RANNÁ ROZCVIČKA :)

$$-\int \frac{-1 \cdot 5}{x \sin^2(3-\ln x)}\,dx = \left| \begin{array}{l} 3-\ln x = t \\ -\frac{1}{x}dx = dt \end{array} \right| = -5\int \frac{1}{\sin^2 t}\,dt = -5\cdot(-\cotg x)+C =$$

$$= 5\cotg t + C = 5\cotg(3-\ln x)+C$$

INTEGROVANIE METÓDOU PER PARTES

- INTEGROVANIE PO ČASTIACH
- POUŽÍVA SA PRE INTEGROVANIE SÚČINU NIEKTORÝCH F-CIÍ

JEDNODUCHÉ ODVODENIE : DERIVÁCIU SÚČINU POZNÁME, ROBÍMO OPAK ⇒ INTEGRUJEME ZDERIVOVANÝ SÚČIN

$$[u(x)\cdot v(x)]' = u'(x).v(x) + u(x).v'(x) \quad / \text{ ZINTEGRUJEM}$$

$$\int [u(x).v(x)]'\,dx = \int u'(x).v(x)\,dx + \int u(x).v'(x)\,dx$$

$$u(x).v(x) = \int u'(x).v(x)\,dx + \int u(x).v'(x)\,dx \quad \Rightarrow \quad \int u(x).v'(x)\,dx = u(x).v(x) - \int u'(x).v(x)\,dx$$

VETA: NECH F-CIE $u(x)$; $v(x)$ MAJÚ SPOJITÉ DERIVÁCIE NA INTERVALE J. POTOM NA INTERVALE J PLATÍ $\int u.v'\,dx = u.v - \int u'.v\,dx$

KEDY URČITE POUŽIJEME PER PARTES:

$$\int P_n(x).e^{kx}\,dx$$
$$\int P_n(x).\cos kx\,dx$$
$$\int P_n(x).\sin kx\,dx$$
$P_n(x) \to u(x)$, the other factor $\to v'(x)$

$$\int P_n(x).\log_a x\,dx$$
$P_n(x) \to v'(x)$, $\log_a x \to u(x)$

PRÍKLAD: VYPOČÍTAJME $\qquad u.v - \int u'.v\,dx$

1) $$\int (4x-7).e^{-3x}\,dx = \left| \begin{array}{ll} u = 4x-7 \xrightarrow{\text{DERIV.}} & u' = 4 \\ v' = e^{-3x} \xrightarrow{\text{INTEGR.}} & v = \frac{e^{-3x}}{-3} \end{array} \right| = (4x-7)\cdot\left(\frac{e^{-3x}}{-3}\right) - \int 4\cdot\left(\frac{e^{-3x}}{-3}\right)dx =$$

$$= (4x-7).\left(\frac{e^{-3x}}{-3}\right) + \frac{4}{3}\int e^{-3x}\,dx = (4x-7).\left(\frac{e^{-3x}}{-3}\right) + \frac{4}{3}\left(\frac{e^{-3x}}{-3}\right) + C = \dots$$

2) $$\int (x^2+4x-5).e^{2x}\,dx = \left| \begin{array}{ll} u = x^2+4x-5 & u' = 2x+4 \\ v' = e^{2x} & v = \frac{e^{2x}}{2} \\ \hline u'.v = (2x+4).\frac{e^{2x}}{2} = (x+2).e^{2x} & \end{array} \right| = (x^2+4x-5)\frac{e^{2x}}{2} - \int (2x+4).\frac{e^{2x}}{2}\,dx =$$

$(x+2)$

$$= (x^2+4x-5).\frac{e^{2x}}{2} - \int (x+2).e^{2x}\,dx = \left| \begin{array}{ll} u = x+2 & u' = 1 \\ v' = e^{2x} & v = \frac{e^{2x}}{2} \\ \hline u'.v = \frac{1}{2}e^{2x} & \end{array} \right| = (x^2+4x-5).\frac{e^{2x}}{2} - \left[(x+2).\frac{e^{2x}}{2} - \frac{1}{2}\int e^{2x}\,dx\right] =$$

$$= (x^2+4x-5).\frac{e^{2x}}{2} - (x+2).\frac{e^{2x}}{2} + \frac{1}{2}\cdot\frac{e^{2x}}{2} + C$$

3) $$\int (3-2x).\cos x\,dx = \left| \begin{array}{ll} u = 3-2x & u' = -2 \\ v' = \cos x & v = \sin x \\ \hline u'.v = -2\sin x & \end{array} \right| = (3-2x)\sin x - (-2)\int \sin x\,dx = (3-2x)\sin x + 2(-\cos x) + C$$

$+2$

4) $$\int (2x+1).\sin\frac{x}{3}\,dx = \left| \begin{array}{ll} u = 2x+1 & u' = 2 \\ v' = \sin\frac{x}{3} & v = \frac{-\cos\frac{x}{3}}{\frac{1}{3}} = -3\cos\frac{x}{3} \\ \hline u.v = (2x+1).(-3\cos\frac{x}{3}) = -3(2x+1)\cos\frac{x}{3} & \\ u'.v = 2.(-3\cos\frac{x}{3}) = -6\cos\frac{x}{3} & \end{array} \right| = -3(2x+1)\cos\frac{x}{3} + 6\int \cos\frac{x}{3}\,dx =$$

$$= -3(2x+1)\cos\frac{x}{3} + 6.\frac{\sin\frac{x}{3}}{\frac{1}{3}} + C$$

5) $$\int 10x^2.\cos 5x\,dx = \left| \begin{array}{ll} u = 10x^2 & u' = 20x \\ v' = \cos 5x & v = \frac{\sin 5x}{5} \end{array} \right| = 2x^2\sin 5x - 4\int x\sin 5x\,dx = \left| \begin{array}{ll} u = x & u' = 1 \\ v' = \sin 5x & v = \frac{-\cos 5x}{5} \end{array} \right| =$$

$$= 2x^2\sin x - 4\left[-\frac{x\cos 5x}{5} + \frac{1}{5}\int \cos 5x\,dx\right] = 2x^2\sin x + \frac{4x\cos 5x}{5} - \frac{4}{5}\cdot\frac{\sin 5x}{5} + C$$

$$u.v = v.u \Rightarrow (\ln x).\left(\frac{3x^2}{2}-x\right) = \left(\frac{3x^2}{2}-x\right).\ln x$$

6) $$\int (3x-1).\ln x\,dx = \left| \begin{array}{ll} u = \ln x & u' = \frac{1}{x} \\ v' = 3x-1 & v = \frac{3x^2}{2}-x \\ \hline u'.v = \frac{1}{x}\left(\frac{3x^2}{2}-x\right) = \frac{3}{2}x-1 & \end{array} \right| = \left(\frac{3x^2}{2}-x\right).\ln x - \int\left(\frac{3}{2}x-1\right)dx =$$

$$= \left(\frac{3x^2}{2}-x\right).\ln x - \left(\frac{3}{2}.\frac{x^2}{2}-x\right) + C$$

7) $$\int (x^2+2)\ln x\,dx = \left| \begin{array}{ll} u = \ln x & u' = \frac{1}{x} \\ v' = x^2+2 & v = \frac{x^3}{3}+2x \\ \hline u'.v = \frac{1}{x}\left(\frac{x^3}{3}+2x\right) = \frac{x^2}{3}+2 & \end{array} \right| = \left(\frac{x^3}{3}+2x\right).\ln x - \int\left(\frac{x^2}{3}+2\right)dx =$$

$$= \left(\frac{x^3}{3}+2x\right)\ln x - \left(\frac{x^3}{9}+2x\right) + C$$

8) $$\int x.\log_2 x\,dx = \left| \begin{array}{ll} u = \log_2 x & u' = \frac{1}{x.\ln 2} \\ v' = x & v = \frac{x^2}{2} \\ \hline u'.v = \frac{1}{x\ln 2}.\frac{x^2}{2} = \frac{1}{2\ln 2}.x & \end{array} \right| = \frac{x^2}{2}\log_2 x - \frac{1}{2\ln 2}\int x\,dx = \frac{x^2}{2}\log_2 x - \frac{1}{2\ln 2}.\frac{x^2}{2} + C$$

9) SUBST. / PER PARTES

$$\frac{1}{3}\int 3x^2.\ln(x^3-1)\,dx = \left| \begin{array}{l} x^3-1 = t \\ 3x^2dx = dt \end{array} \right| = \frac{1}{3}\int 1.\ln t\,dt = \left| \begin{array}{ll} u = \ln t & u' = \frac{1}{t} \\ v' = 1 & v = t \\ \hline u'.v = \frac{1}{t}.t = 1 & \end{array} \right| =$$

$$= \frac{1}{3}\left[t.\ln t - \int 1\,dt\right] = \frac{1}{3}\left[t\ln t - t\right] + C = \frac{1}{3}\left[(x^3-1).\ln(x^3-1) - (x^3-1)\right] + C$$

10) $$\int \frac{\ln x}{x^5}\,dx = \int \frac{1}{x^5}.\ln x\,dx = \left| \begin{array}{ll} u = \ln x & u' = \frac{1}{x} \\ v' = \frac{1}{x^5} = x^{-5} & v = \frac{x^{-4}}{-4} = \frac{-1}{4x^4} \\ \hline u'.v = \frac{1}{x}.\left(\frac{-1}{4x^4}\right) = \frac{-1}{4}.\frac{1}{x^5} = \frac{-1}{4}x^{-5} & \end{array} \right| =$$

$$= \frac{-1}{4x^4}\ln x + \frac{1}{4}\int x^{-5}\,dx = \frac{-1}{4x^4}\ln x + \frac{1}{4}.\left(\frac{x^{-4}}{-4}\right) + C = \frac{-\ln x}{4x^4} - \frac{1}{16x^4} + C$$