Additional Exercises

One-Sample Parametric Tests | Exercises 5.1–5.6

Instructions for students

For each exercise, state the given information and what is required, H_0 and H_1, the test and its assumptions, the test statistic, the critical region or p-value, the decision, and an interpretation in context.

PROBLEM STATEMENTS

Exercise 5.1

The following failure-free operating times of a production machine were recorded [hours]: 300, 320, 315, 298, 310, 322, 318, 286, 270, 270, 302, 298. Assuming an exponential distribution, test H_0: $\lambda = 300$ against H_1: $\lambda > 300$ at significance level $\alpha = 0.05$.

Exercise 5.2

The tin content (%) of ore samples was measured. The results are shown below, where z_i denotes a representative class value:

z_i	30	35	40	45	50	55	60	65	70	75
n_i	1	3	4	10	15	20	11	5	3	2

The population variance is known: $\sigma^2 = 85$. At $\alpha = 0.05$, test H_0: $\mu = 52$ against H_1: $\mu \neq 52$.

Exercise 5.3

The average amount of iron waste at a factory was 25.5 tonnes. After waste-reduction measures, the following values were recorded over two weeks: 23.8; 24.5; 26.1; 22.8; 25.0; 21.9; 24.0; 23.5; 25.2; 22.0; 23.0; 25.0. At $\alpha = 0.05$, test the claim that the average amount of iron waste has decreased.

Exercise 5.4

A city bus route has an average rush-hour speed of 8 km/h in the city centre. A new route was tested on ten randomly selected days, giving the following speeds: 8.5; 9.5; 7.8; 8.2; 9.0; 7.5; 8.2; 7.8; 9.0; 8.5. Assess whether changing the route increases the average speed at a) $\alpha = 0.01$; b) $\alpha = 0.05$.

Exercise 5.5

Milk fat content (%) was measured: 3.84; 4.06; 3.67; 3.97; 4.16; 3.98; 3.76; 4.02; 3.82; 3.71; 3.94; 4.04; 4.07; 3.61; 4.01. At $\alpha = 0.05$, test H_0: $\sigma^2 = 0.03$ against H_1: $\sigma^2 \neq 0.03$.

Exercise 5.6

The precision of an automatic machine tool is characterised by the variance of the lengths of the parts it produces. If this variance exceeds 20 mm², the machine must be readjusted. Measurements [mm]: 792, 803, 790, 804, 801, 803, 798, 799, 806, 797, 802, 796, 802, 801, 798, 799, 806, 809, 797, 803. At $\alpha = 0.05$, assess whether the machine needs to be readjusted.

Common assumption for Exercises 5.2–5.6

Assume that the population is normally distributed. Exercise 5.1 explicitly assumes an exponential distribution.