

Matematika 1 – 5.cvičenie

príprava

Asymptoty funkcie

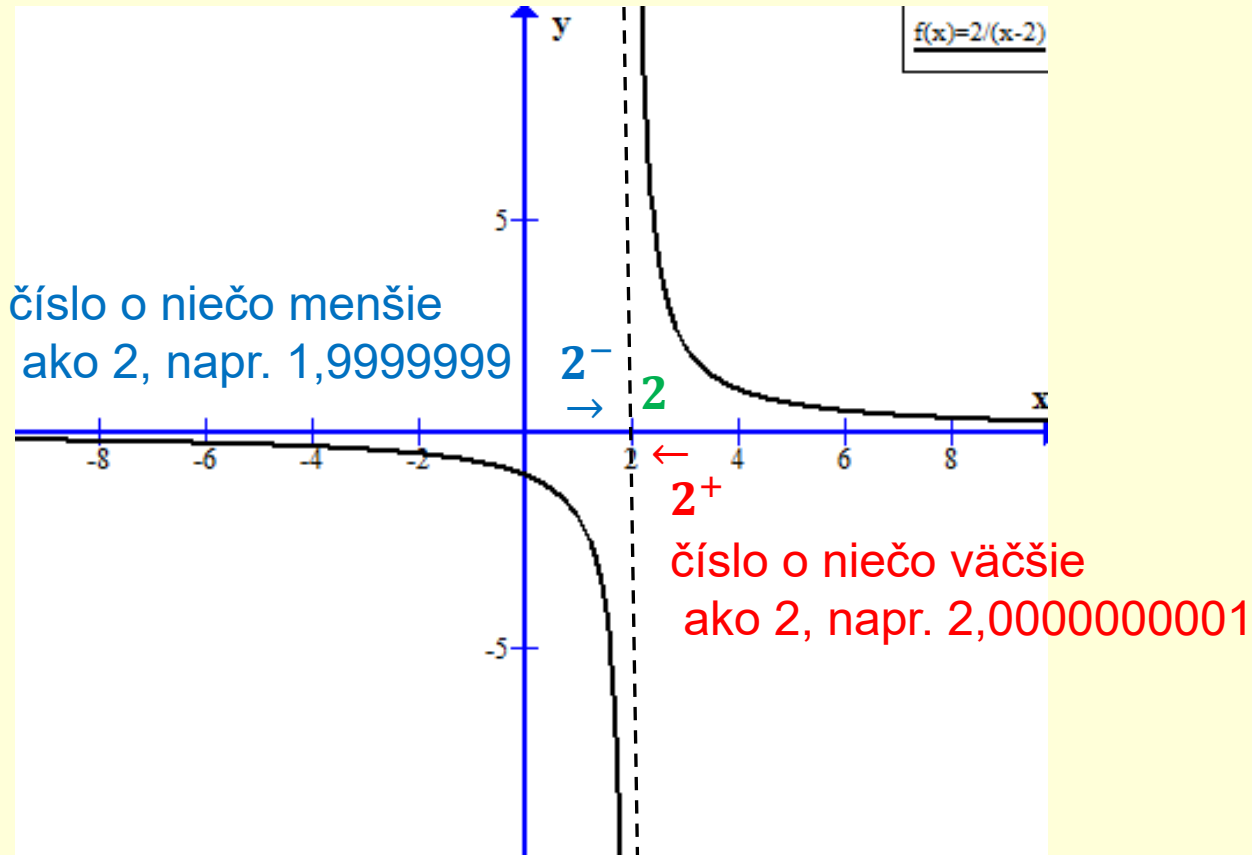
Jednostranné limity:

limita pre x idúce k a sprava

$$\lim_{x \rightarrow a^+} f(x)$$

limita pre x idúce k a zľava

$$\lim_{x \rightarrow a^-} f(x)$$



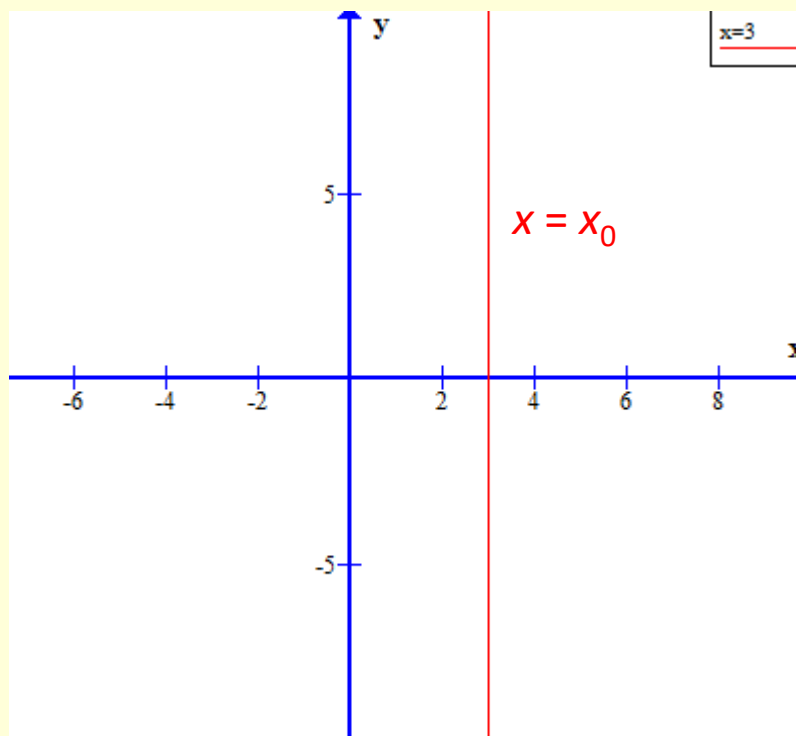
Asymptota je priamka, ktorá opisuje správanie sa krivky.

Asymptota bez smernice (ABS): vypočítame jednostranné limity v **bode nespojitosti** x_0 (bod v ktorom funkcie nie je definovaná), ak vyjdú nevlastné čísla $\pm\infty$ (stačí jedna z limít rovná $\pm\infty$, potom funkcia má ABS)

$$\lim_{x \rightarrow x_0^+} f(x) = \pm\infty$$

$$\lim_{x \rightarrow x_0^-} f(x) = \pm\infty$$

potom **ABS** je $x = x_0$



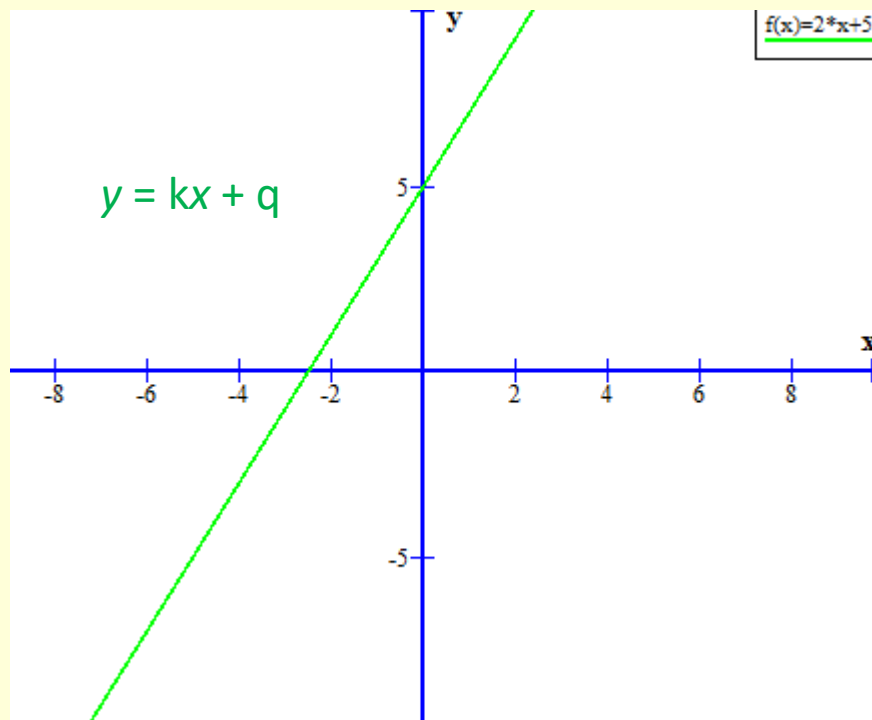
Asymptota so smernicou (ASS): priamka $y = kx + q$

$$k = \lim_{x \rightarrow \pm\infty} \frac{f(x)}{x}$$

$$q = \lim_{x \rightarrow \pm\infty} [f(x) - kx]$$

vypočítame k , q , ak vyjdú vlastné čísla, potom

ASS je $y = k_1x + q_1$, $y = k_2x + q_2$



Derivácia funkcie

Označenie derivácie funkcie v bode x_0 : $f'(x_0)$

Pravidlá pre výpočet derivácie funkcie:

$$[cf(x)]' = cf'(x) \quad c \in \mathbb{R}$$

$$[f(x) + g(x)]' = f'(x) + g'(x)$$

$$[f(x)g(x)]' = f'(x)g(x) + f(x)g'(x)$$

$$\left[\frac{f(x)}{g(x)}\right]' = \frac{f'(x)g(x) - f(x)g'(x)}{g^2(x)}.$$

$$[f(g(x))]' = f'(g(x))g'(x).$$

$$\left[f(x)^{g(x)}\right]' = \left[e^{g(x) \cdot \ln f(x)}\right]'$$

Napríklad:

$$(2x)'$$

$$(2x^2 + 5 - \arcsin x)'$$

$$(x \cdot \ln x)'$$

$$\left(\frac{3x}{\sin x}\right)'$$

$$(\ln \sin 2x)'$$

$$(\sin x)^x = \left[e^{x \cdot \ln(\sin x)}\right]'$$

Derivácie elementárnych funkcií:

- $[c]' = 0$ $[x^1]' = 1$
- $[x^\alpha]' = \alpha x^{\alpha-1}, \alpha \in \mathbb{R}$
- $[\sin x]' = \cos x$
- $[\cos x]' = -\sin x$
- $[\operatorname{tg} x]' = \frac{1}{\cos^2 x}$
- $[\operatorname{cotg} x]' = -\frac{1}{\sin^2 x}$
- $[\arcsin x]' = \frac{1}{\sqrt{1-x^2}}$
- $[\arccos x]' = -\frac{1}{\sqrt{1-x^2}}$
- $[\operatorname{arctg} x]' = \frac{1}{1+x^2}$
- $[\operatorname{arccotg} x]' = -\frac{1}{1+x^2}$
- $[e^x]' = e^x$
- $[a^x]' = a^x \ln a$
- $[\ln x]' = \frac{1}{x}$
- $[\log_a x]' = \frac{1}{x \ln a}$

Ukážky použitia:

$$[c]' = 0 \quad c - \text{konštanta, bez } x \text{ v zápise}$$

$$(2)' = 0$$

$$(-8)' = 0$$

$$(\ln 5)' = 0$$

$$(e^8)' = 0$$

$$(100^2)' = 0$$

$$\left(\frac{1}{50}\right)' = 0$$

$$[cf(x)]' = cf'(x) \quad c \in \mathbb{R}$$

$$(2x)' = 2x'$$

$$(-8 \cdot \ln x)' = -8 \cdot (\ln x)'$$

$$(30 \cdot \sin x)' = 30 \cdot (\sin x)'$$

$$\left(\frac{1}{50} e^x\right)' = \frac{1}{50} (e^x)'$$

$$[x^\alpha]' = \alpha x^{\alpha-1}, \alpha \in \mathbb{R}$$

$$(x^3)' = 3x^{3-1} = 3x^2$$

$$(2x)' = 2x' = 2 \cdot 1 = 2$$

$$[x^1]' = 1$$

Úprava výrazov na mocninový tvar

! 2^x nie je typ x^α !

$$\left(\frac{1}{x^2}\right)' = (x^{-2})'$$

$$(\sqrt{x^3})' = (x^{\frac{3}{2}})'$$

$$\left(\frac{1}{\sqrt[5]{x}}\right)' = (x^{-\frac{1}{5}})'$$

Ukážky použitia:

$$[f(x)g(x)]' = f'(x)g(x) + f(x)g'(x)$$

$$(\sin x \cdot \ln x)' = (\sin x)' \cdot \ln x + \sin x \cdot (\ln x)'$$

$$(e^x \cdot x^3)' = (e^x)' \cdot x^3 + e^x \cdot (x^3)'$$

$$((x+1) \cdot \operatorname{tg} x)' = (x+1)' \cdot \operatorname{tg} x + (x+1) \cdot (\operatorname{tg} x)'$$

Toto nederivujeme ako súčin funkcií:

$$(3(x+5))' = 3 \cdot (x+5)'$$

$$(2 \cdot \cotg x)' = 2(\cotg x)'$$

$$\left(\frac{\ln x}{2}\right)' = \frac{1}{2} \cdot (\ln x)'$$

$$\left[\frac{f(x)}{g(x)}\right]' = \frac{f'(x)g(x) - f(x)g'(x)}{g^2(x)}$$

$$\left(\frac{\ln x}{x^2}\right)' = \frac{(\ln x)' \cdot x^2 - \ln x \cdot (x^2)'}{(x^2)^2}$$

$$\left(\frac{x}{\cos x}\right)' = \frac{(x)' \cdot \cos x - x \cdot (\cos x)'}{(x^2)^2}$$

$$\left(\frac{\operatorname{arctg} x}{\sqrt[5]{x}}\right)' = \frac{(\operatorname{arctg} x)' \cdot \sqrt[5]{x} - \operatorname{arctg} x \cdot (\sqrt[5]{x})'}{(\sqrt[5]{x})^2}$$

Toto nederivujeme ako podiel funkcií:

$$\left(\frac{\ln x}{2}\right)' = \frac{1}{2} (\ln x)'$$

$$\left(\frac{3}{x^5}\right)' = 3 (x^{-5})'$$

$$\left(\frac{1}{\sin x}\right)' = (\sin x^{-1})'$$