Additional Exercises in Estimation Theory

PART A PROBLEM STATEMENTS

In Exercises 4.1–4.5, assume that the population is normally distributed. For data grouped into intervals, use the interval midpoints as the representative values.

Exercise 4.1 High Jump

The heights achieved by students in the high jump, in centimetres, are given below:

z_j	120	130	140	150	160	170	180	190	200	210
n_j	3	5	7	11	12	6	2	2	1	1

The population standard deviation is known: $\sigma = 20$ cm. Determine:

- the 95% two-sided confidence interval for the mean μ;
- the 95% one-sided upper confidence interval for μ;
- the 95% one-sided lower confidence interval for μ.

Exercise 4.2 Required Sample Size

We wish to estimate the mean duration of a telephone call with a confidence level of 0.99 and a margin of error of at most ±10 seconds. The standard deviation of call duration is $\sigma = 2.5$ minutes. Determine the minimum required number of randomly selected calls.

Exercise 4.3 Fibre Strength

The strength of a fibre [kg/mm²] was measured for ten specimens: 10.3; 8.8; 9.7; 9.6; 11.2; 10.7; 9.1; 9.5; 10.3; 9.3. Determine:

- point estimates of μ and σ;
- the 95% two-sided confidence interval for μ;
- the 99% one-sided upper confidence interval for μ;
- the 90% one-sided lower confidence interval for μ.

Exercise 4.4 Groundwater Level

The following values were measured in centimetres: 159.53; 159.49; 159.61; 159.71; 159.88; 161.08; 160.98; 161.09; 160.91; 160.79; 161.02; 160.96; 160.80. Determine:

- point estimates of μ and σ;
- the 95% two-sided confidence intervals for the variance σ^2 and standard deviation σ;
- the 99% one-sided upper confidence interval for the variance σ^2;
- the 90% one-sided lower confidence interval for the standard deviation σ.

Exercise 4.5 Data Grouped into Intervals

l_i	15–17	17–19	19–21	21–23	23–25	25–27
n_i	10	30	50	70	60	30

Determine:

- the 95% two-sided confidence interval for the mean μ;
- a 95% lower confidence bound for the mean μ;
- the 95% two-sided confidence interval for the standard deviation σ;
- a 95% upper confidence bound for the variance σ^2.

Exercise 4.6 Failure-Free Operating Time

The failure-free operating times [hours] are: 412, 482, 134, 115, 839, 357, 405, 219, 172, 141, 644, 194, 212, 909, 1065, 349, 21, 539, 268, 749, 143, 684. Assume an exponential distribution with parameter $\lambda = E(X)$. Determine a point estimate of λ and the 95% two-sided confidence interval for λ.