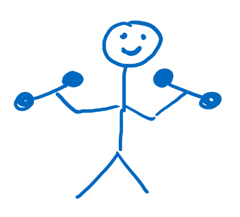
RANNÁ ROZCVIČKA



PRÍKLAD: VYPOČÍTAJME

1) $\int_1^2 (6x^2-3)\,dx = \left[6\frac{x^3}{3} - 3x\right]_1^2 = (2\cdot 2^3 - 3\cdot 2) - (2\cdot 1^3 - 3\cdot 1) = (16-6)-(2-3) = \underline{\underline{11}}$

$(2x^3-3x)$

2) $\int_0^1 \frac{10^x + 8^x}{2^x}\,dx = \int_0^1 \left(\frac{10^x}{2^x} + \frac{8^x}{2^x}\right)dx = \int_0^1 \left[\left(\frac{10}{2}\right)^x + \left(\frac{8}{2}\right)^x\right]dx = \int_0^1 (5^x + 4^x)\,dx =$

$= \left[\frac{5^x}{\ln 5} + \frac{4^x}{\ln 4}\right]_0^1 = \left(\frac{5^1}{\ln 5} + \frac{4^1}{\ln 4}\right) - \left(\frac{5^0}{\ln 5} + \frac{4^0}{\ln 4}\right) = \underline{\underline{\frac{5}{\ln 5} + \frac{4}{\ln 4} - \frac{1}{\ln 5} - \frac{1}{\ln 4}}}$

VLASTNOSTI URČITÉHO INTEGRÁLU

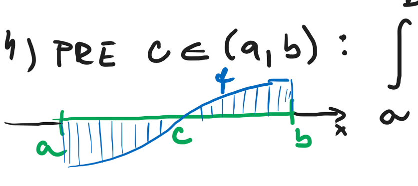
VETA: NECH $a<b$; $f(x)$ JE INTEGROVATEĽNÁ NA $\langle a,b\rangle$, POTOM PLATÍ:

1) $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

2) $\int_a^a f(x)\,dx = 0$

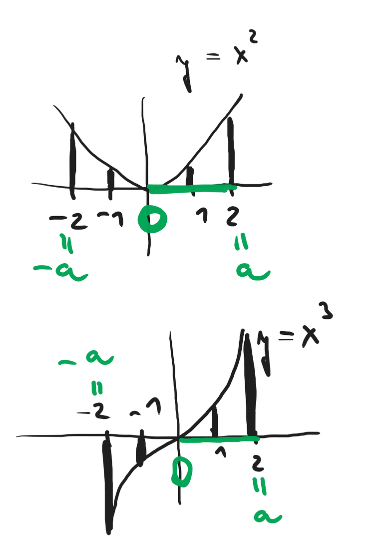
3) PRE $k_1, k_2 \in R$: $\int_a^b [k_1\cdot f(x) + k_2\cdot g(x)]\,dx = k_1\int_a^b f(x)\,dx + k_2\int_a^b g(x)\,dx$

4) PRE $c \in (a,b)$: $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$



5) PRE $f(x)$-PÁRNU NA $\langle -a,a\rangle$: $\int_{-a}^a f(x)\,dx = 2\cdot\int_0^a f(x)\,dx$

6) PRE $f(x)$-NEPÁRNU NA $\langle -a,a\rangle$: $\int_{-a}^a f(x)\,dx = 0$



SUBSTITUČNÁ METÓDA

VETA: NECH $\varphi: \langle a,b\rangle \to \langle \alpha,\beta\rangle$ MÁ SPOJITÚ DERIVÁCIU. NECH $F(t)$ JE PRIMITÍVNA K $f(t)$ NA $\langle\alpha,\beta\rangle$. POTOM FCIA $F[\varphi(x)]$ JE PRIMITÍVNA K $f[\varphi(x)]\cdot\varphi'(x)$ NA $\langle a,b\rangle$ A PLATÍ

$$\int_a^b f[\varphi(x)]\cdot\varphi'(x)\,dx = \left|\begin{array}{l}\varphi(x)=t\\ \varphi'(x)\,dx = dt\\ a\to\varphi(a)=\alpha\\ b\to\varphi(b)=\beta\end{array}\right| = \int_\alpha^\beta f(t)\,dt$$

PRÍKLAD: VYPOČÍTAJME

1) $\frac{1}{3}\int_0^1 3x^2\cdot e^{x^3+4}\,dx = \left|\begin{array}{l}x^3+4=t\\ 3x^2\,dx = dt\\ 0\to 0^3+4=4\\ 1\to 1^3+4=5\end{array}\right| = \frac{1}{3}\int_4^5 e^t\,dt = \frac{1}{3}\left[e^t\right]_4^5 = \frac{1}{3}e^5 - \frac{1}{3}e^4 = \underline{\underline{\frac{1}{3}(e^5-e^4)}}$

2) $-\frac{1}{3}\int_0^{\pi/2} \frac{-3\sin x}{5+3\cos x}\,dx = \left|\begin{array}{l}5+3\cos x = t\\ -3\sin x\,dx = dt\\ 0\to 5+3\cos 0 = 8\\ \frac{\pi}{2}\to 5+3\cos\frac{\pi}{2} = 5\end{array}\right| = -\frac{1}{3}\int_8^5 \frac{1}{t}\,dt = +\frac{1}{3}\int_5^8 \frac{1}{t}\,dt = \frac{1}{3}\left[\ln|t|\right]_5^8 =$

$= \underline{\underline{\frac{1}{3}(\ln 8 - \ln 5)}} = \frac{1}{3}\ln\frac{8}{5}$

3) $\int_0^{\pi/2} (3 - 2\sin x + 3\sin^2 x)\cdot\cos x\,dx = \left|\begin{array}{l}\sin x = t\\ \cos x\,dx = dt\\ 0\to\sin 0 = 0\\ \frac{\pi}{2}\to\sin\frac{\pi}{2} = 1\end{array}\right| = \int_0^1 (3-2t+3t^2)\,dt =$

$= \left[3t - \cancel{2}\frac{t^2}{\cancel{2}} + \cancel{3}\frac{t^3}{\cancel{3}}\right]_0^1 = \left[3t - t^2 + t^3\right]_0^1 = (3\cdot 1 - 1^2 + 1^3) - (3\cdot 0 - 0^2 + 0^3) = \underline{\underline{3}}$

4) $\int_1^e \frac{\sqrt{2+\ln x}}{x}\,dx = \left|\begin{array}{l}2+\ln x = t\\ \frac{1}{x}\,dx = dt\\ 1\to 2+\ln 1 = 2\\ e\to 2+\ln e = 3\end{array}\right| = \int_2^3 \sqrt{t}\,dt = \int_2^3 t^{\frac{1}{2}}\,dt = \left[\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right]_2^3 =$

$= \left[\frac{2}{3}\sqrt{t^3}\right]_2^3 = \underline{\underline{\frac{2}{3}\left(\sqrt{27}-\sqrt{8}\right)}}$

METÓDA PER PARTES

VETA: NECH FCIE $u(x)$ a $v(x)$ MAJÚ NA $\langle a,b\rangle$ SPOJITÉ DERIVÁCIE.
POTOM PLATÍ

$$\int_a^b u\cdot v'\,dx = \left[u\cdot v\right]_a^b - \int_a^b u'\cdot v\,dx$$

PRÍKLAD: VYPOČÍTAJME

1) $\int_1^e 6x^5\cdot\ln x\,dx = \left|\begin{array}{l}u = \ln x \xrightarrow{\text{DERIV.}} u' = \frac{1}{x}\\ v' = 6x^5 \xrightarrow{\text{INTEGR.}} v = \cancel{6}\frac{x^6}{\cancel{6}} = x^6\\ u'\cdot v = \frac{1}{x}\cdot x^6 = x^5\end{array}\right| = \left[x^6\cdot\ln x\right]_1^e - \int_1^e x^5\,dx =$

$= \left[x^6\ln x - \frac{x^6}{6}\right]_1^e = \left(e^6\underset{1}{\ln e} - \frac{e^6}{6}\right) - \left(1^6\underset{0}{\ln 1} - \frac{1^6}{6}\right) = e^6 - \frac{e^6}{6} + \frac{1}{6} = \frac{5}{6}e^6 + \frac{1}{6}$

2) $\int_1^2 (3x-1)\cdot e^x\,dx = \left|\begin{array}{ll}u = 3x-1 & u' = 3\\ v' = e^x & v = e^x\end{array}\right| = \left[(3x-1)\cdot e^x\right]_1^2 - 3\int_1^2 e^x\,dx =$

$= \left[(3x-1)e^x - 3e^x\right]_1^2 = (5\cdot e^2 - 3\cdot e^2) - (2\cdot e - 3\cdot e) = \underline{\underline{2e^2 + e}}$

3) $\int_0^{\pi/2} (2x+3)\cdot\cos 3x\,dx = \left|\begin{array}{ll}u = 2x+3 & u' = 2\\ v' = \cos 3x & v = \frac{\sin 3x}{3}\end{array}\right| =$

$= \left[(2x+3)\cdot\frac{\sin 3x}{3}\right]_0^{\frac{\pi}{2}} - \frac{2}{3}\int_0^{\frac{\pi}{2}} \sin 3x\,dx = \left[(2x+3)\frac{\sin 3x}{3} - \frac{2}{3}\cdot\left(-\frac{\cos 3x}{3}\right)\right]_0^{\frac{\pi}{2}} =$

$= \left[\frac{1}{3}(2x+3)\sin 3x + \frac{2}{9}\cos 3x\right]_0^{\frac{\pi}{2}} = \left(\frac{1}{3}(\pi+3)\underset{-1}{\sin\frac{3}{2}\pi} + \frac{2}{9}\underset{0}{\cos\frac{3}{2}\pi}\right) - \left(\frac{1}{\cancel{3}}\cancel{3}\underset{0}{\sin 0} + \frac{2}{9}\underset{1}{\cos 0}\right) =$

$= \underline{\underline{-\frac{1}{3}(\pi+3) - \frac{2}{9}}}$