_r$
$0+2+1=3$
$1+0+4=5$

$2 \quad 4 \quad \boxed{8} = \|B\|_s$

b) $B=\begin{pmatrix}0{,}3\\0{,}7\\-0{,}8\end{pmatrix}$ 0,3 0,7 0,8

$\|B\|_r=0{,}8$

Definícia: Štvorcovú maticu A ($\to n\times n$) nazývame **diagonálne dominantná** práve vtedy, keď je splnená aspoň jedna z nasledujúcich podmienok:

1.) pre všetky $i=1,2,\dots,n$ platí

$$|a_{ii}|>\sum_{\substack{j=1\\ j\neq i}}^{n}|a_{ij}| \qquad \text{(cez riadky)}$$

2.) pre všetky $j=1,2,\dots,n$ platí

$$|a_{jj}|>\sum_{\substack{i=1\\ i\neq j}}^{n}|a_{ij}| \qquad \text{(cez stĺpce)}$$

PRÍKLAD

$$A=\begin{pmatrix}\boxed{7}&2&-3\\0&\boxed{2}&-1\\-1&0&\boxed{-4}\end{pmatrix}$$

$7>2+3$
$2>0+1$
$4>1+0$

JE DIAG. DOMINANTNÁ (CEZ RIADKY)

$7>0+1 \quad 2>2+0 \quad 4>3+1$

NIE JE DIAG. DOMINANTNÁ – CEZ STĹPCE

ZÁVER – MATICA A JE DIAGONÁLNE DOMINANTNÁ

$A\vec{x}=\vec{b} \iff (*)$

Na zabezpečenie diagonálnej dominancie matice A v sústave (*) niekedy nebude stačiť vzájomná výmena riadkov.
Budeme musieť sústavu (*) upraviť pomocou ekvivalentných úprav s maticami (pripočítame nenulový násobok nejakého riadku k inému riadku).

PRÍKLAD Upravme sústavu tak, aby jej matica A bola diagonálne dominantná.

$$\begin{array}{rl} 8x_1+3x_2-x_3=10 & \\ 6x_1-7x_2+x_3=0 & /-R_1\\ -9x_1-3x_2-4x_3=-16 & /+R_1\end{array} \qquad A=\begin{pmatrix}8&3&-1\\6&-7&1\\-9&-3&-4\end{pmatrix}$$

$8>3+1$
$7\not>6+1$
$4\not>9+3$

A – nie je diag. dominantná

$$\begin{array}{l} 8x_1+3x_2-x_3=10\\ -2x_1-10x_2+2x_3=-10\\ -x_1\qquad\quad -5x_3=-6\end{array} \longrightarrow A=\begin{pmatrix}8&3&-1\\-2&-10&2\\-1&0&-5\end{pmatrix}$$

$8>3+1$
$10>2+2$
$5>1+0$

A je diag. dominantná

Majme $n=3$.

(1)
$$\begin{array}{l} a_{11}x+a_{12}y+a_{13}z=b_1\\ a_{21}x+a_{22}y+a_{23}z=b_2\\ a_{31}x+a_{32}y+a_{33}z=b_3\end{array} \iff$$

(1) $A\vec{x}=\vec{b}$

$$A=\begin{pmatrix}a_{11}&a_{12}&a_{13}\\a_{21}&a_{22}&a_{23}\\a_{31}&a_{32}&a_{33}\end{pmatrix}$$

$$\vec{x}=\begin{pmatrix}x\\y\\z\end{pmatrix} \quad \vec{b}=\begin{pmatrix}b_1\\b_2\\b_3\end{pmatrix}$$

(2)
$$\begin{array}{l} x=\alpha_{11}x+\alpha_{12}y+\alpha_{13}z+\beta_1\\ y=\alpha_{21}x+\alpha_{22}y+\alpha_{23}z+\beta_2\\ z=\alpha_{31}x+\alpha_{32}y+\alpha_{33}z+\beta_3\end{array} \iff \vec{x}=\alpha\vec{x}+\beta$$

(3)
$$\begin{array}{l} x^{(k+1)}=\alpha_{11}x^{(k)}+\alpha_{12}y^{(k)}+\alpha_{13}z^{(k)}+\beta_1\\ y^{(k+1)}=\alpha_{21}x^{(k)}+\alpha_{22}y^{(k)}+\alpha_{23}z^{(k)}+\beta_2\\ z^{(k+1)}=\alpha_{31}x^{(k)}+\alpha_{32}y^{(k)}+\alpha_{33}z^{(k)}+\beta_2\end{array} \iff \vec{x}^{(k+1)}=\alpha\vec{x}^{(k)}+\beta$$

(3)

VETA Ak v sústava (2) norma (riadková) matice $\|\alpha\| < 1$, tak iteračný proces (3) konverguje nezávislé od voľby počiatočného priblíženia.

Pozn. Vhodné je $\vec{x}^{(0)} = \beta$

STOPTEST: $\|\vec{x} - \vec{x}^{(k)}\| \le \frac{\|\alpha\|}{1-\|\alpha\|} \cdot \|\vec{x}^{(k)} - \vec{x}^{(k-1)}\| < \varepsilon$

$\vec{x}$ ← presné riešenie; $\vec{x}^{(k)}$ ← približné riešenie

PRÍKLAD:

$$\begin{aligned} x - 3y - 20z &= 9 \\ 3x - 10y + z &= -38 \\ 30x + y + 2z &= 32 \end{aligned}$$

$\varepsilon = 0{,}005$

$$A = \begin{pmatrix} \boxed{1} & -3 & -20 \\ 3 & \boxed{-10} & 1 \\ 30 & 1 & \boxed{2} \end{pmatrix}$$

$$\begin{aligned} 30\boxed{x} + y + 2z &= 32 \\ 3x - 10\boxed{y} + z &= -38 \\ x - 3y - 20\boxed{z} &= 9 \end{aligned}$$

$$A = \begin{pmatrix} \boxed{30} & 1 & 2 \\ 3 & \boxed{-10} & 1 \\ 1 & -3 & \boxed{-20} \end{pmatrix}$$

$30 > 1+2$

$10 > 3+1$

$20 > 1+3$

A je diagonálne dominantná

$$\begin{aligned} 30\boxed{x} &= -y - 2z + 32 && \mid :30 \\ -10\boxed{y} &= -3x - z - 38 && \mid :(-10) \\ -20\boxed{z} &= -x + 3y + 9 && \mid :(-20) \end{aligned}$$

$$\left\{\begin{aligned} x^{(k+1)} &= -\frac{1}{30}y^{(k)} - \frac{1}{15}z^{(k)} + \frac{16}{15} \\ y^{(k+1)} &= 0{,}3x^{(k)} + 0{,}1z^{(k)} + 3{,}8 \\ z^{(k+1)} &= 0{,}05x^{(k)} - 0{,}15y^{(k)} - 0{,}45 \end{aligned}\right. \quad (3)$$

$$\alpha = \begin{pmatrix} 0 & -\frac{1}{30} & -\frac{1}{15} \\ 0{,}3 & 0 & 0{,}1 \\ 0{,}05 & -0{,}15 & 0 \end{pmatrix}$$

$$\|\alpha\| = \max\left\{\frac{1}{30} + \frac{1}{15};\ 0{,}3 + 0{,}1;\ 0{,}05 + 0{,}15\right\} = \underline{\underline{0{,}4}}$$

$\|\alpha\| < 1 \rightarrow (3)$ konverguje

$$\beta = \begin{pmatrix} \frac{16}{15} \\ 3{,}8 \\ -0{,}45 \end{pmatrix} \qquad \frac{16}{15} = 1{,}06667$$

nech $x^{(0)} = 1{,}06667$

$y^{(0)} = 3{,}8$

$z^{(0)} = -0{,}45$

MODE – MATICE $\boxed{AC}$

SHIFT + 4 → DIM → MAT A $\boxed{AC}$

SHIF + 4 → DIM → MAT B $\boxed{AC}$

$$A = \begin{pmatrix} 0 & -\frac{1}{30} & -\frac{1}{15} \\ 0{,}3 & 0 & 0{,}1 \\ 0{,}05 & -0{,}15 & 0 \end{pmatrix}$$

$$B = \begin{pmatrix} \frac{32}{30} \\ 3{,}8 \\ -0{,}45 \end{pmatrix}$$

k	$x^{(k)}$	$y^{(k)}$	$z^{(k)}$
0	1,06667	3,8	−0,45
1	0,97	4,075	−0,96667
2	0,99528	3,99433	−1,01275
3	1,00104	3,99731	−0,99939
4	1,00005	4,00037	−0,99954

MATA × MATB + MATB = MAT ANS

$\text{MAT A} \times \underbrace{\text{MAT ANS}}_{\text{ďalšie približ.}} + \text{MAT B}$

$$\|\vec{x}^{(4)} - \vec{x}^{(3)}\| = \left\|\begin{pmatrix} -0{,}00099 \\ 0{,}00306 \\ -0{,}00015 \end{pmatrix}\right\| = 0{,}00306$$

$\varepsilon = 0{,}005$

$\|\alpha\| = 0{,}4$

$$\frac{\|\alpha\|}{1-\|\alpha\|}\|\vec{x}^{(4)} - \vec{x}^{(3)}\| = 0{,}00204 < \varepsilon \Rightarrow \text{STOP}$$

približné rieš. je $\begin{pmatrix} 1{,}00005 \\ 4{,}00037 \\ -0{,}99954 \end{pmatrix}$

ZRÝCHLENÁ JACOBIHO ITERAČNÁ METÓDA

$$\begin{aligned} x^{(k+1)} &= \alpha_{11}x^{(k)} + \alpha_{12}y^{(k)} + \alpha_{13}z^{(k)} + \beta_1 \\ y^{(k+1)} &= \alpha_{21}x^{(k+1)} + \alpha_{22}y^{(k)} + \alpha_{23}z^{(k)} + \beta_2 \\ z^{(k+1)} &= \alpha_{31}x^{(k+1)} + \alpha_{32}y^{(k+1)} + \alpha_{33}z^{(k)} + \beta_3 \end{aligned}$$

ak kalkulačka je bez MODE – MATICE

– počítame s využitím pamäte

PRÍKLAD

$$\begin{aligned} -x - 2y + 8z &= 6{,}6 \\ 10x + y - 2z &= 18{,}1 \\ 3y + 9z &= 12{,}3 && \mid :3 \end{aligned}$$

$\varepsilon = 0{,}05$

$$\begin{aligned} 10x + y - 2z &= 18{,}1 \\ -x - 2y + 8z &= 6{,}6 && \mid -3R_3 \\ y + 3z &= 4{,}1 \end{aligned}$$

$$\begin{aligned} \underline{10}x + y - 2z &= 18{,}1 \\ -x - \underline{5}y - z &= -5{,}7 \\ y + \underline{3}z &= 4{,}1 \end{aligned}$$

$10 > 1+2$

$5 > 1+1$

$3 > 0+1$

A je diagonálne dominantná

$$\begin{aligned} 10x &= -y + 2z + 18{,}1 && \mid :10 \\ -5y &= x + z - 5{,}7 && \mid :(-5) \\ 3z &= -y + 4{,}1 && \mid :3 \end{aligned}$$

$$\left\{\begin{array}{l} x^{(k+1)} = -0,1y^{(k)} + 0,2z^{(k)} + 1,81 \\ y^{(k+1)} = -0,2x^{(k+1)} - 0,2z^{(k)} + 1,14 \\ z^{(k+1)} = -0,333y^{(k+1)} + 1,3667 \end{array}\right.$$

$$\alpha = \begin{pmatrix} 0 & -0,1 & 0,2 \\ -0,2 & 0 & -0,2 \\ 0 & -0,333 & 0 \end{pmatrix} \begin{array}{l} 0,3 \\ 0,4 \\ 0,333 \end{array}$$

$\|\alpha\| = 0,4 < 1 \Rightarrow$ konverg.

TAB PRE ZRÝCHLENÚ JACOBIHO M.

k	$x^{(k)}$	$y^{(k)}$	$z^{(k)}$
0	1,81	1,14	1,3667
1	1,96934	0,47279	1,20912
2	2,00455	0,49727	1,20111
3	2,000495	0,49968	1,20031

← zrýchlená

$\|\vec{x}^{(3)} - \vec{x}^{(2)}\| = \left\|\begin{pmatrix} -0,004055 \\ 0,00241 \\ -0,0008 \end{pmatrix}\right\| = 0,004055$

1,81 (SHIFT)(RCL) (STO) (−)(A) → uloží 1,81 do A

(ALPHA) (−) (A) + 1 = →

1,14 B

1,3667 C

ZAKAŽDÝM JE V PAMATI NOVÁ HODNOTA

→ zrýchlená jacobiho m.

STOPTEST: $\|\vec{x} - \vec{x}^{(3)}\| \leq \frac{\|\alpha\|}{1-\|\alpha\|} \cdot \|\vec{x}^{(3)} - \vec{x}^{(2)}\| =$

$= \frac{0,4}{1-0,4} \cdot 0,004055 = 0,0027 < \varepsilon$

Riešenie s presnosťou ε je $\begin{pmatrix} 2,000495 \\ 0,49968 \\ 1,20031 \end{pmatrix}$.

PRÍKLAD:

$$\begin{array}{ll} 5x - 4y + 3z = -140 & \mid + R_2 \quad 5 \not> 4+3 \\ x + 4y - 2z = -63 & 4 > 1+2 \\ 3x + 5y + z = -85 & \mid -2R_2 \quad 1 \not> 3+5 \end{array}$$

3 kroky jacobiho iter. m.

$$\begin{array}{ll} \textcircled{6}x + z = -203 & 6 > 1 \\ x + \textcircled{4}y - 2z = -63 & 4 > 1+2 \\ x - 3y + \textcircled{5}z = 41 & 5 > 1+3 \end{array}$$

A je diagonálne dominant.

$$\begin{array}{ll} 6x = -z - 203 & \mid : 6 \\ 4y = -x + 2z - 63 & \mid : 4 \\ 5z = -x + 3y + 41 & \mid : 5 \end{array}$$

$$\begin{array}{l} x^{(k+1)} = -0,16667z^{(k)} - 33,83333 \\ y^{(k+1)} = -0,25x^{(k)} + 0,5z^{(k)} - 15,75 \\ z^{(k+1)} = -0,2x^{(k)} + 0,6y^{(k)} + 8,2 \end{array}$$

$\|\alpha\| = \max\{0,16667; 0,75; 0,8\} = 0,8 < 1$

k	$x^{(k)}$	$y^{(k)}$	$z^{(k)}$
0	−33,83333	−15,75	8,2
1	−35,20002	−3,19167	5,51667
2	−34,75279	−4,19166	13,325
3	−36,05421	−0,3993	12,63556

DOMÁCA ÚLOHA

JACOBIHO ITERAČNOU METÓDOU RIEŠTE SÚSTAVY ROVNÍC S PRESNOSŤOU $0,05 = \varepsilon$

a) $21x - 2y - 3z = 1$
$19x + 18y - 2z = 4$
$3x - 2y - 33z = 227$

b) $-2x + y - 15z = 15$
$20x + 2y + z = 11$
$2x + 21y - z = 23$

Newtonovou metódou riešte sústavu rovníc s presnosťou $0,02 = \varepsilon$ z bodu $[x_0, y_0]$

a) $2\cos(xy) = 1$
$2\sin(x+y) = 1$
$[x_0, y_0] = [2,5; 0,25]$

b) $(x-1,2)^2 + (y-0,6)^2 = 1$
$4,2x^2 + 8,8y^2 = 1,42$
$[x_0, y_0] = [0,4; 0,2]$

c) $x^3 - xy^2 - 1 = 0$
$y^3 - 2x^2y + 2 = 0$
$[x_0, y_0] = [-1,5; 2]$