URČITÝ INTEGRÁL

Vieme

1° derivácia $F(x) = \sin x$ ... $F'(x) = \cos x = f(x)$

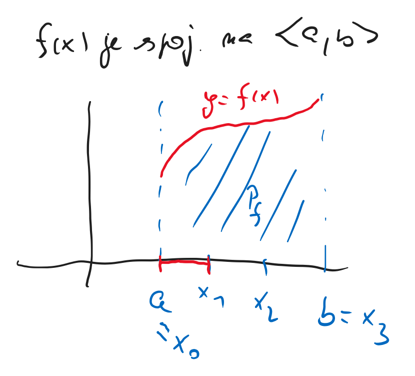
2° PF PF k $\cos x$ je $\sin x$ } FCIE

3° Neurč. int. $\int f(x)\,dx = F(x) + C$

4° URČITÝ INT. $\int_a^b f(x)\,dx \longrightarrow$ číslo

Motivácia = Aplikácia + Def.

$f(x)$ je spoj. na $\langle a,b\rangle$



Úloha nájsť plochu pod grafom $f(x)$ na $\langle a,b\rangle$

Postup [4 KROKY]

1. Delenie

mn. $D = \{x_0, x_1, \dots, x_{p_n}\}$ sa nazýva delenie $\langle a,b\rangle$ ak

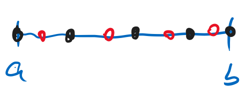
$a = x_0 < x_1 < x_2 \dots < x_{p_n} = b$

$\Delta x_1 = x_1 - x_0$

$\Delta x_i = x_i - x_{i-1}$ – dĺžka i-teho podintervalu

norma $\|D\| = \max \Delta x_i$ dĺžka najväčšieho podintervalu

Uvažuj postupnosť $\{D_n\}_{n=1}^{\infty}$ → zahusťujem delenie (pridávam deliace body)



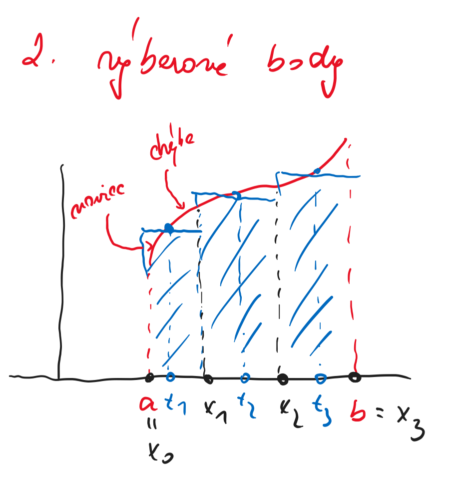
D_1 čierne body

D_2 červené body + čierne body

Požadujeme aby $\|D_n\| \to 0$ pre $n \to \infty$

geom. $[x_{i-1}, x_i] \xrightarrow{n\to\infty}$ • bod

2. výberové body $t_i \in \langle x_{i-1}, x_i\rangle$

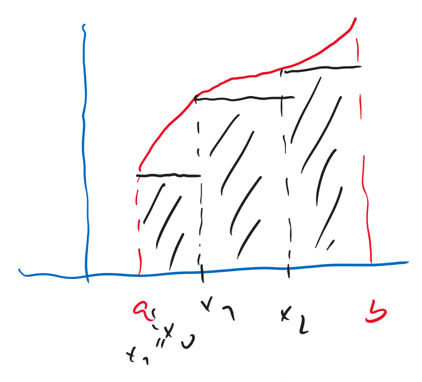


3. integrálny súčet = APROXIMÁCIA ÚLOHY

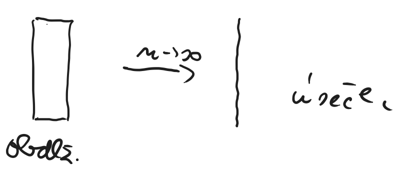
zostrojíme obdĺžniky jedna strana $f(t_i)$ $(f(x) > 0)$
druhá strana Δx_i

$P_f \approx \sum_{i=1}^{p_n} f(t_i)\Delta x_i$ (súčet obsahov obdĺžnikov)

Pre rastúcu fciu zober $t_i = x_{i-1}$ výberový ľavý konc. bod intervalu



chyba pri aproximácii je značná
ale pre $n \to \infty$ chyba → 0



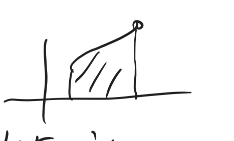
4. definícia integrálu

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{p_n} f(t_i)\Delta x_i = P_f$$

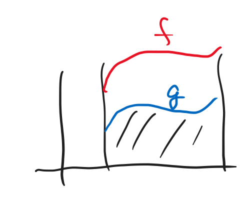
definícia prostr. apl.

ZÁKL. VLASTNOSTI

V1 ak $f(x) \geq 0$, tak $\int_a^b f(x)\,dx \geq 0$
↳ plocha nemôže byť záporná

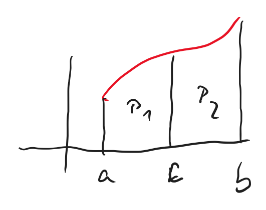


V2



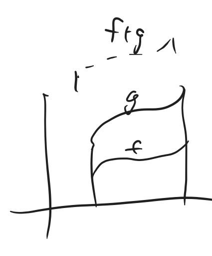
ak $f(x) \geq g(x)$ na $\langle a,b\rangle$, tak $\int_a^b f(x)\,dx \geq \int_a^b g(x)\,dx$

V3.



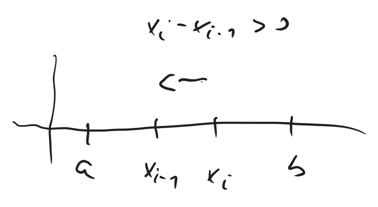
$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$
$P = P_1 + P_2$

✓ V4: $\int_a^b (f(x) + g(x))\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$



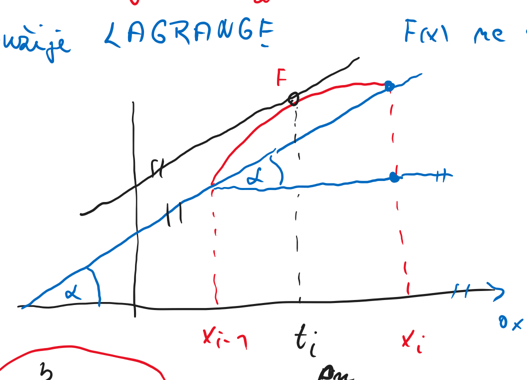
✓ V5: $\int_a^b \alpha f(x)\,dx = \alpha \int_a^b f(x)\,dx$

V6: $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$



Výpočet $\int_a^b f(x)\,dx$

Použije LAGRANGE $F(x)$ na $\langle x_{i-1}, x_i\rangle$



smernica sečnice

$k_s = \operatorname{tg}\alpha = \frac{F(x_i) - F(x_{i-1})}{\Delta x_i} = F'(t_i)$

$F(x_i) - F(x_{i-1}) = F'(t_i)\Delta x_i$

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{p_n} f(t_i)\Delta x_i = \left| \begin{matrix} \text{TRIK} \\ F \text{ je PF k } f \\ \text{t.j. } F' = f \end{matrix} \right| = \lim_{n\to\infty} \sum_{i=1}^{p_n} F'(t_i)\Delta x_i =$$

int. príst. x_i – delíce
t_i – výberové sú presne tie, ktoré dáva LAGR. veta

$x_0 = a$

$$= \lim_{n\to\infty} \sum_{i=1}^{p_n} (F(x_i) - F(x_{i-1})) = \lim_{n\to\infty} \left(\begin{matrix} F(x_1) - F(x_0) \\ F(x_2) - F(x_1) \\ F(x_3) - F(x_2) \\ \vdots \\ F(x_{p_n}) - F(x_{p_n - 1}) \end{matrix} \right) = \lim_{n\to\infty} (F(b) - F(a))$$

rozpíš

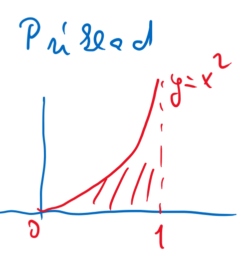
$x_{p_n} = b$

$= F(b) - F(a)$
$= F(x)\big|_a^b$
diferencia

Veta o výpočte.
Nech $f(x)$ je spoj. na $\langle a,b\rangle$. Nech $F(x)$ je PF k $f(x)$. Potom

$$\int_a^b f(x)\,dx = F(x)\Big|_a^b \quad (F(b) - F(a))$$

Príklad

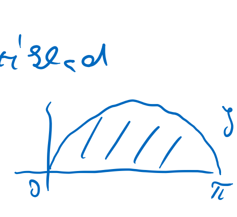


$P_f = \int_0^1 x^2\,dx = \frac{x^3}{3}\Big|_0^1 = 1/3$

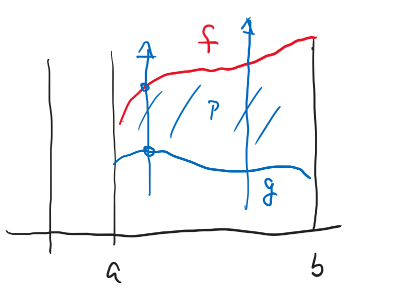
Výpočet U.I. vedie na hľadanie PF t.j. $\int f(x)\,dx$

$\int_0^1 x^2\,dx = \left(\frac{x^3}{3} + c\right)\Big|_0^1 = \left(\frac{1}{3} + c\right) - \left(\frac{0}{3} + c\right) = 1/3$

Príklad



$P_f = \int_0^{\pi} \sin x\,dx = -\cos x\Big|_0^{\pi} = 2$

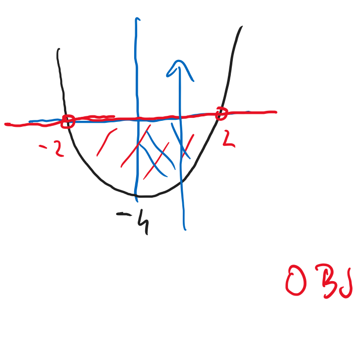


$P = P_f - P_g = \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$

$P = \int_a^b (f(x) - g(x))\,dx$ platí všeob. $f(x) \geq g(x)$

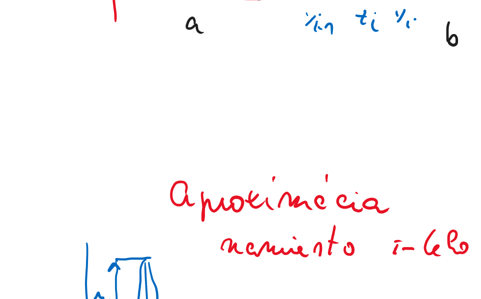
Horná f Dolná f

Pr. $y = x^2 - 4$; $y = 0$

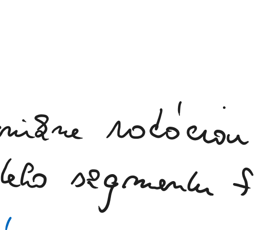


$P = \int_{-2}^{2} HF - DF\,dx = 2\int_0^2 0 - (x^2 - 4)\,dx = 2\left[-\frac{x^3}{3} + 4x\right]_0^2 = \dots$

OBJEM ROTAČNÉHO TELESA (okolo osi x)



$f(x) > 0$ na $\langle a,b\rangle$
rotuje okolo osi x
vznikne teleso ? V = ?



$\{D_n\}$ post. delení ako pri def $\int_a^b$

$V = \sum_{i=1}^{p_n} V_i$ V_i – objem, vznikne rotáciou i-teho segmentu f

Aproximácia
namiesto i-teho segmentu necháme rotovať úsečku čím vznikne valec

$V_i \approx V(\text{valca}) = \pi r^2 v = \pi f^2(t_i)\Delta x_i$

$V \approx \sum_{i=1}^{p_n} \pi f^2(t_i)\Delta x_i$

$$V = \lim_{n\to\infty} \sum_{i=1}^{p_n} \pi f^2(t_i)\Delta x = \int_a^b \pi f^2(x)\,dx = \pi \int_a^b f^2(x)\,dx$$