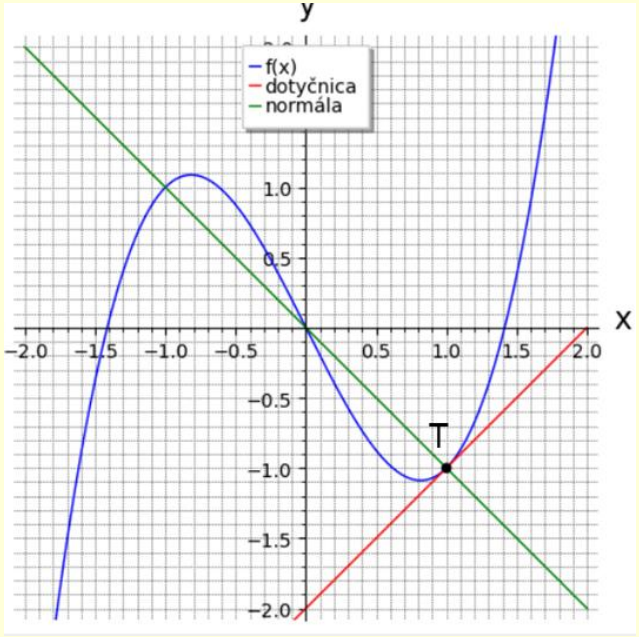
Matematika 1 – 7.cvičenie

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Geometrický význam derivácie funkcie



dotykový bod $T\,[x_0, y_0]$ ku grafu funkcie $f(x)$

Smernica dotyčnice(tangenty) = derivácia $f'(x_0)$ v dotykovom bode T ku grafu $f(x)$

$$k_t = f'(x_0)$$

Rovnica dotyčnice v dotykovom bode T ku grafu $f(x)$

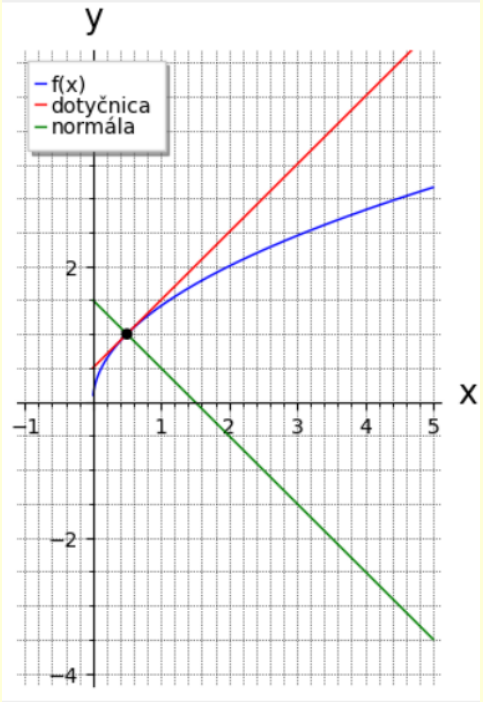
$$t:\ y - y_0 = k_t\,(x - x_0)$$

Smernica normály (kolmice) $\quad k_n = -\frac{1}{f'(x_0)} = -\frac{1}{k_t}$

Rovnica normály v dotykovom bode T ku grafu $f(x)$ kolmo na dotyčnicu

$$n:\ y - y_0 = k_n\,(x - x_0)$$

Pr. 1 – 29 / 5: Nájdite rovnicu dotyčnice a normály ku grafu $f(x) = \sqrt{2x}$ v dotykovom bode $T = \left[\frac{1}{2}, ?\right]$



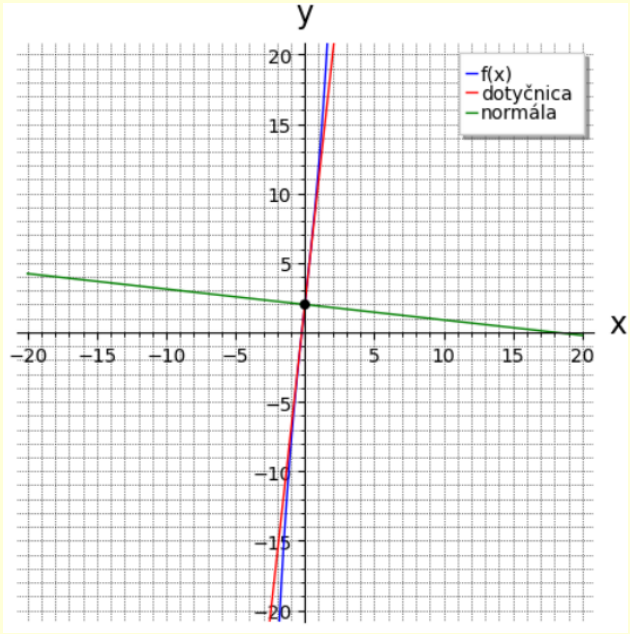
Pr. 2 – 29 / 3: Nájdite rovnicu dotyčnice a normály ku grafu $f(x) = x^3 + 9x + 2$ v dotykovom bode $T = [0, ?]$

$t: 9x - y + 2 = 0$
$n: x + 9y - 18 = 0$

$x_0 = 0, \qquad y_0 = f(x_0) = 0^3 + 9.0 + 2 = 2 \qquad T = [0,2]$

$f'(x) = 3x^2 + 9 \qquad \boldsymbol{k_t = f'(x_0)} = 3.0^2 + 9 = 9$

$$\boldsymbol{k_n = -\frac{1}{f'(x_0)} = -\frac{1}{k_t} = -\frac{1}{9}}$$



$\boldsymbol{t}$: $\boldsymbol{y - y_0 = k_t\,(x - x_0)} \qquad y - 2 = 9\,(x - 0)$

$y - 2 = 9x$

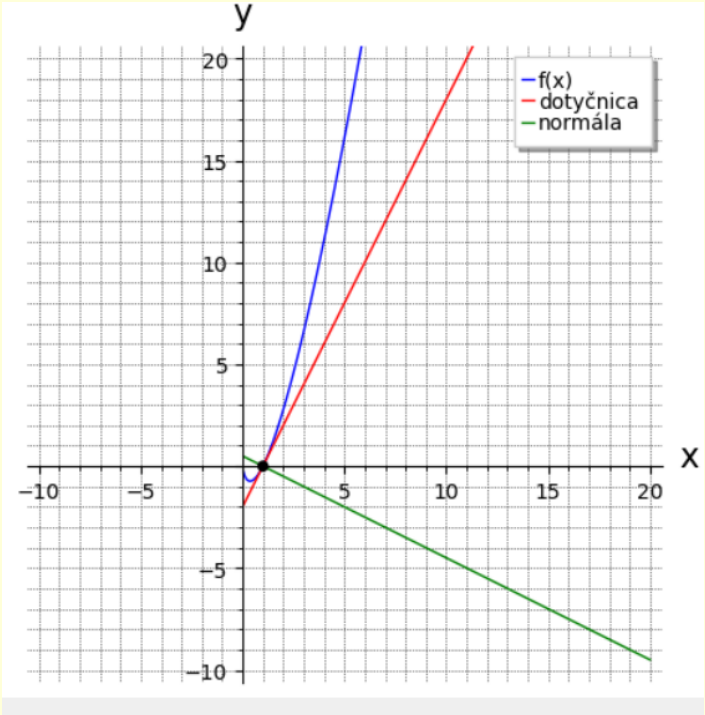
$\underline{t: 0 = 9x - y + 2}$

$\boldsymbol{n}$: $\boldsymbol{y - y_0 = k_n\,(x - x_0)} \qquad y - 2 = -\frac{1}{9}\,(x - 0)$

$-9y + 18 = x$

$\underline{n: 0 = x + 9y - 18}$

Pr. 3 – 29 / 7: Nájdite rovnicu dotyčnice a normály ku grafu $f(x) = 2x \ln x$ v dotykovom bode $T = [1, ?]$



Pr. 4 – 29 / 9: Nájdite rovnicu dotyčnice a normály ku grafu $f(x) = \frac{e^x}{2}$ v dotykovom bode $T = [0, ?]$

$t: x - 2y + 3 = 0$

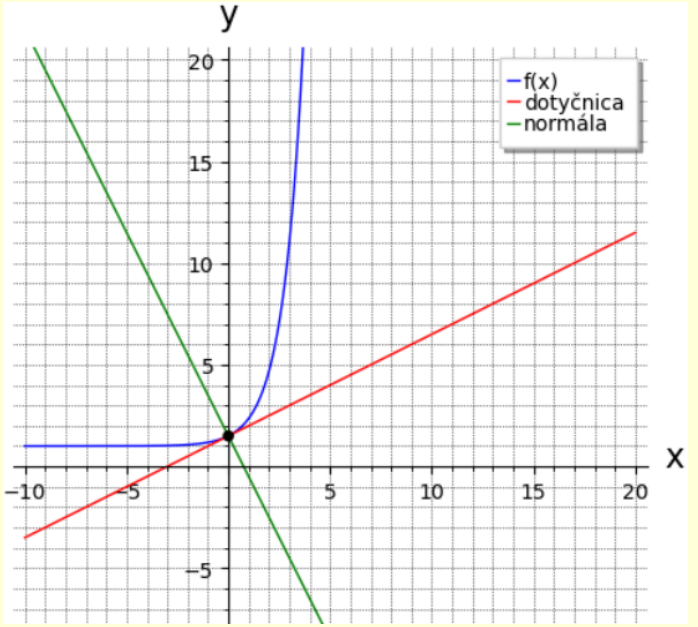
$n: 4x + 2y - 3 = 0$

$x_0 = 0, \qquad y_0 = f(x_0) = \frac{e^0}{2} + 1 = \frac{1}{2} + 1 = \frac{3}{2} \qquad T = [0, \frac{3}{2}]$

$e^0 = 1$

$f'(x) = \frac{e^x}{2} \qquad k_t = f'(x_0) = \frac{e^0}{2} = \frac{1}{2}$

$k_n = -\frac{1}{f'(x_0)} = -\frac{1}{k_t} = -2$



$t: \; y - y_0 = k_t\,(x - x_0) \qquad y - \frac{3}{2} = \frac{1}{2}(x - 0)$

$2y - 3 = x$

$t: 0 = x - 2y + 3$

$n: \; y - y_0 = k_n\,(x - x_0) \qquad y - \frac{3}{2} = -2\,(x - 0)$

$-2y + 3 = 4x$

$n: 0 = 4x + 2y - 3$

Dú – str. 29 / 4, 6, 8, 10

L´ Hospitalovo pravidlo

Pre výpočet limít, kde sa použije derivácia. Platí za určitých podmienok

$$\lim_{x\to a}\frac{f(x)}{g(x)}=\lim_{x\to a}\frac{f'(x)}{g'(x)}$$

1. limity s neurčitosťou $\frac{\infty}{\infty}$, $\frac{0}{0}$ - priamo použijeme toto pravidlo

Pr. 1: $\lim_{x\to 0}\frac{\cos x-\cos^2 x}{x^2}$

Pr. 2 – 34 / 5:

$$\lim_{x\to 0}\frac{6^x-3^x}{x} \qquad\qquad \ln 2$$

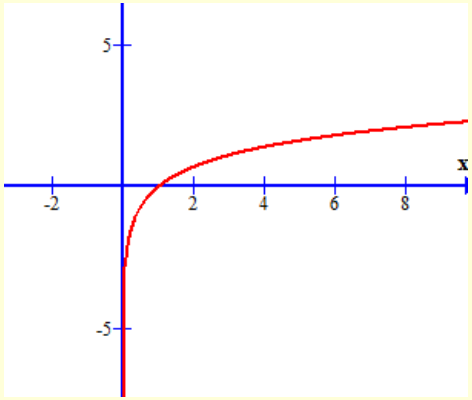
$$\lim_{x\to 0}\frac{6^x-3^x}{x}\overset{\frac{0}{0}\ L'H}{=}\lim_{x\to 0}\frac{(6^x-3^x)'}{x'}=\lim_{x\to 0}\frac{6^x\ln 6-3^x\ln 3}{1}=6^0\ln 6-3^0\ln 3=\ln\frac{6}{3}=\ln 2$$

$$\ln a-\ln b=\ln\frac{a}{b}$$

Pr. 3 – 35 / 13:

$$\lim_{x\to\infty}\frac{\ln x}{\sqrt{x}}$$

$$\lim_{x\to\infty}\frac{\ln x}{\sqrt{x}} \overset{\frac{\infty}{\infty}\ L'H}{=} \lim_{x\to\infty}\frac{(\ln x)'}{(\sqrt{x})'} = \lim_{x\to\infty}\frac{\frac{1}{x}}{\frac{1}{2}x^{\frac{-1}{2}}} = \lim_{x\to\infty}\frac{2\sqrt{x}}{x} \overset{\frac{\infty}{\infty}\ L'H}{=} \lim_{x\to\infty}\frac{(2\sqrt{x})'}{x'}\lim_{x\to\infty}\frac{2\frac{1}{2}x^{\frac{-1}{2}}}{1} = \lim_{x\to\infty}\frac{1}{\sqrt{x}} = 0$$

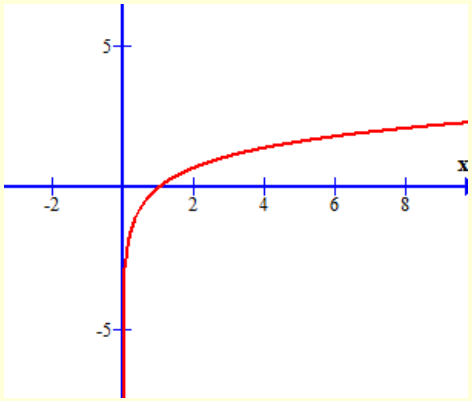


Dú – str. 34, 35 / 3, 4, 6, 7, 9, 11, 12

2. limity s neurčitosťou $\infty - \infty$, upravíme na spoločného menovateľa, dostaneme neurčitosť $\frac{\infty}{\infty}$ alebo $\frac{0}{0}$ a potom použijeme toto pravidlo

Pr. 4 – 35 / 16:

$$\lim_{x \to 1+} \left(\frac{x}{x-1} - \frac{1}{\ln x} \right)$$



Pr. 5 – 35 / 19:

$$\lim_{x\to 0+}\left(\frac{1}{2x}-\frac{1}{\sin x}\right) \qquad -\infty$$

$$\lim_{x\to 0^+}\left(\frac{1}{2x}-\frac{1}{\sin x}\right) \overset{\infty-\infty}{=} \lim_{x\to 0^+}\frac{\sin x-2x}{2x.\sin x} \overset{\frac{0}{0}\ L'H}{=} \lim_{x\to 0^+}\frac{[\sin x-2x]'}{[2x.\sin x]'} =$$

$$\lim_{x\to 0^+}\frac{\cos x\ -2}{2.\sin x+2x.\cos x} = \frac{\cos 0^+-2}{2.\sin 0^++2.0^+.\cos 0^+} = \frac{1-2}{0^++0^+} = \frac{-1}{0^+} = -\infty$$

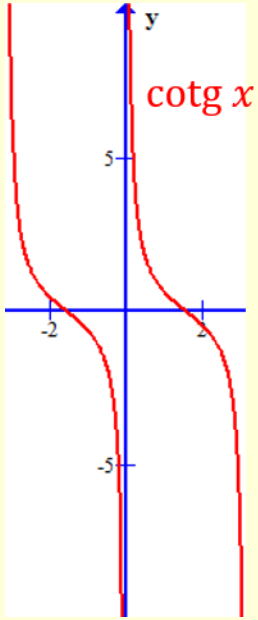
Dú – str. 34, 35 / 17, 18, 20

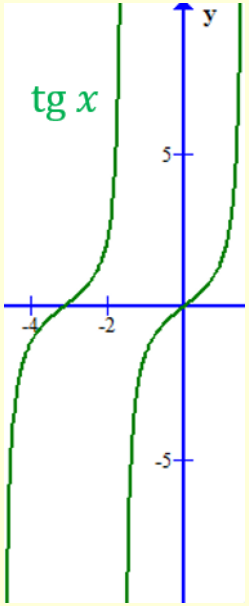
3. limity s neurčitosťou $0.\infty$, <u>upravíme limitu na tvar</u>

$$\lim_{x\to a} f(x)\cdot g(x) = \lim_{x\to a} \frac{f(x)}{\frac{1}{g(x)}} \quad \textbf{alebo} \quad \lim_{x\to a} f(x)\cdot g(x) = \lim_{x\to a} \frac{g(x)}{\frac{1}{f(x)}}$$

dostaneme neurčitosť $\frac{\infty}{\infty}$ alebo $\frac{0}{0}$ a potom použijeme toto pravidlo

Pr. 6: $\lim_{x\to 0^+} (1-\cos x).\operatorname{cotg} x$

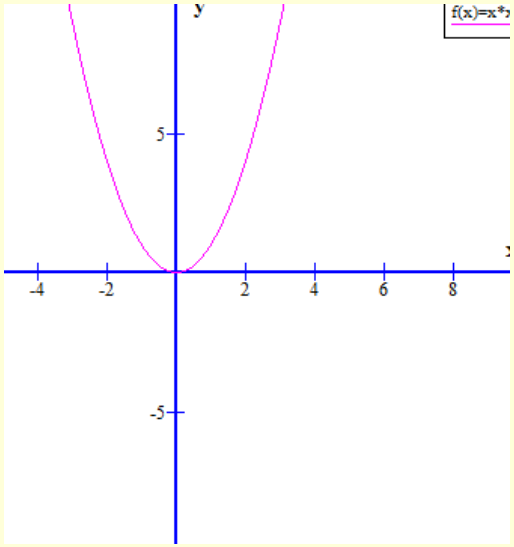


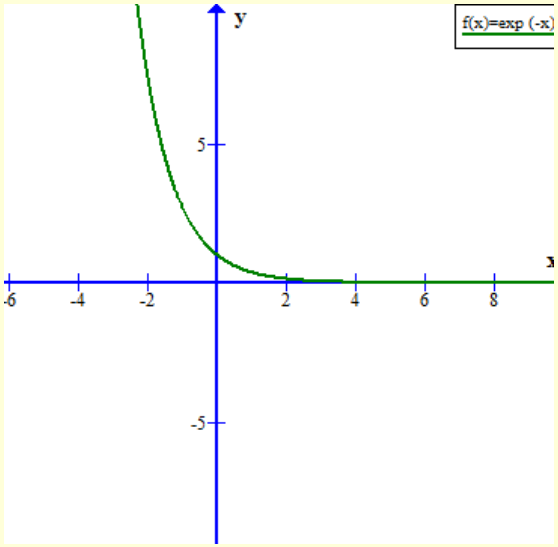


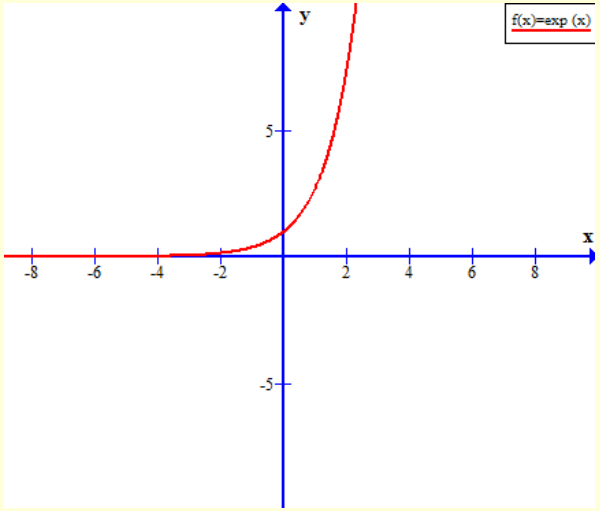
Pr. 7 – 35 / 21:

$$\lim_{x\to\infty} x^2 e^{-x} \qquad 0$$

$$\lim_{x\to\infty} x^2 e^{-x} \overset{\infty.0}{=} \lim_{x\to\infty} \frac{x^2}{\frac{1}{e^{-x}}} \overset{\frac{\infty}{\infty}\ L'H}{=} \lim_{x\to\infty} \frac{(x^2)'}{(e^x)'} = \lim_{x\to\infty} \frac{2x}{e^x} \overset{\frac{\infty}{\infty}\ L'H}{=} \lim_{x\to\infty} \frac{(2x)'}{(e^x)'} = \lim_{x\to\infty} \frac{2}{e^x} = \frac{2}{\infty} = 0$$







Dú – str. 34, 35 / 22