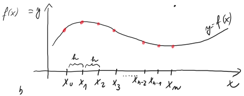
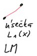
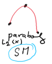
SIMPSONOVA METÓDA







3 body

⇓

K vytváraniu trojíc potrebujeme párny počet intervalov

⇓

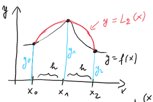
m – PÁRNE

$$\int_a^b f(x)\,dx$$

FUNKCIU $f(x)$ na intervale $\langle x_0, x_2\rangle$ $\langle x_2, x_4\rangle \dots \langle x_{n-2}, x_n\rangle$ aproximujeme časťami parabol (t.j. $L_2(x)$.) Paraboly prechádzajú 3 bodmi x_0, x_1, x_2 $\quad x_2, x_3, x_4$ $\quad \dots$

$$\int_a^b f(x)\,dx = \int_{x_0}^{x_m} f(x)\,dx = \int_{x_0}^{x_2} f(x)\,dx + \int_{x_2}^{x_4} f(x)\,dx + \dots + \int_{x_{n-2}}^{x_n} f(x)\,dx$$

Majme interval $\langle x_0, x_2\rangle$



x_i	x_0	x_1	x_2
y_i	y_0	y_1	y_2

$$f(x) \approx L_2(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}y_0 + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}y_1 + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}y_2 =$$

$$= \frac{(x-x_1)(x-x_2)}{-h\cdot(-2h)}y_0 + \frac{(x-x_0)(x-x_2)}{h\cdot(-h)}y_1 + \frac{(x-x_0)(x-x_1)}{2h\cdot h}y_2$$

Položme: $\boxed{\frac{x-x_0}{h} = t}$

$\frac{1}{h}dx = dt$

$dx = h\,dt$

$x = x_0 \Rightarrow t = 0$

$x = x_2 \Rightarrow t - 2$

} ZMENA HRANÍC INTEGRÁLU

$$L_2(x) = \frac{1}{2}(t-1)(t-2)\,y_0 - t(t-2)\,y_1 + \frac{1}{2}t(t-1)\,y_2$$

$$\underbrace{\int_{x_0}^{x_2} f(x)\,dx}_{} \approx \int_0^2 \left[\frac{1}{2}\underbrace{(t-1)(t-2)}_{t^2-3t+2}\underset{\substack{\downarrow\\ \text{konšt}}}{y_0} - \underbrace{t(t-2)}_{t^2-2t}\underset{\substack{\downarrow\\ \text{konšt}}}{y_1} + \frac{1}{2}\underbrace{t(t-1)}_{t^2-t}\underset{\substack{\downarrow\\ \text{konšt}}}{y_2}\right]\underset{\substack{\downarrow\\ \text{konšt}}}{h}\,dt =$$

$$\int t^n dt = \frac{t^{n+1}}{n+1}$$

$$= h\left[\frac{1}{2}y_0\left(\frac{t^3}{3} - 3\frac{t^2}{2} - 2t\right) - y_1\left(\frac{t^3}{3} - \frac{2t^2}{2}\right) + \frac{1}{2}y_2\left(\frac{t^3}{3} - \frac{t^2}{2}\right)\right]_0^2$$

$$= \dots = h\left(\frac{y_0}{3} + \frac{4}{3}y_1 + \frac{y_2}{3}\right) = \underbrace{\frac{h}{3}(y_0 + 4y_1 + y_2)}_{}$$

$$\int_{x_0}^{x_2} f(x)\,dx \approx \frac{h}{3}(y_0 + 4y_1 + y_2)$$

$$\int_{x_2}^{x_4} f(x)\,dx \approx \frac{h}{3}(y_2 + 4y_3 + y_4)$$

$$\vdots$$

$$\int_{x_{m-4}}^{x_{m-2}} f(x)\,dx \approx \frac{h}{3}(y_{m-4} + 4y_{m-3} + y_{m-2})$$

$$\int_{x_{m-2}}^{x_m} f(x)\,dx \approx \frac{h}{3}(y_{m-2} + 4y_{m-1} + y_m)$$

$$\int_{x_0}^{x_m} f(x) \approx \frac{h}{3}\left(y_0 + 4y_1 + y_2 + y_2 + 4y_3 + y_4 + \dots + y_{m-4} + 4y_{n-3} + y_{m-2} + y_{m-2} + 4y_{m-1} + y_m\right) =$$

$$= \frac{h}{3}\left(y_0 + y_m + 2y_2 + 2y_4 + \dots + 2y_{m-2} + 4y_1 + 4y_3 + \dots + 4y_{m-3} + 4y_{m-1}\right)$$

$$= \frac{h}{3}\left[y_0 + y_m + 2(y_2 + y_4 + y_6 + \dots + y_{m-2}) + 4(y_1 + y_3 + y_5 + \dots y_{m-1})\right]$$

n_páre

párne indexy

$y_k = f(x_k)$

nepárne indexy

$$\int_{x_0}^{x_m} f(x)\,dx = \frac{h}{3}\left[y_0 + y_n + \textcircled{2}(y_{\textcircled{2}} + y_4 + \dots + y_{n-2}) + 4(y_1 + y_3 + \dots y_{n-1})\right] + |R_S|$$

AKEJ CHYBY SME SA DOPUSTILI ?

$$|R_S| \le \frac{(b-a)^5 \cdot M_4}{180 \cdot n^4} \le \varepsilon \quad \Rightarrow \quad \frac{(b-a)^5 M_4}{180 \cdot \varepsilon} \le n^4$$

$$n \ge \sqrt[4]{\frac{(b-a)^5 M_4}{180 \cdot \varepsilon}}$$

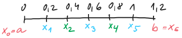
$$M_4 \ge \max_{x \in \langle a, b\rangle} |f^{(IV)}(x)|$$

PRÍKLAD

Simpsonovou metódou vypočítajme $\int_0^{1,2} \ln(1+x)\,dx$ pre $n=6$ a odhadnime chybu, ktorej sme sa dopustili.

$$n = 6 \Rightarrow h = \frac{b-a}{n}$$

$$h = \frac{1,2}{6} = 0,2$$



$$\boxed{f(x) = \ln(1+x)}$$

$$\int_0^{1,2} \ln(1+x)\,dx = \frac{h}{3}\left[f(x_0) + f(x_6) + 2\left(f(x_2) + f(x_4)\right) + 4\left(f(x_1) + f(x_3) + f(x_5)\right)\right] + |R_S|$$

$$= \frac{0,2}{3}\left[\ln(1+0) + \ln(1+1,2) + 2\underbrace{\left(\ln(1+0,4) + \ln(1+0,8)\right)}_{0,92426} + 4\underbrace{\left(\ln(1+0,2) + \ln(1+0,6) + \ln(1+1)\right)}_{1,34547}\right] + |R_S|$$

$$= 0,53459 + |R_S|$$

$$|R_S| = ?$$

$$f'(x) = \frac{1}{1+x} = (1+x)^{-1}$$

$$f''(x) = -(1+x)^{-2}$$

$$f'''(x) = 2(1+x)^{-3}$$

$$f^{(IV)}(x) = -6(1+x)^{-4} = \frac{-6}{(1+x)^4}$$

$$\max_{x \in \langle a, b\rangle} |f^{(IV)}(x)| = \max_{x \in \langle 0; 1,2\rangle} \left|\frac{-6}{\underset{\text{najmenšie ak } x=0}{(1+x)^4}}\right| = \underline{\underline{6}}$$

$$M_4 = 6$$

$$|R_S| \le \frac{(1,2-0)^5 \cdot 6}{180 \cdot 6^4} = \underline{\underline{0,000064}} \approx \varepsilon$$

PRÍKLAD

Simpsonovou metódou vypočítajme $\int_0^1 e^{x^2}\,dx$, $\varepsilon = 0,05$.

$$f(x) = e^{x^2}$$

$$f'(x) = e^{x^2} \cdot 2x$$

$$f''(x) = e^{x^2} \cdot 2x \cdot 2x + e^{x^2} \cdot 2 = e^{x^2}(4x^2 + 2)$$

$$f'''(x) = e^{x^2} \cdot 2x\,(4x^2 + 2) + e^{x^2}(8x) = e^{x^2}(8x^3 + 4x + 8x) =$$

$$= e^{x^2}(8x^3 + 12x)$$

$$f^{(IV)}(x) = e^{x^2} \cdot 2x\,(8x^3 + 12x) + e^{x^2}(24x^2 + 12) =$$

$$= e^{x^2}(16x^4 + 24x^2 + 24x^2 + 12) = e^{x^2}(16x^4 + 48x^2 + 12)$$

$$\max_{\substack{x\in\langle 0;1\rangle \\ \text{nezáporné}}} \left|e^{x^2}\left(16x^4+48x^2+12\right)\right| = e^{1^2}\left(16.1^4+48.1^2+12\right)=76e=M_4$$

$|a.b|=|a||b|$

$$n \geq \sqrt[4]{\frac{(b-a)^5 M_4}{180.\varepsilon}} \qquad n \geq \sqrt[4]{\frac{1^5 . 76e}{180.0{,}05}} \Rightarrow n \geq 2{,}19$$

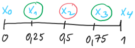
& n-párne

$$\Rightarrow \boxed{n=4}$$

$$\Downarrow$$

$$h=\frac{b-a}{n}$$

$$h=\frac{1}{4}=0{,}25$$



$$\int_0^1 e^{x^2}dx \approx \frac{h}{3}\left[y_0+y_4+2.y_2+4\left(y_1+y_3\right)\right]$$

$$=\frac{0{,}25}{3}\left[e^{0^2}+e^{1^2}+2.e^{0{,}5^2}+4\left(e^{0{,}25^2}+e^{0{,}75^2}\right)\right]=$$

$$=\underline{\underline{1{,}46371}}$$

PRÍKLAD

$$\int_0^1 \cos x^2 dx \qquad \varepsilon=0{,}001$$

$f(x)=\cos x^2$

$f'(x)=-\sin x^2 . 2x$

$\vdots$

$f^{(IV)}(x)=\left(16x^4-12\right)\cos x^2+48x^2 . \sin x^2$

$$\max_{x\in\langle 0,1\rangle} \left|\left(16x^4-12\right)\cos x^2+48x^2 . \sin x^2\right| \leq$$

$$\leq \max_{x\in\langle 0,1\rangle}\left(\left|\underbrace{\left(16x^4-12\right)}_{\leq|16x^4|+|12|,\ x=1} . \overset{x=0}{\cos x^2}\right|+\left|48\overset{x=1}{x^2}\sin \overset{x=1}{x^2}\right|\right) \leq (16.1+12)\cos 0 +$$

$$+48.1.\sin 1=$$

$$=68{,}39$$

Nech $M_4=68{,}4$

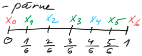
$$n \geq \sqrt[4]{\frac{(b-a)^5.M_4}{180.\varepsilon}}$$

$$n \geq \sqrt[4]{\frac{68{,}4}{180.0{,}001}}$$

$n \geq 4{,}41$ & n-párne

Nech $n=6$

$\Downarrow$ $h=\frac{1}{6}$



$$\int_0^1 \cos x^2 dx \approx \frac{h}{3}\left[y_0+y_6+2\left(y_2+y_4\right)+4\left(y_1+y_3+y_5\right)\right]=$$

$$=\frac{\frac{1}{6}}{3}\left[\cos 0^2+\cos 1^2+2\left(\overbrace{\cos\left(\tfrac{1}{3}\right)^2+\cos\left(\tfrac{2}{3}\right)^2}^{1{,}51056}\right)+4\left(\overbrace{\cos\left(\tfrac{1}{6}\right)^2+\cos\left(\tfrac{1}{2}\right)^2+\cos\left(\tfrac{5}{6}\right)^2}^{2{,}19476}\right)\right]=\ldots$$

$\frac{1}{18}$

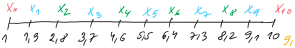
PRÍKLAD

$$\int_1^{10}\sqrt{x}\,dx \qquad n=10 \qquad + \text{ ODHAD CHYBY}$$

$$h=\frac{10-1}{10}=0{,}9$$

$$f(x)=x^{\frac{1}{2}} \ldots\ldots f^{(IV)}(x)=-\frac{15}{16}\frac{1}{\sqrt{x^7}}$$

$$\max_{x\in\langle 1,10\rangle}\left|-\frac{15}{16}\frac{1}{\underset{x=1}{\sqrt{x^7}}}\right|=\underline{\underline{\frac{15}{16}=M_4}}$$



$$\int_1^{10}\sqrt{x}\,dx=\frac{0{,}9}{3}\left[\sqrt{1}+\sqrt{10}+2\overbrace{\left(\sqrt{2{,}8}+\sqrt{4{,}6}+\sqrt{6{,}4}+\sqrt{8{,}2}\right)}^{9{,}21146}+\right.$$

$$\left.+4\underbrace{\left(\sqrt{1{,}9}+\sqrt{3{,}7}+\sqrt{5{,}5}+\sqrt{7{,}3}+\sqrt{9{,}1}\right)}_{11{,}36562}\right]$$

$$=20{,}41431+|R_S|$$

$$|R_S| \leq \frac{(b-a)^5.M_4}{180n^4}=\frac{9^5.\frac{15}{16}}{180.10^4}=\underline{\underline{0{,}03075}}\approx\varepsilon$$

POZNÁMKA

$$\max_{x\in\langle a,b\rangle}\frac{☁}{□}$$

Budeme dosadzovať za x
tie hodnoty z intervalu $\langle a,b\rangle$,
pre ktoré je ☁ čo najväčšie
a zároveň □ je čo najmenšie

$$\max_{x\in\langle a,b\rangle}\left(☁ . □\right)$$

☁ - najväčší
□ - najväčší

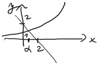
PRÍKLADY MOŽNÝCH ÚLOH NA ZÁPOČTOVEJ PÍSOMKE

①. Graficky odseparujte všetky korene, určte a overte intervaly separácie pre rovnicu $e^x + x - 2 = 0$.

$e^x = 2 - x$

$y = 2 - x$

x	0	2
y	2	0



1 koreň → 1 interval separácie

$f(x) = e^x + x - 2$

$\left.\begin{matrix} f(0) < 0 \\ f(1) > 0 \end{matrix}\right\} \alpha \in \langle 0, 1\rangle$

②. Pre rovnicu $e^x + x - 2 = 0$ overte na intervale $\langle 0; 0{,}5\rangle$ podmienky konvergencie Newtonovej metódy a z krajných bodov intervalu $\langle 0; 0{,}5\rangle$ vyberte vhodný štartovací bod.

$\alpha \in \langle 0; 0{,}5\rangle$ $\qquad |f''(x)| > 0$ pre $x \in \langle 0; 0{,}5\rangle$

$f(x) = e^x + x - 2$

$f'(x) = e^x + 1$

$f''(x) = e^x > 0$ pre ľubovoľné $x \in \mathbb{R}$

Teda $|f''(x)| > 0$

$x_0 : \; f(x_0) \cdot \underset{+}{f''(x_0)} > 0$ $\qquad f(0) < 0$

$f(0{,}5) > 0$

$x_0 = 0{,}5$

③. Overte, že $x_k = 0{,}93023$ je riešenie rovnice $\ln(x+2) - \frac{1}{x} = 0$ s presnosťou $\varepsilon = 0{,}001$

$f(x_k - \varepsilon) \cdot f(x_k + \varepsilon) \overset{?}{<} 0$ $\qquad f(x) = \ln(x+2) - \frac{1}{x}$

$f(\underset{\ominus}{0{,}92923}) \cdot f(\underset{\oplus}{0{,}93123}) < 0$ $\qquad \alpha = 0{,}93023 \pm 0{,}001$

④. Newtonovou metódou s presnosťou $\varepsilon = 10^{-2} = 0{,}01$ riešme rovnicu $\underbrace{e^x + x - 2}_{= f(x)} = 0$. Začnite v bode $x_0 = 0{,}8$.

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$\boxed{x_{n+1} = x_n - \frac{e^{x_n} + x_n - 2}{e^{x_n} + 1}}$$

ITERAČNÝ PROCES

$x_0 = 0{,}8$

$x_1 = 0{,}48206$

$x_2 = 0{,}44332$

$x_3 = 0{,}44285$

STOPTEST

$f(x_3 - \varepsilon) \cdot f(x_3 + \varepsilon) = f(\underset{\ominus}{0{,}43285}) \cdot f(\underset{\oplus}{0{,}45285}) < 0$

$\Downarrow$

$\alpha = 0{,}44285 \pm \varepsilon$

⑤. Iteračnou metódou s presnosťou $\varepsilon = 10^{-2}$ riešte rovnicu $e^{\boxed{x}} + \boxed{x} - 2 = 0$. Začnite v bode $x_0 = 0{,}8$.

$e^x = 2 - x$ $\qquad x = \underbrace{2 - e^x}_{\varphi(x)}$

$x = \underbrace{\ln(2 - x)}_{\varphi(x)}$

$\boxed{x_{n+1} = 2 - e^{x_n}}$ iteračný proces

$\boxed{x_{n+1} = \ln(2 - x_n)}$

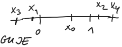
$x_0 = 0{,}8$

$x_1 = -0{,}2255$

$x_2 = 1{,}2019$ ....

$x_3 = -1{,}326$

$x_4 = 1{,}73$



DIVERGUJE

$x_0 = 0{,}8$

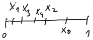
$x_1 = 0{,}18232$

$x_2 = 0{,}59756$

$x_3 = 0{,}3382$

$x_4 = 0{,}50789$

$x_5 = 0{,}400189$



$x_6 = 0{,}469885$

$x_7 = 0{,}42534$

$x_8 = 0{,}45404$

$x_9 = 0{,}435646$

$x_{10} = 0{,}44747$

$x_{11} = 0{,}43988$

$x_{12} = 0{,}44476$

$x_{13} = 0{,}44163$

STOPTEST

$f(x_{13} - \varepsilon) \cdot f(x_{13} + \varepsilon) = f(\underset{\ominus}{0{,}43163}) \cdot f(\underset{\oplus}{0{,}45163}) < 0$

$\Rightarrow \alpha = x_{13} \pm \varepsilon$

⑥. Určme riadkovú normu matice $A = \begin{pmatrix} 0{,}8 & -1 & -0{,}3 \\ -0{,}4 & 1{,}2 & -1{,}1 \\ 1{,}5 & -0{,}2 & 0 \end{pmatrix}$

$$\|A\|_r = \max\{0,8+1+0,3\,;\ 0,4+1,2+1,1\,;\ 1,5+0,2+0\} =$$
$$= \max\{2,1;\ 2,7;\ 1,7\} = \underline{\underline{2,7}}$$

⑦ Overte, že matica B je riadkovo diagonalne dominantná, ak

$$B = \begin{pmatrix} \boxed{-1,2} & 0,5 & -0,6 \\ -1,2 & \boxed{1,8} & 0,1 \\ -0,1 & -0,2 & \boxed{-0,8} \end{pmatrix}$$

$|-1,2| > |0,5| + |-0,6|$ $\qquad 1,2 > 1,1$

$|1,8| > |-1,2| + |0,1|$ $\qquad 1,8 > 1,3$

$|-0,8| > |-0,1| + |-0,2|$ $\qquad 0,8 > 0,3$

⑧ Pre sústavu rovníc vytvorte pomocou Jacobiho metódy konvergentný iteračný proces a zo štartovacieho bodu $[0,0,0]$ urobte 2 iterácie

$-x - y + 8z = 9$ III.

$10x + y + z = -9$ I.

$x + 8y - z = -2$ II.

$A \cdot \vec{x} = \vec{b}$ $\qquad$ A – musí byť diagonálne dominantná

$$\begin{pmatrix} \boxed{-1} & -1 & 8 \\ 10 & \boxed{1} & 1 \\ 0 & 8 & \boxed{-1} \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 9 \\ -9 \\ -2 \end{pmatrix}$$

$$\left.\begin{aligned} 10\underline{x} + y + z &= -9 \\ x + 8\underline{y} - z &= -2 \\ -x - y + 8\underline{z} &= 9 \end{aligned}\right\} \Rightarrow \quad A = \begin{pmatrix} 10 & 1 & 1 \\ 1 & 8 & -1 \\ -1 & -1 & 8 \end{pmatrix} \quad \begin{aligned} 10 &> 1+1 \\ 8 &> 1+1 \\ 8 &> 1+1 \end{aligned}$$

$$\Rightarrow \left.\begin{aligned} x &= \frac{1}{10}(-y - z - 9) \\ y &= \frac{1}{8}(-x + z - 2) \\ z &= \frac{1}{8}(x + y + 9) \end{aligned}\right\} \Rightarrow \boxed{\begin{aligned} x_{k+1} &= -0,1y_k - 0,1z_k - 0,9 \\ y_{k+1} &= -0,125x_k + 0,125z_k - 0,25 \\ z_{k+1} &= 0,125x_k + 0,125y_k + 1,125 \end{aligned}}$$

A je diagonálne dominantná

iteračný proces

k	x_k	y_k	z_k
0	0	0	0
1	−0,9	−0,25	1,125
2	−0,9875	0,003125	0,98125

$x_2 = \dots$

$y_2 = -0,125 \cdot x_1 + 0,125 \cdot 1,125 - 0,25$

$z_2 = \dots$

⑨ Pre sústavu rovníc s iteračným procesom

$$x_{k+1} = -0,1y_k - 0,1z_k - 0,9$$
$$y_{k+1} = -0,125x_k + 0,125z_k - 0,25$$
$$z_{k+1} = 0,125x_k + 0,125y_k + 1,125$$

zo štartovacieho bodu $[0,0,0]$ urobte 2 iterácie a odhadnite chybu, ktorej sme sa dopustili

$$\vec{x}_{k+1} = \alpha\,\vec{x}_k + \beta \qquad \alpha = \begin{pmatrix} 0 & -0,1 & -0,1 \\ -0,125 & 0 & 0,125 \\ 0,125 & 0,125 & 0 \end{pmatrix}$$

$$\|\alpha\|_r = \max\{0,2;\ 0,25;\ 0,25\} = \underline{\underline{0,25}}$$

k	x_k	y_k	z_k
0	0	0	0
1	−0,9	−0,125	1,125
2	−0,9875	0,003125	0,98125

ODHAD CHYBY:

mový vzorec

$$\|\vec{x}_2 - \vec{x}^*\| \le \frac{\|\alpha\|_r}{1-\|\alpha\|_r} \cdot \|\vec{x}_2 - \vec{x}_1\| = \frac{0,25}{0,75} \cdot 0,253125 =$$
$$= 0,084375$$

PRESNÉ RIEŠENIE

$$\vec{x}_2 = \begin{pmatrix} -0,9875 \\ 0,003125 \\ 0,98125 \end{pmatrix} \qquad \vec{x}_1 = \begin{pmatrix} -0,9 \\ -0,25 \\ 1,125 \end{pmatrix} \qquad \|\vec{x}_2 - \vec{x}_1\|_r = \left\|\begin{pmatrix} -0,0875 \\ 0,253125 \\ -0,14375 \end{pmatrix}\right\|_r = 0,253125$$

⑩ Riešme sústavu rovníc s iteračným procesom

$$x_{k+1} = -0,1y_k - 0,1z_k - 0,9$$
$$y_{k+1} = -0,125x_k + 0,125z_k - 0,25 \qquad \text{s presnosťou } \varepsilon = 0,1$$
$$z_{k+1} = 0,125x_k + 0,125y_k + 1,125$$

$[x_0, y_0, z_0] = [0,0,0]$

KEDY KONČÍ VÝPOČET ?

Ak $\dfrac{\|\alpha\|_r}{1-\|\alpha\|_r} \cdot \|\vec{x}_{k+1} - \vec{x}_k\|_r < \varepsilon$

$k=1$ $\qquad \underbrace{\qquad}_{0{,}084375} < 0{,}1$

riešenie s presnosťou $\varepsilon = 0{,}1$ je $\vec{x}_2 = \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix} = \begin{pmatrix} \\ \end{pmatrix}$

⑪ Newtonovou metódou s presnosťou $\varepsilon = 0{,}25$ riešme sústavu rovníc
$$x^2+4x-y^2-2y-1=0$$
$$x^2+5y-4=0$$
zo štartovacieho bodu $[\underset{x_0}{0{,}5}; \underset{y_0}{0{,}5}]$

$F(x,y) = x^2+4x-y^2-2y-1$

$G(x,y) = x^2+5y-4$

$$\left(\begin{array}{cc|c} F'_x & F'_y & -F \\ G'_x & G'_y & -G \end{array}\right) \qquad \left(\begin{array}{cc|c} 2x+4 & -2y-2 & -(x^2+4y-y^2-2y-1) \\ 2x & 5 & -(x^2+5y-4) \end{array}\right)$$

dosadíme za x, y hodnoty x_0, y_0 $\rightarrow \left(\begin{array}{cc|c} 5 & -3 & 0 \\ 1 & 5 & -\frac{5}{4} \end{array}\right)$ $\quad D = \begin{vmatrix} 5 & -3 \\ 1 & 5 \end{vmatrix} = 28$

$D_1 = \begin{vmatrix} 0 & -3 \\ -\frac{5}{4} & 5 \end{vmatrix} = \frac{15}{4}$

$x_1 = x_0 + \frac{D_1}{D} = 0{,}63393$

$D_2 = \begin{vmatrix} 5 & 0 \\ 1 & -\frac{5}{4} \end{vmatrix} = \frac{25}{4}$

$y_1 = y_0 + \frac{D_2}{D} = 0{,}723214$

$|x_1 - x_0| = 0{,}13395 < \varepsilon$

$|y_1 - y_0| = 0{,}223214 < \varepsilon$

STOP

približné riešenie je $[0{,}63393; 0{,}723214]$

⑫ Vypočítajme $\int_0^{0,5} \sin x^2 \, dx$.

Použite LM s počtom krokov $n=5$

(+odhad chyby.)

⑬ Vypočítajte $\int_0^{0,4} \sin x^2 \, dx$

Použite SM s počtom krokov $n=8$

(+ odhad chyby)

⑭ S akým najmenším n musíte počítať $\int_0^{0,5} \sin x^2 \, dx$ lich. m., aby ste zabezpečili presnosť $\varepsilon = 10^{-3}$

⑮ S akým najmenším n musíte počítať $\int_2^3 \frac{1}{1+x} \, dx$ simpsonovou m., aby ste zabezpečili presnosť $\varepsilon = 10^{-8}$

⑯ $f(x)$

x_i	-1	1	2
y_i	3	1	3

$f(x) \approx L_2(x)$

⑰ $f(x)$

x_i	-1	0	2	4
y_i	-2	$-0{,}5$	$5{,}4$	$3{,}5$

$f(1) \approx L_3(1)$

$x = 1$

⑱ Aproximácia pomocou MNŠ