NÁVRAT K URČITÝ INTEGRÁL

- def urč. int. $\int_a^b f(x)\,dx$ sú 2 predpoklady

1. $\langle a,b\rangle$ KONEČNÝ

2. f je ohr. na $\langle a,b\rangle$

AK NIEKTORÁ PODMIENKA NEPLATÍ $=$

Nevl. int. II

$\lim_{x\to c^+} |f(x)| = \infty$

$f(c) = \pm\infty$

Nevlastný integrál I

$\langle a,\infty)$ (resp. $(-\infty,b\rangle$ resp. $(-\infty,\infty)$)

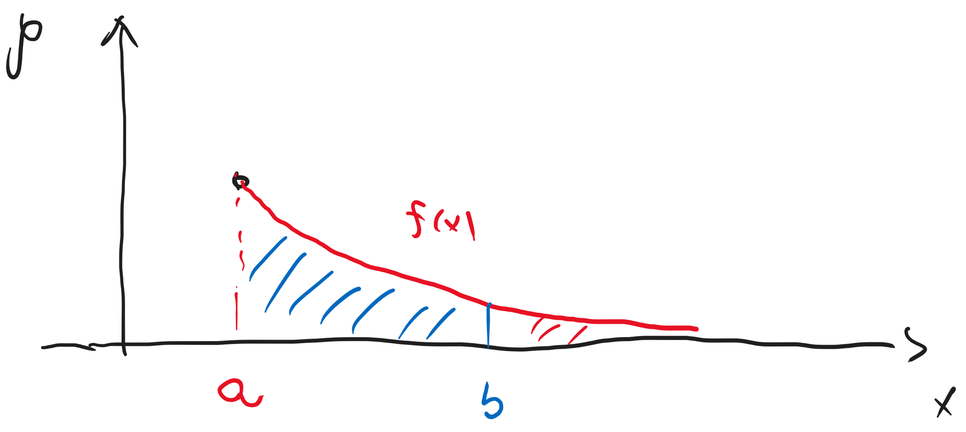
Máme f def na $\langle a,\infty)$ a integrovateľnú na $\langle a,b\rangle$, pre každé $b>a$

AKO DEFINOVAŤ

KRÁTKA

$$\boxed{\int_a^\infty f(x)\,dx = \lim_{b\to\infty}\int_a^b f(x)\,dx}$$

geometricky plocha medzi grafom f a osou x



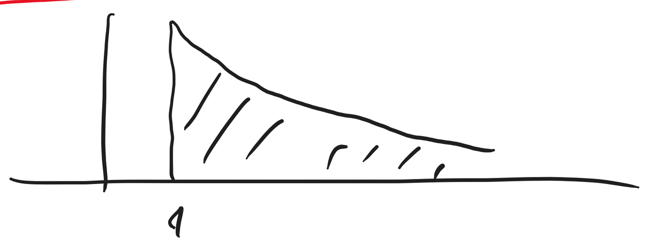
DLHÁ DEF

1. nech f je def na $\langle a,\infty)$
2. nech f je integr. na $\langle a,b\rangle$, pre $\forall b>a$
3. nech existuje vlastná

$$\lim_{b\to\infty}\int_a^b f(x)\,dx$$

Potom $$\int_a^\infty f(x)\,dx = \lim_{b\to\infty}\int_a^b f(x)\,dx$$

Pr. Vyp. $\int_1^\infty \frac{1}{x^2}\,dx$



dlhá verzia
$$\int_1^\infty \frac{1}{x^2}\,dx = \lim_{b\to\infty}\int_1^b \frac{1}{x^2}\,dx = \lim_{b\to\infty}\left(-\frac{1}{x}\Big|_1^b\right) = \lim_{b\to\infty}\left(-\frac{1}{b}+1\right) = 1$$

$$\int_a^\infty f(x)\,dx = \lim_{b\to\infty}\int_a^b f(x)\,dx = \lim_{b\to\infty}\Big(F(b)-F(a)\Big) = F(\infty)-F(a)$$

$$\int_a^\infty f(x)\,dx = F(x)\Big|_a^\infty$$

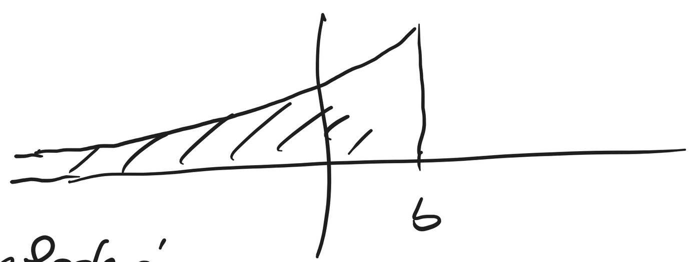
VÝPOČET LIMITY

$$\int_1^\infty \frac{1}{x^2}\,dx = -\frac{1}{x}\Big|_1^\infty = 1$$

Pre pokročilých

ANALOGICKY
$$\int_{-\infty}^b f(x)\,dx = \lim_{c\to-\infty}\int_c^b f(x)\,dx$$

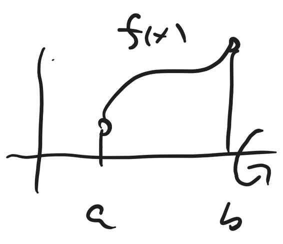
ak táto limita je vlastná KONVERG.



Potom
$$\int_{-\infty}^\infty f(x)\,dx = \int_{-\infty}^a f(x)\,dx + \int_a^\infty f(x)\,dx$$

ak oba integrály KONVERGUJÚ

HLAVOLAM



$V = \pi\int_a^b f^2(x)\,dx$ objem

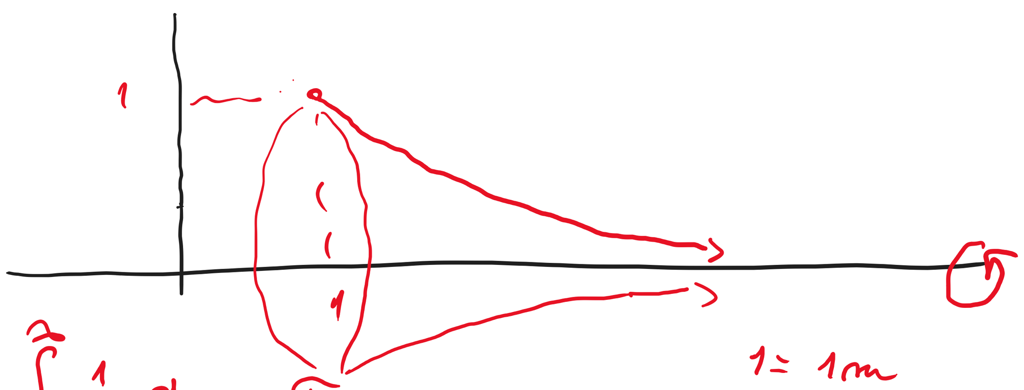
$S = 2\pi\int_a^b f(x)\sqrt{1+(f'(x))^2}\,dx$ povrch

platia aj pre nevlastné integrály $(b=\infty)$

TORRICELLIHO TRÚBKA

GABRIELOVA ROH

$f(x) = \frac{1}{x}$ na $\langle 1,\infty)$



rotujeme $\frac{1}{x}$ okolo osi x

$$V = \pi\int_1^\infty f^2(x)\,dx = \pi\int_1^\infty \frac{1}{x^2}\,dx = \pi$$

$1 = 1\,m$

$V = \pi\,l = 3{,}14\,l$

KONEČNÝ OBJEM

$$P = 2\pi\int_1^\infty \frac{1}{x}\sqrt{1+\left(\frac{-1}{x^2}\right)^2}\,dx > 2\pi\int_1^\infty \frac{1}{x}\,dx = 2\pi\ln x\Big|_1^\infty = \infty$$

$1+\frac{1}{x^4}\ge 1$

NEKONEČNÝ POVRCH

? PARADOX ?

MAĽUJEME