

Matematika 1 – 5.cvičenie

RNDr. Z. Gibová, PhD.

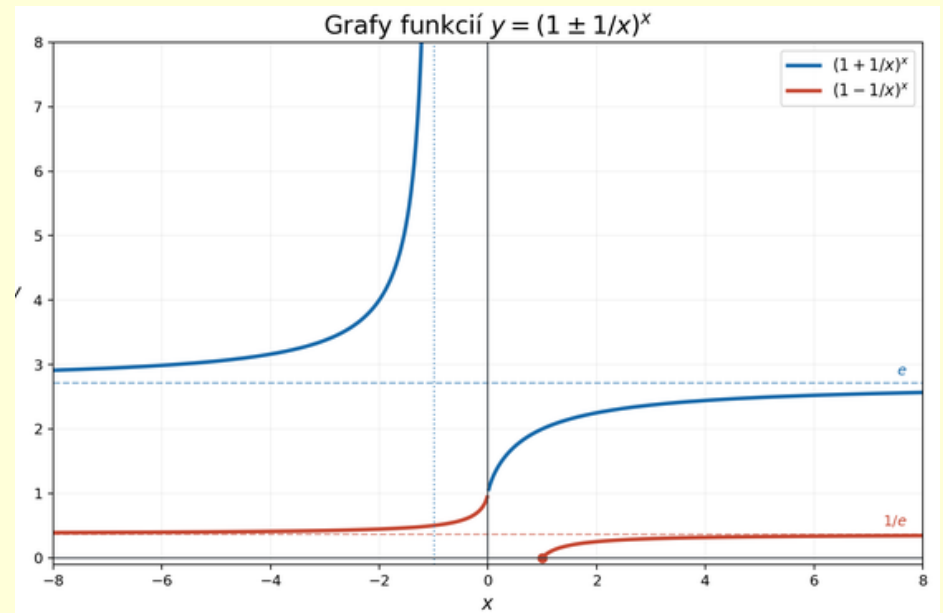
b - 6) dosadíme, ak **neurčitost** 1^∞ , použijeme jeden zo vzorcov

$$\lim_{x \rightarrow \infty} \left(1 \pm \frac{1}{x}\right)^x = e^{\pm 1}$$

$$\lim_{x \rightarrow \infty} \left(1 + \frac{k}{f(x)}\right)^{f(x)} = e^k, \quad f(x) \rightarrow \infty$$

Pr. 14 – 22 / 21:

$$\lim_{x \rightarrow \infty} \left(1 + \frac{2}{x+3}\right)^x$$



Pr. 15:

$$\lim_{x \rightarrow \infty} \left(1 + \frac{3}{x-5}\right)^{2x} \quad e^6$$

$$\left(1 + \frac{2}{\infty}\right)^{\infty} = 1^{\infty}$$

$$\lim_{x \rightarrow \infty} \left(1 + \frac{3}{x-5}\right)^{2x} \stackrel{1^{\infty}}{=} \lim_{x \rightarrow \infty} \left(1 + \frac{3}{x-5}\right)^{2x \frac{(x-5)}{(x-5)}} = \lim_{x \rightarrow \infty} \left[\left(1 + \frac{3}{x-5}\right)^{x-5} \right]^{\frac{2x}{x-5}} =$$

$$e^{3 \cdot \lim_{x \rightarrow \infty} \frac{2x}{x-5}} = e^{3 \cdot \frac{2}{1-0}} = e^6$$

$$\lim_{x \rightarrow \infty} \left(1 + \frac{3}{x+1}\right)^{x-1} \quad \text{Dú} \quad e^3$$

$$\lim_{x \rightarrow \infty} \left(1 - \frac{1}{x^2}\right)^x \quad \text{Dú} \quad 1$$

$$\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x-2}\right)^{x+1} \quad \text{Dú} \quad e^1$$

Asymptoty funkcie

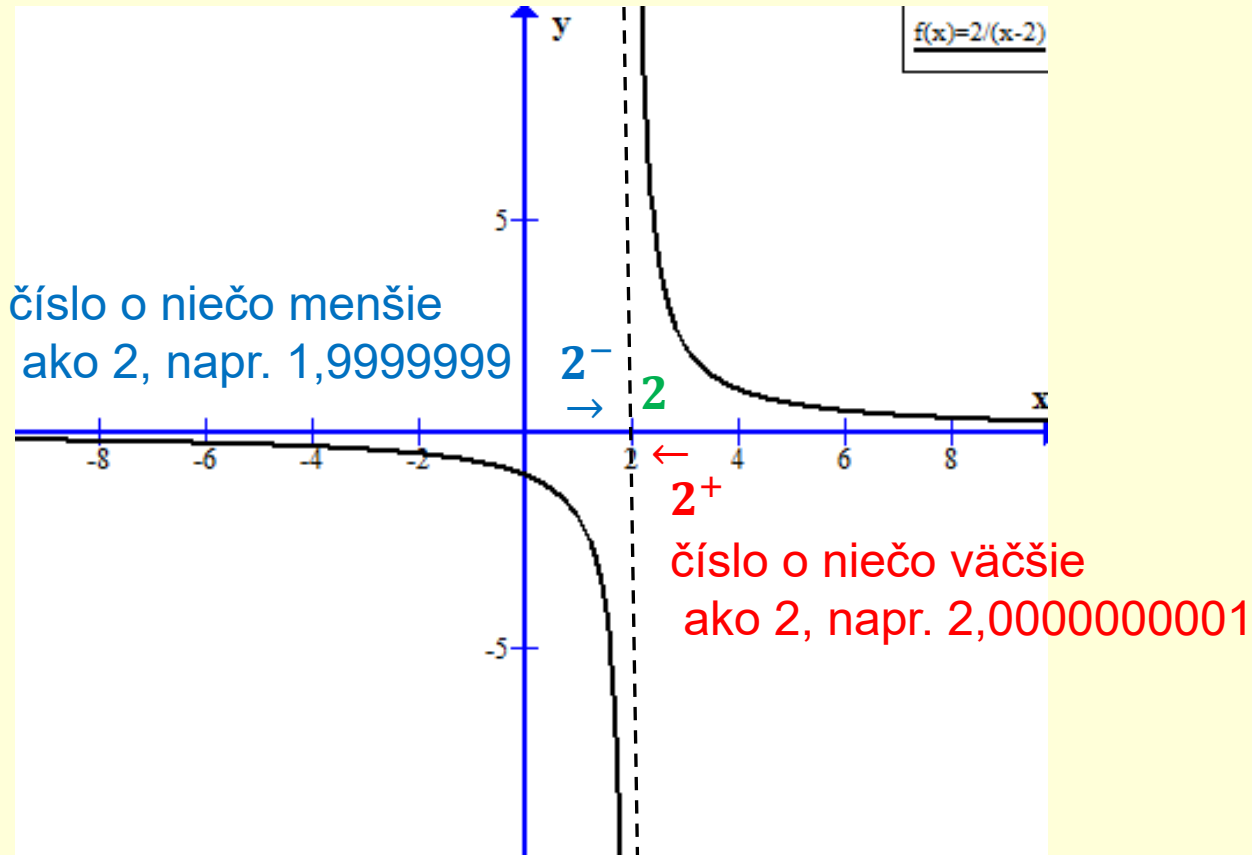
Jednostranné limity:

limita pre x idúce k a sprava

$$\lim_{x \rightarrow a^+} f(x)$$

limita pre x idúce k a zľava

$$\lim_{x \rightarrow a^-} f(x)$$



Asymptota je priamka, ktorá opisuje správanie sa krivky.

Asymptota bez smernice (ABS): vypočítame jednostranné limity v **bode nespojitosti** x_0 (bod v ktorom funkcie nie je definovaná), ak vyjdú nevlastné čísla $\pm\infty$ (stačí jedna z limit rovná $\pm\infty$, potom funkcia má ABS)

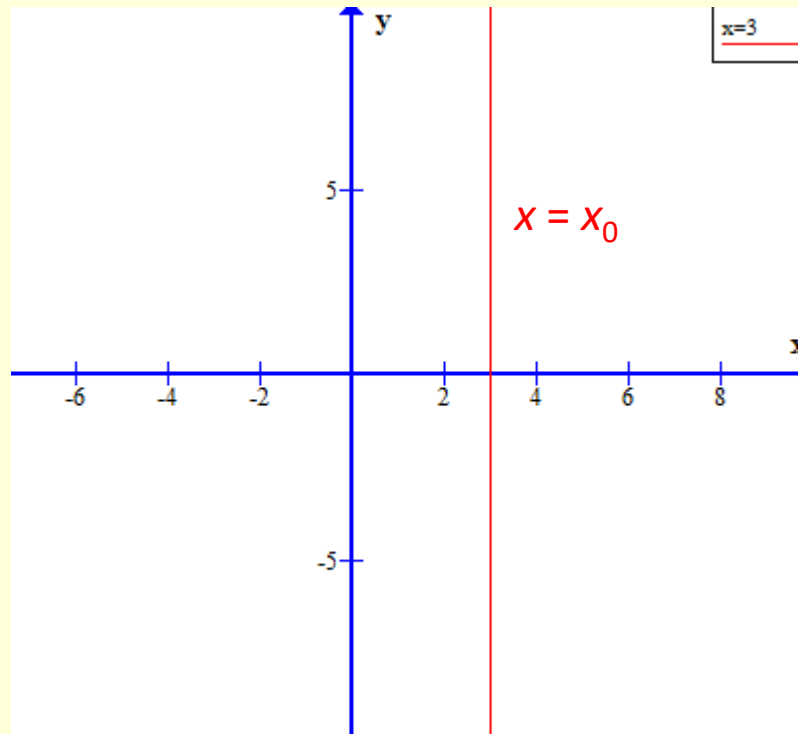
$$\lim_{x \rightarrow x_0^+} f(x) = \pm\infty$$

$$\lim_{x \rightarrow x_0^-} f(x) = \pm\infty$$

potom **ABS** je $x = x_0$

$$\lim_{x \rightarrow 0^+} \frac{1}{x} = +\infty$$

$$\lim_{x \rightarrow 0^-} \frac{1}{x} = -\infty$$



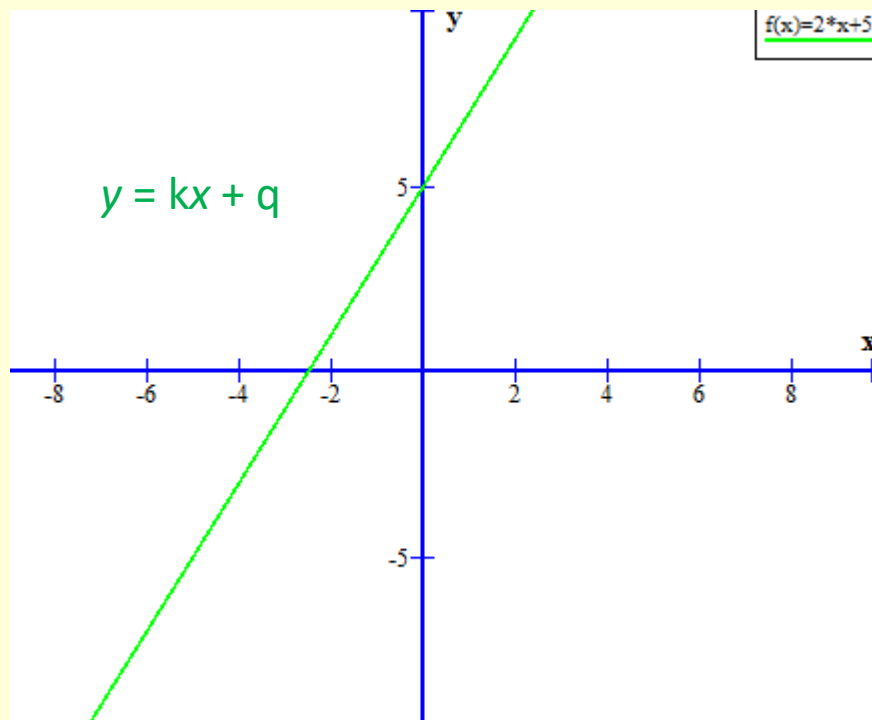
Asymptota so smernicou (ASS): priamka $y = kx + q$

$$k = \lim_{x \rightarrow \pm\infty} \frac{f(x)}{x} - \text{smernica}$$

$$q = \lim_{x \rightarrow \pm\infty} [f(x) - kx]$$

vypočítame k , q , ak vyjdú vlastné čísla, potom

ASS je $y = k_1x + q_1$, $y = k_2x + q_2$



Pr. 1: Určte ASS a ABS funkcie

$$y = \frac{3(x-1)^2}{x-2}$$

1. Určiť D(f) a bod nespojitosti.

$$x - 2 \neq 0$$
$$x \neq 2, \quad D(f) = R - \{2\}$$

bod nespojitosti $x_0 = 2$

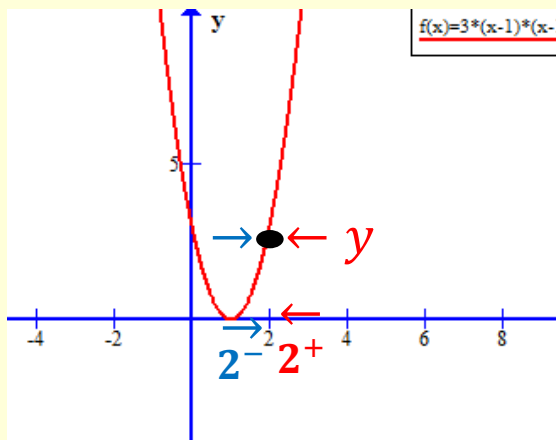
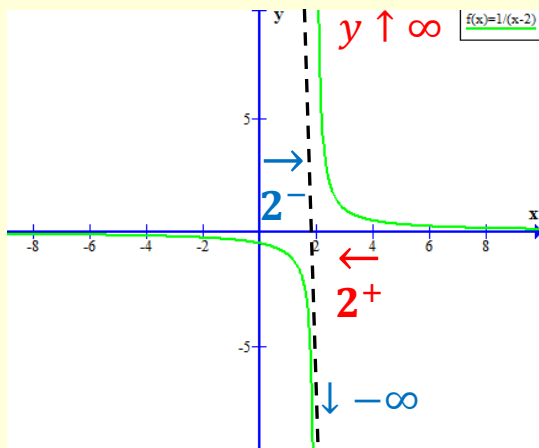
2. Určiť ABS v bode nespojitosti

$$\lim_{x \rightarrow x_0^+} f(x) = \lim_{x \rightarrow 2^+} \frac{3(x-1)^2}{x-2} = \lim_{x \rightarrow 2^+} \frac{1}{x-2} \cdot \lim_{x \rightarrow 2^+} 3(x-1)^2 = +\infty$$

$\frac{1}{2^+ - 2} = \frac{1}{0^+}$ $3(2^+ - 1)^2 = + \text{ číslo}$

$$\lim_{x \rightarrow x_0^-} f(x) = \lim_{x \rightarrow 2^-} \frac{3(x-1)^2}{x-2} = \lim_{x \rightarrow 2^-} \frac{1}{x-2} \cdot \lim_{x \rightarrow 2^-} 3(x-1)^2 = -\infty$$

$\frac{1}{2^- - 2} = \frac{1}{0^-}$ $3(2^- - 1)^2 = + \text{ číslo}$



ABS: $x = 2$

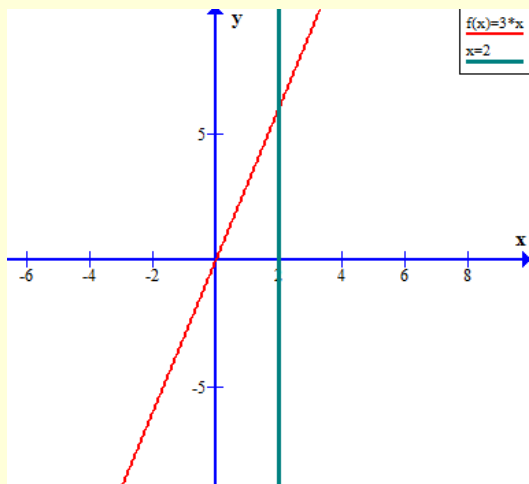
3. Určit' ASS

$$k = \lim_{x \rightarrow \infty} \frac{f(x)}{x} = \lim_{x \rightarrow \infty} \frac{3(x-1)^2}{x(x-2)} = \lim_{x \rightarrow \infty} \frac{3x^2 - 6x + 3}{x^2 - 2x} = 3$$

$$k = \lim_{x \rightarrow -\infty} \frac{f(x)}{x} = \lim_{x \rightarrow -\infty} \frac{3(x-1)^2}{x(x-2)} = \lim_{x \rightarrow -\infty} \frac{3x^2 - 6x + 3}{x^2 - 2x} = 3$$

$$q = \lim_{x \rightarrow \infty} [f(x) - kx] = \lim_{x \rightarrow \infty} \left[\frac{3(x-1)^2}{(x-2)} - 3x \right] = \lim_{x \rightarrow \infty} \left[\frac{3x^2 - 6x + 3 - 3x^2 + 6x}{(x-2)} \right]$$

$$= \lim_{x \rightarrow \infty} \left[\frac{3}{(x-2)} \right] = 0 \quad q = \lim_{x \rightarrow -\infty} \left[\frac{3}{(x-2)} \right] = 0 \quad \text{ASS: } y = 3x \quad \text{pre } x \rightarrow \pm\infty$$



ASS: $y = 3x$ pre $x \rightarrow \pm\infty$

ABS: $x = 2$

Pr. 2 – 47/ 2: Určte ASS a ABS funkcie

$$y = x^3 + 3x^2 - 2$$

Pr. 3: Určte ASS a ABS funkcie

$$y = \frac{2x^2}{2x - 1}$$

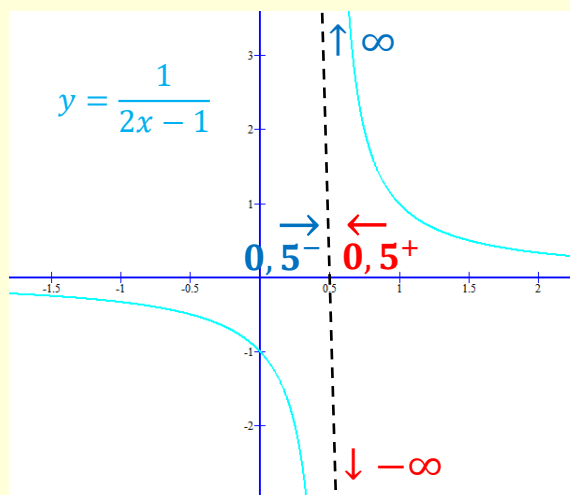
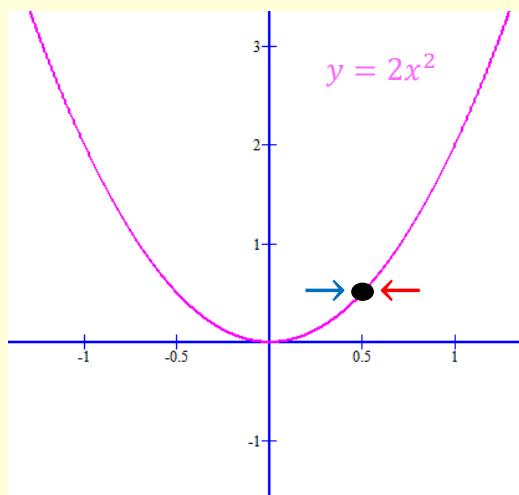
$$2x - 1 \neq 0$$

$$x \neq \frac{1}{2}, \quad D(f) = R - \left\{\frac{1}{2}\right\}$$

bod nespojitosti $x_0 = 0,5$

$$\lim_{x \rightarrow x_0^+} f(x) = \lim_{x \rightarrow 0,5^+} \frac{2x^2}{2x - 1} = \lim_{x \rightarrow 0,5^+} \frac{1}{2x - 1} \cdot \lim_{x \rightarrow 0,5^+} 2x^2 = \frac{1}{0^+} \cdot (+ \text{ číslo}) = +\infty$$

$$\lim_{x \rightarrow x_0^-} f(x) = \lim_{x \rightarrow 0,5^-} \frac{2x^2}{2x - 1} = \lim_{x \rightarrow 0,5^-} \frac{1}{2x - 1} \cdot \lim_{x \rightarrow 0,5^-} 2x^2 = \frac{1}{0^-} \cdot (+ \text{ číslo}) = -\infty$$

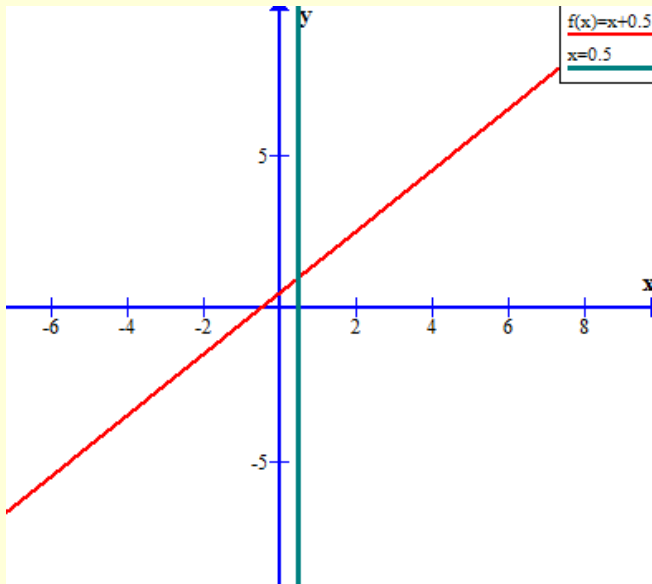


ABS: $x = 0,5$

$$k = \lim_{x \rightarrow \pm\infty} \frac{f(x)}{x} = \lim_{x \rightarrow \pm\infty} \frac{2x^2}{2x-1} = \lim_{x \rightarrow \pm\infty} \frac{2x^2}{2x^2-x} = 1$$

$$q = \lim_{x \rightarrow \pm\infty} [f(x) - kx] = \lim_{x \rightarrow \pm\infty} \left[\frac{2x^2}{2x-1} - x \right] = \lim_{x \rightarrow \pm\infty} \left[\frac{2x^2 - 2x^2 + x}{2x-1} \right] = \frac{1}{2}$$

ASS: $y = x + 0,5$ pre $x \rightarrow \pm\infty$



ASS: $y = x + 0,5$ pre $x \rightarrow \pm\infty$

ABS: $x = 0,5$

Pr. 4 – 47 / 12: Určte ASS a ABS funkcie

$$y = \frac{x^2}{4 - x^2}$$

$$\lim_{x \rightarrow x_0^+} f(x) = \lim_{x \rightarrow -2^+} \frac{x^2}{4 - x^2} = \lim_{x \rightarrow -2^+} \frac{1}{4 - x^2} \cdot \lim_{x \rightarrow -2^+} x^2 = \frac{1}{0^+} \cdot (+\text{číslo}) = \infty$$

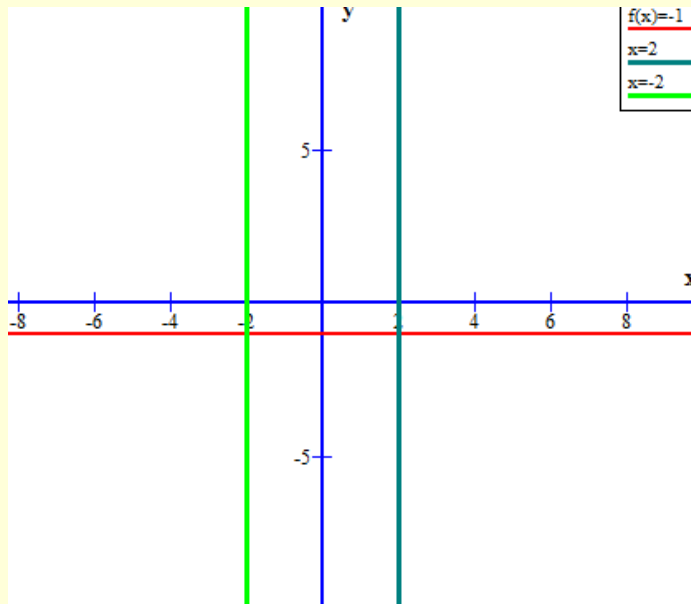
$$\lim_{x \rightarrow x_0^-} f(x) = \lim_{x \rightarrow -2^-} \frac{x^2}{4 - x^2} = \lim_{x \rightarrow -2^-} \frac{1}{4 - x^2} \cdot \lim_{x \rightarrow -2^-} x^2 = \frac{1}{0^-} \cdot (+\text{číslo}) = -\infty$$

ABS: $x = -2$

$$k = 0$$

$$q = \lim_{x \rightarrow \pm\infty} [f(x) - kx] = \lim_{x \rightarrow \pm\infty} \left[\frac{x^2}{4 - x^2} \right] = -1$$

ASS: $y = -1$ pre $x \rightarrow \pm\infty$



ASS: $y = -1$ pre $x \rightarrow \pm\infty$

ABS: $x = -2$

ABS: $x = 2$

Pr. 5 – 47 / 11: Určte ASS a ABS funkcie

$$y = x - \frac{1}{x}$$

Dú – 47 / 1, 5, 6, 8, 9, 12, 13, 16