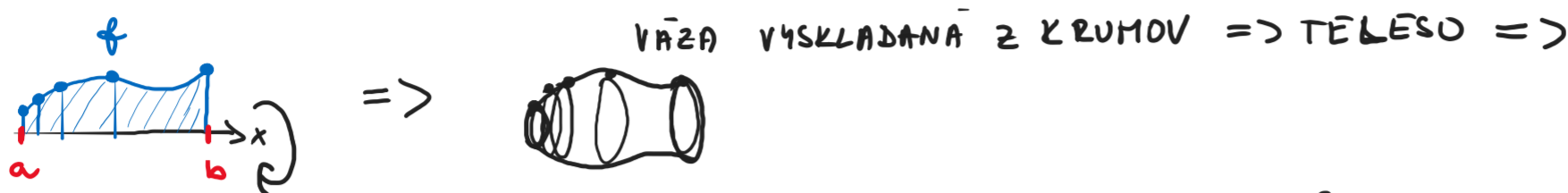
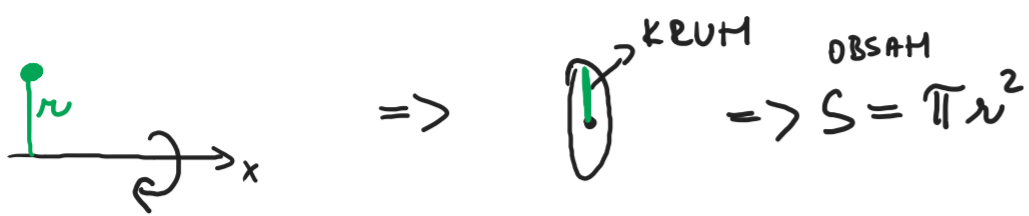


OBJEM ROTAČNĚHO TELESÁ

AKO VYPOČÍTÁME OBJEM TELESÁ, KTORÉHO OSOU SÚMERNOSTI JE OS Ox ?

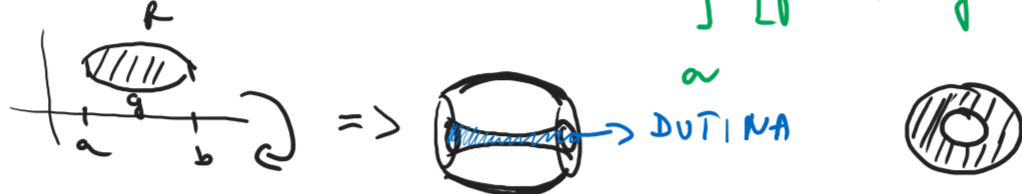


SÚČET OBSAHOV KRUHOV $\Rightarrow \pi r_1^2 + \pi r_2^2 + \dots + \pi r_n^2 = \pi \sum_{i=1}^n (r_i)^2$; KDE $r_i = f(x_i) \Rightarrow V = \pi \int_a^b f^2(x) dx$

ZÁVER: OBJEM TELESÁ, KTORÉ VZNIKNE ROTÁCIOU OKOLO OSI Ox ELEMENTÁRNEJ

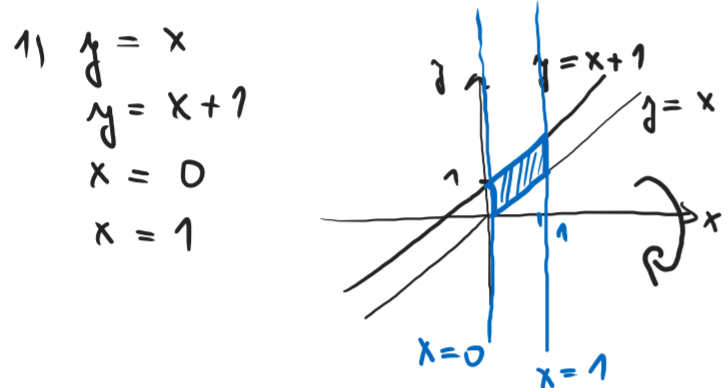
OBLASTI $a \leq x \leq b$ VYPOČÍTAME TAKTO $V = \pi \int_a^b [f^2(x) - g^2(x)] dx$

$0 \leq g(x) \leq y \leq f(x)$



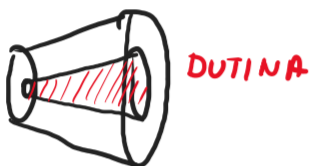
PRÍKLAD: VYPOČÍTAME OBJEM TELESÁ, KTORÉ VZNIKNE ROTÁCIOU OKOLO Ox

OBLASTI OHRANIČENÉJ KRVKAMI

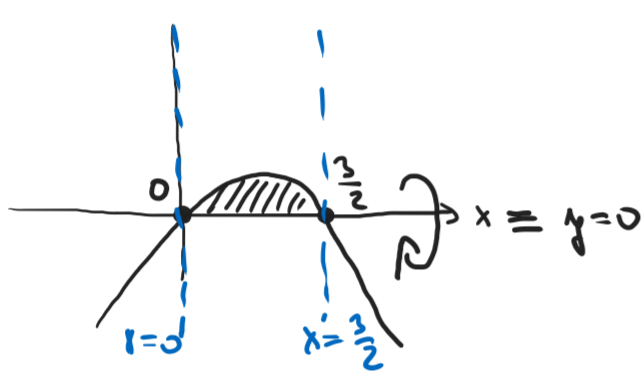


$0 \leq x \leq 1$
 $x \leq y \leq x+1$

$V = \pi \int_0^1 [(x+1)^2 - (x)^2] dx = \pi \int_0^1 (x^2 + 2x + 1 - x^2) dx =$
 $= \pi \int_0^1 (2x+1) dx = \pi [x^2 + x]_0^1 = \pi(1+1) - \pi(0+0) = \underline{\underline{2\pi}}$

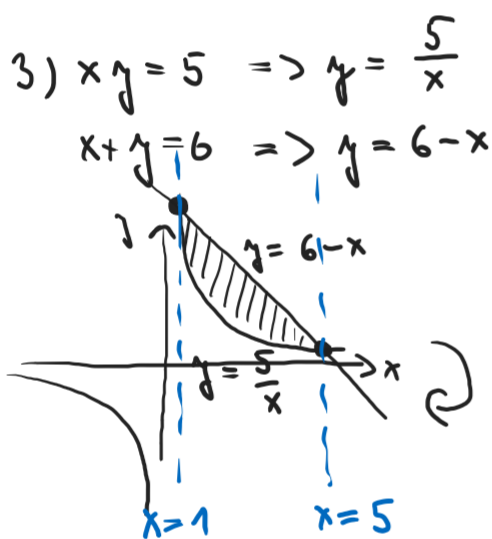


2) $y = 3x - 2x^2 = x(3-2x)$
 $y = 0$
 $x_1 = 0$ v $x_2 = \frac{3}{2}$



$0 \leq x \leq \frac{3}{2}$
 $0 \leq y \leq 3x - 2x^2$

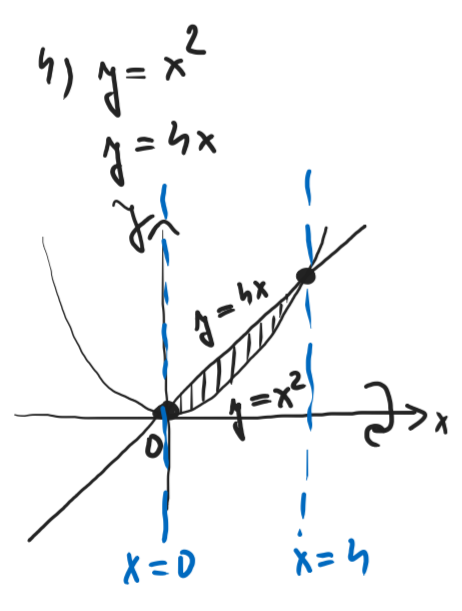
$V = \pi \int_0^{3/2} [(3x-2x^2)^2 - (0)^2] dx = \pi \int_0^{3/2} (9x^2 - 12x^3 + 4x^4) dx =$
 $= \pi [3x^3 - 3x^4 + \frac{4}{5}x^5]_0^{3/2} = \pi(3 \cdot \frac{27}{8} - 3 \cdot \frac{81}{16} + \frac{4}{5} \cdot \frac{243}{32}) - \pi(0-0+0) = \dots$



$y = y$
 $\frac{5}{x} = 6 - x \quad / \cdot x \neq 0$
 $5 = 6x - x^2$
 $x^2 - 6x + 5 = 0$
 $(x-1)(x-5) = 0$
 $x_1 = 1$ v $x_2 = 5$

$1 \leq x \leq 5$
 $\frac{5}{x} \leq y \leq 6 - x$

$V = \pi \int_1^5 [(6-x)^2 - (\frac{5}{x})^2] dx =$
 $= \pi \int_1^5 (36 - 12x + x^2 - 25 \cdot x^{-2}) dx = \pi [36x - 6x^2 + \frac{x^3}{3} - 25 \frac{x^{-1}}{-1}]_1^5 =$
 $= \pi [36 \cdot 5 - 6 \cdot 25 + \frac{125}{3} + \frac{25}{5}] - \pi [36 - 6 + \frac{1}{3} + 25] =$
 $= \frac{64}{3} \pi$

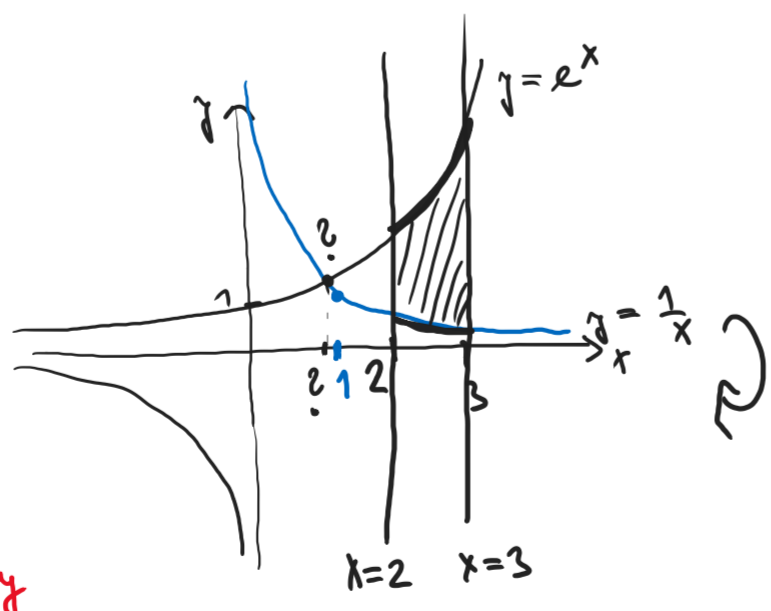


$y = y$
 $x^2 = 4x$
 $x^2 - 4x = 0$
 $x(x-4) = 0$
 $x_1 = 0$ v $x_2 = 4$

$0 \leq x \leq 4$
 $x^2 \leq y \leq 4x$

$V = \pi \int_0^4 [(4x)^2 - (x^2)^2] dx = \pi \int_0^4 (16x^2 - x^4) dx =$
 $= \pi [16 \frac{x^3}{3} - \frac{x^5}{5}]_0^4 = \pi (\frac{16}{3} \cdot 4^3 - \frac{4^5}{5}) = \dots$

5) $y = e^x$
 $y = \frac{1}{x}$
 $x = 2$
 $x = 3$



$2 \leq x \leq 3$
 $\frac{1}{x} \leq y \leq e^x$

$V = \pi \int_2^3 [(e^x)^2 - (\frac{1}{x})^2] dx = \pi \int_2^3 (e^{2x} - x^{-2}) dx = \pi [\frac{e^{2x}}{2} - \frac{x^{-1}}{-1}]_2^3 = \pi [\frac{e^{2x}}{2} + \frac{1}{x}]_2^3 =$
 $= \pi (\frac{e^6}{2} + \frac{1}{3}) - \pi (\frac{e^4}{2} + \frac{1}{2}) = \underline{\underline{\pi (\frac{e^6 - e^4}{2} - \frac{1}{6})}}$