Technical University of Košice

Faculty of Electrical Engineering and Informatics



Applied Statistics

Exercise 5 - Worked Solutions in R

Jana Fortes

Version 2

```r
# EXERCISE 5.6
# Two protective coatings are applied to 24 different objects:
# 10 receive coating 1 and 14 receive coating 2. Wear is measured
# after a certain time. Mean wear is 62 for coating 1 and 60 for
# coating 2. The known population variances are 16 and 15.
# Assume independent random samples from normal populations.
# a) Test equal effectiveness at alpha = 0.05.
# b) Test greater effectiveness of coating 1 at alpha = 0.01.
# c) Test lower effectiveness of coating 1 at alpha = 0.1.
# Lower mean wear means greater effectiveness.

n1=10
m1=62
n2=14
m2=60
s1=16  # Known population variance of sample 1
s2=15  # Known population variance of sample 2
ss1=sqrt(s1)
ss2=sqrt(s2)

# Two-sample Z-test of equal means: population variances are known.
TS = (m1-m2)*sqrt(n1*n2)/sqrt(n2*s1+n1*s2)
TS

# a) H0: mu1 = mu2; H1: mu1 != mu2
alpha1 = 0.05  # Significance level
KH1 = qnorm(1-alpha1/2, 0, 1)  # Critical value
cat("Critical region: (-Inf,", -KH1, ") union (",
    KH1, ", Inf)\n")
# Decision using the critical region
if (TS < -KH1 || TS > KH1) {
  cat("Statistic in the critical region: reject H0.\n")
} else {
  cat("Statistic outside the critical region: do not reject H0.\n")
}

# Check using the corresponding test (BSDA package)
# install.packages("BSDA")  # Run once if the package is needed.
if (requireNamespace("BSDA", quietly = TRUE)) {
  test_result = BSDA::zsum.test(
      mean.x = m1, sigma.x = ss1, n.x = n1,
      mean.y = m2, sigma.y = ss2, n.y = n2,
      alternative = "two.sided", mu = 0,
      conf.level = 1 - alpha1)
  print(test_result)
  p = test_result$p.value
  if (p > alpha1) {
    cat("p > alpha: do not reject H0.\n")
  } else {
    cat("p <= alpha: reject H0.\n")
  }
} else {
  cat("BSDA check skipped. The direct calculation is complete.\n")
}
# Interpretation: at alpha = 0.05, do not reject H0.
# There is insufficient evidence of different mean wear.
# Non-rejection does not establish equal effectiveness.

# b) H0: mu1 = mu2; H1: mu1 < mu2
alpha2 = 0.01  # Significance level
KH2 = qnorm(1-alpha2, 0, 1)  # Critical value
cat("Critical region: (-Inf,", -KH2, ")\n")
# Decision using the critical region
if (TS < -KH2) {
  cat("Statistic in the critical region: reject H0.\n")
} else {
  cat("Statistic outside the critical region: do not reject H0.\n")
}
```

```r
# EXERCISE 5.6 (CONTINUED)

# Check using the corresponding test (BSDA package)
# install.packages("BSDA")  # Run once if the package is needed.
if (requireNamespace("BSDA", quietly = TRUE)) {
  test_result = BSDA::zsum.test(
      mean.x = m1, sigma.x = ss1, n.x = n1,
      mean.y = m2, sigma.y = ss2, n.y = n2,
      alternative = "less", mu = 0,
      conf.level = 1 - alpha2)
  print(test_result)
  p = test_result$p.value
  if (p > alpha2) {
    cat("p > alpha: do not reject H0.\n")
  } else {
    cat("p <= alpha: reject H0.\n")
  }
} else {
  cat("BSDA check skipped. The direct calculation is complete.\n")
}
# Interpretation: at alpha = 0.01, do not reject H0.
# There is insufficient evidence that coating 1 is more effective.

# c) H0: mu1 = mu2; H1: mu1 > mu2
alpha3 = 0.1  # Significance level
KH3 = qnorm(1-alpha3, 0, 1)  # Critical value
cat("Critical region: (", KH3, ", Inf)\n")
# Decision using the critical region
if (TS > KH3) {
  cat("Statistic in the critical region: reject H0.\n")
} else {
  cat("Statistic outside the critical region: do not reject H0.\n")
}

# Check using the corresponding test (BSDA package)
# install.packages("BSDA")  # Run once if the package is needed.
if (requireNamespace("BSDA", quietly = TRUE)) {
  test_result = BSDA::zsum.test(
      mean.x = m1, sigma.x = ss1, n.x = n1,
      mean.y = m2, sigma.y = ss2, n.y = n2,
      alternative = "greater", mu = 0,
      conf.level = 1 - alpha3)
  print(test_result)
  p = test_result$p.value
  if (p > alpha3) {
    cat("p > alpha: do not reject H0.\n")
  } else {
    cat("p <= alpha: reject H0.\n")
  }
} else {
  cat("BSDA check skipped. The direct calculation is complete.\n")
}
# Interpretation: at alpha = 0.10, do not reject H0.
# There is insufficient evidence that coating 1 is less effective.

# If the original observations x and y are supplied, the check is:
# BSDA::z.test(x, y, alternative = "two.sided", mu = 0,
#              sigma.x = ss1, sigma.y = ss2, conf.level = 0.95)
# This is a template only: raw x and y were not given in Exercise 5.6.
```

```
# EXERCISE 5.7
# Two protective coatings are applied to 24 different objects:
# 10 receive coating 1 and 14 receive coating 2. Wear is measured
# after a certain time. Mean wear is 62 for coating 1 and 60 for
# coating 2. The sample variances are 16 and 15.
# Assume independent random samples from normal populations.
# a) Test equal effectiveness at alpha = 0.05.
# b) Test greater effectiveness of coating 1 at alpha = 0.01.
# c) Test lower effectiveness of coating 1 at alpha = 0.1.
# Lower mean wear means greater effectiveness.

n1=10
m1=62
n2=14
m2=60
s1=16  # Sample variance, not standard deviation
s2=15  # Sample variance, not standard deviation
ss1=sqrt(s1)
ss2=sqrt(s2)

# a) First test H0: sigma1^2 = sigma2^2 against inequality.
alpha1 = 0.05
TS=s1/s2
KH11=qf(alpha1/2,n1-1,n2-1)
KH21=qf(1-alpha1/2,n1-1,n2-1)
cat("F-test critical region: (0,", KH11, ") union (",
    KH21, ", Inf)\n")
# Decision using the critical region
if (TS < KH11 || TS > KH21) {
  cat("Statistic in the critical region: reject H0.\n")
} else {
  cat("Statistic outside the critical region: do not reject H0.\n")
}
# Interpretation: do not reject equality of the variances.
# Following the textbook, assume equal variances for the pooled test.
# Non-rejection alone does not prove that the variances are equal.

# Two-sample t-test: unknown variances assumed equal.
TS1=((m1-m2)/sqrt((n1-1)*s1+(n2-1)*s2))*
    sqrt((n1*n2*(n1+n2-2))/(n1+n2))
TS1

# a) H0: mu1 = mu2; H1: mu1 != mu2
alpha1 = 0.05  # Significance level
KH1 = qt(1-alpha1/2, n1+n2-2)  # Critical value
cat("Critical region: (-Inf,", -KH1, ") union (",
    KH1, ", Inf)\n")
# Decision using the critical region
if (TS1 < -KH1 || TS1 > KH1) {
  cat("Statistic in the critical region: reject H0.\n")
} else {
  cat("Statistic outside the critical region: do not reject H0.\n")
}
```

```
# EXERCISE 5.7 (CONTINUED)

# Check using the corresponding test (BSDA package)
# install.packages("BSDA")  # Run once if the package is needed.
if (requireNamespace("BSDA", quietly = TRUE)) {
  test_result = BSDA::tsum.test(
      mean.x = m1, s.x = ss1, n.x = n1,
      mean.y = m2, s.y = ss2, n.y = n2,
      alternative = "two.sided", mu = 0, var.equal = TRUE,
      conf.level = 1 - alpha1)
  print(test_result)
  p = test_result$p.value
  if (p > alpha1) {
    cat("p > alpha: do not reject H0.\n")
  } else {
    cat("p <= alpha: reject H0.\n")
  }
} else {
  cat("BSDA check skipped. The direct calculation is complete.\n")
}
# Interpretation: at alpha = 0.05, do not reject H0.
# There is insufficient evidence of different mean wear.
# This does not prove equal effectiveness.

# b) First test H0: sigma1^2 = sigma2^2 against inequality.
alpha2 = 0.01
TS=s1/s2
KH12=qf(alpha2/2,n1-1,n2-1)
KH22=qf(1-alpha2/2,n1-1,n2-1)
cat("F-test critical region: (0,", KH12, ") union (",
    KH22, ", Inf)\n")
# Decision using the critical region
if (TS < KH12 || TS > KH22) {
  cat("Statistic in the critical region: reject H0.\n")
} else {
  cat("Statistic outside the critical region: do not reject H0.\n")
}
# Interpretation: do not reject equality of the variances.
# Following the textbook, assume equal variances for the pooled test.
# Non-rejection alone does not prove that the variances are equal.

# Two-sample t-test: unknown variances assumed equal.
TS1=((m1-m2)/sqrt((n1-1)*s1+(n2-1)*s2))*
    sqrt((n1*n2*(n1+n2-2))/(n1+n2))
TS1

# b) H0: mu1 = mu2; H1: mu1 < mu2
alpha2 = 0.01  # Significance level
KH2 = qt(1-alpha2, n1+n2-2)  # Critical value
cat("Critical region: (-Inf,", -KH2, ")\n")
# Decision using the critical region
if (TS1 < -KH2) {
  cat("Statistic in the critical region: reject H0.\n")
} else {
  cat("Statistic outside the critical region: do not reject H0.\n")
}
```

```
# EXERCISE 5.7 (CONTINUED)

# Check using the corresponding test (BSDA package)
# install.packages("BSDA")  # Run once if the package is needed.
if (requireNamespace("BSDA", quietly = TRUE)) {
  test_result = BSDA::tsum.test(
      mean.x = m1, s.x = ss1, n.x = n1,
      mean.y = m2, s.y = ss2, n.y = n2,
      alternative = "less", mu = 0, var.equal = TRUE,
      conf.level = 1 - alpha2)
  print(test_result)
  p = test_result$p.value
  if (p > alpha2) {
    cat("p > alpha: do not reject H0.\n")
  } else {
    cat("p <= alpha: reject H0.\n")
  }
} else {
  cat("BSDA check skipped. The direct calculation is complete.\n")
}
# Interpretation: at alpha = 0.01, do not reject H0.
# There is insufficient evidence that coating 1 is more effective.

# c) First test H0: sigma1^2 = sigma2^2 against inequality.
alpha3 = 0.1
TS=s1/s2
KH13=qf(alpha3/2,n1-1,n2-1)
KH23=qf(1-alpha3/2,n1-1,n2-1)
cat("F-test critical region: (0,", KH13, ") union (",
    KH23, ", Inf)\n")
# Decision using the critical region
if (TS < KH13 || TS > KH23) {
  cat("Statistic in the critical region: reject H0.\n")
} else {
  cat("Statistic outside the critical region: do not reject H0.\n")
}
# Interpretation: do not reject equality of the variances.
# Following the textbook, assume equal variances for the pooled test.
# Non-rejection alone does not prove that the variances are equal.

# Two-sample t-test: unknown variances assumed equal.
TS1=((m1-m2)/sqrt((n1-1)*s1+(n2-1)*s2))*
    sqrt((n1*n2*(n1+n2-2))/(n1+n2))
TS1

# c) H0: mu1 = mu2; H1: mu1 > mu2
alpha3 = 0.1  # Significance level
KH3 = qt(1-alpha3, n1+n2-2)  # Critical value
cat("Critical region: (", KH3, ", Inf)\n")
# Decision using the critical region
if (TS1 > KH3) {
  cat("Statistic in the critical region: reject H0.\n")
} else {
  cat("Statistic outside the critical region: do not reject H0.\n")
}
```

```r
# EXERCISE 5.7 (CONTINUED)

# Check using the corresponding test (BSDA package)
# install.packages("BSDA")  # Run once if the package is needed.
if (requireNamespace("BSDA", quietly = TRUE)) {
  test_result = BSDA::tsum.test(
      mean.x = m1, s.x = ss1, n.x = n1,
      mean.y = m2, s.y = ss2, n.y = n2,
      alternative = "greater", mu = 0, var.equal = TRUE,
      conf.level = 1 - alpha3)
  print(test_result)
  p = test_result$p.value
  if (p > alpha3) {
    cat("p > alpha: do not reject H0.\n")
  } else {
    cat("p <= alpha: reject H0.\n")
  }
} else {
  cat("BSDA check skipped. The direct calculation is complete.\n")
}
# Interpretation: at alpha = 0.1, do not reject H0.
# There is insufficient evidence that coating 1 is less effective.

# OPTIONAL TEMPLATE: WHEN RAW OBSERVATIONS x AND y ARE GIVEN
# The original exercise gives summary statistics, not raw x and y.
# The following commands are therefore comments, not executed code.
# alpha = 0.05
# var.test(x, y, conf.level = 1-alpha)
# p1 = var.test(x, y, conf.level = 1-alpha)$p.value
# if (p1 > alpha) {
#   test_result = t.test(x, y, var.equal = TRUE,
#       conf.level = 1-alpha, alternative = "two.sided")
# } else {
#   test_result = t.test(x, y, var.equal = FALSE,
#       conf.level = 1-alpha, alternative = "two.sided")
# }
# print(test_result)
# p = test_result$p.value
# if (p > alpha) cat("Do not reject H0.\n") else cat("Reject H0.\n")
# This follows the textbook's preliminary F-test procedure.
# Non-rejection of equal variances does not prove that they are equal.
```

```r
# EXERCISE 5.8
# Two protective coatings are applied to 24 different objects:
# 10 receive coating 1 and 14 receive coating 2. Wear is measured
# after a certain time. Mean wear is 62 for coating 1 and 60 for
# coating 2. The sample variances are 4 and 16.
# Assume independent random samples from normal populations.
# a) Test equal effectiveness at alpha = 0.05.
# b) Test greater effectiveness of coating 1 at alpha = 0.1.
# c) Test lower effectiveness of coating 1 at alpha = 0.05.
# Lower mean wear means greater effectiveness.

n1=10
m1=62
n2=14
m2=60
s1=4  # Sample variance, not standard deviation
s2=16  # Sample variance, not standard deviation
ss1=sqrt(s1)
ss2=sqrt(s2)

# a) First test H0: sigma1^2 = sigma2^2 against inequality.
alpha1 = 0.05
TS=s1/s2
KH11=qf(alpha1/2,n1-1,n2-1)
KH21=qf(1-alpha1/2,n1-1,n2-1)
cat("F-test critical region: (0,", KH11, ") union (",
    KH21, ", Inf)\n")
# Decision using the critical region
if (TS < KH11 || TS > KH21) {
  cat("Statistic in the critical region: reject H0.\n")
} else {
  cat("Statistic outside the critical region: do not reject H0.\n")
}
# Interpretation: reject equality of the variances.
# Use the Aspin-Welch test, as in the textbook.

# Aspin-Welch test: unknown variances, equality not assumed.
TS1=(m1-m2)/sqrt(s1/n1+s2/n2)
m=(s1/n1+s2/n2)^2/((s1/n1)^2/(n1-1)+(s2/n2)^2/(n2-1))
TS1; m  # Retain full precision in the degrees of freedom.

# a) H0: mu1 = mu2; H1: mu1 != mu2
alpha1 = 0.05  # Significance level
KH1 = qt(1-alpha1/2, m)  # Critical value
cat("Critical region: (-Inf,", -KH1, ") union (",
    KH1, ", Inf)\n")
# Decision using the critical region
if (TS1 < -KH1 || TS1 > KH1) {
  cat("Statistic in the critical region: reject H0.\n")
} else {
  cat("Statistic outside the critical region: do not reject H0.\n")
}
```

```
# EXERCISE 5.8 (CONTINUED)

# Check using the corresponding test (BSDA package)
# install.packages("BSDA")  # Run once if the package is needed.
if (requireNamespace("BSDA", quietly = TRUE)) {
  test_result = BSDA::tsum.test(
      mean.x = m1, s.x = ss1, n.x = n1,
      mean.y = m2, s.y = ss2, n.y = n2,
      alternative = "two.sided", mu = 0, var.equal = FALSE,
      conf.level = 1 - alpha1)
  print(test_result)
  p = test_result$p.value
  if (p > alpha1) {
    cat("p > alpha: do not reject H0.\n")
  } else {
    cat("p <= alpha: reject H0.\n")
  }
} else {
  cat("BSDA check skipped. The direct calculation is complete.\n")
}
# Interpretation: at alpha = 0.05, do not reject H0.
# There is insufficient evidence of different mean wear.
# This does not prove equal effectiveness.

# b) First test H0: sigma1^2 = sigma2^2 against inequality.
alpha2 = 0.1
TS=s1/s2
KH12=qf(alpha2/2,n1-1,n2-1)
KH22=qf(1-alpha2/2,n1-1,n2-1)
cat("F-test critical region: (0,", KH12, ") union (",
    KH22, ", Inf)\n")
# Decision using the critical region
if (TS < KH12 || TS > KH22) {
  cat("Statistic in the critical region: reject H0.\n")
} else {
  cat("Statistic outside the critical region: do not reject H0.\n")
}
# Interpretation: reject equality of the variances.
# Use the Aspin-Welch test, as in the textbook.

# Aspin-Welch test: unknown variances, equality not assumed.
TS1=(m1-m2)/sqrt(s1/n1+s2/n2)
m=(s1/n1+s2/n2)^2/((s1/n1)^2/(n1-1)+(s2/n2)^2/(n2-1))
TS1; m  # Retain full precision in the degrees of freedom.

# b) H0: mu1 = mu2; H1: mu1 < mu2
alpha2 = 0.1  # Significance level
KH2 = qt(1-alpha2, m)  # Critical value
cat("Critical region: (-Inf,", -KH2, ")\n")
# Decision using the critical region
if (TS1 < -KH2) {
  cat("Statistic in the critical region: reject H0.\n")
} else {
  cat("Statistic outside the critical region: do not reject H0.\n")
}
```

```
# EXERCISE 5.8 (CONTINUED)

# Check using the corresponding test (BSDA package)
# install.packages("BSDA")  # Run once if the package is needed.
if (requireNamespace("BSDA", quietly = TRUE)) {
  test_result = BSDA::tsum.test(
      mean.x = m1, s.x = ss1, n.x = n1,
      mean.y = m2, s.y = ss2, n.y = n2,
      alternative = "less", mu = 0, var.equal = FALSE,
      conf.level = 1 - alpha2)
  print(test_result)
  p = test_result$p.value
  if (p > alpha2) {
    cat("p > alpha: do not reject H0.\n")
  } else {
    cat("p <= alpha: reject H0.\n")
  }
} else {
  cat("BSDA check skipped. The direct calculation is complete.\n")
}
# Interpretation: at alpha = 0.1, do not reject H0.
# There is insufficient evidence that coating 1 is more effective.

# c) First test H0: sigma1^2 = sigma2^2 against inequality.
alpha3 = 0.05
TS=s1/s2
KH13=qf(alpha3/2,n1-1,n2-1)
KH23=qf(1-alpha3/2,n1-1,n2-1)
cat("F-test critical region: (0,", KH13, ") union (",
    KH23, ", Inf)\n")
# Decision using the critical region
if (TS < KH13 || TS > KH23) {
  cat("Statistic in the critical region: reject H0.\n")
} else {
  cat("Statistic outside the critical region: do not reject H0.\n")
}
# Interpretation: reject equality of the variances.
# Use the Aspin-Welch test, as in the textbook.

# Aspin-Welch test: unknown variances, equality not assumed.
TS1=(m1-m2)/sqrt(s1/n1+s2/n2)
m=(s1/n1+s2/n2)^2/((s1/n1)^2/(n1-1)+(s2/n2)^2/(n2-1))
TS1; m  # Retain full precision in the degrees of freedom.

# c) H0: mu1 = mu2; H1: mu1 > mu2
alpha3 = 0.05  # Significance level
KH3 = qt(1-alpha3, m)  # Critical value
cat("Critical region: (", KH3, ", Inf)\n")
# Decision using the critical region
if (TS1 > KH3) {
  cat("Statistic in the critical region: reject H0.\n")
} else {
  cat("Statistic outside the critical region: do not reject H0.\n")
}
```

```r
# EXERCISE 5.8 (CONTINUED)

# Check using the corresponding test (BSDA package)
# install.packages("BSDA")  # Run once if the package is needed.
if (requireNamespace("BSDA", quietly = TRUE)) {
  test_result = BSDA::tsum.test(
      mean.x = m1, s.x = ss1, n.x = n1,
      mean.y = m2, s.y = ss2, n.y = n2,
      alternative = "greater", mu = 0, var.equal = FALSE,
      conf.level = 1 - alpha3)
  print(test_result)
  p = test_result$p.value
  if (p > alpha3) {
    cat("p > alpha: do not reject H0.\n")
  } else {
    cat("p <= alpha: reject H0.\n")
  }
} else {
  cat("BSDA check skipped. The direct calculation is complete.\n")
}
# Interpretation: at alpha = 0.05, do not reject H0.
# There is insufficient evidence that coating 1 is less effective.

# OPTIONAL TEMPLATE: WHEN RAW OBSERVATIONS x AND y ARE GIVEN
# The original exercise gives summary statistics, not raw x and y.
# The following commands are therefore comments, not executed code.
# alpha = 0.05
# var.test(x, y, conf.level = 1-alpha)
# p1 = var.test(x, y, conf.level = 1-alpha)$p.value
# if (p1 > alpha) {
#   test_result = t.test(x, y, var.equal = TRUE,
#       conf.level = 1-alpha, alternative = "two.sided")
# } else {
#   test_result = t.test(x, y, var.equal = FALSE,
#       conf.level = 1-alpha, alternative = "two.sided")
# }
# print(test_result)
# p = test_result$p.value
# if (p > alpha) cat("Do not reject H0.\n") else cat("Reject H0.\n")
# This follows the textbook's preliminary F-test procedure.
# Non-rejection of equal variances does not prove that they are equal.
```

```
# EXERCISE 5.9
# Eight chemical specimens are analysed using two methods.
# Each row contains measurements of the same specimen.
# Specimen    Polarographic (x)    Titration (y)
# 1           18.60                18.58
# 2           27.60                27.37
# 3           27.50                27.27
# 4           25.00                24.64
# 5           24.50                24.10
# 6           26.80                26.33
# 7           29.70                29.33
# 8           26.63                26.50
# At alpha = 0.01, test equality of the two population mean results.
# H0: mean(x-y) = 0; H1: mean(x-y) != 0.
# The data are paired. Assume independent pairs and normal differences.

x=c(18.60,27.60,27.50,25.00,24.50,26.80,29.70,26.63)
y=c(18.58,27.37,27.27,24.64,24.10,26.33,29.33,26.50)

n=length(x)
alpha = 0.01
di=x-y
d=(1/n)*sum(di)
sd=(1/(n-1))*sum((di-d)^2)
# In the original notation, sd stores the VARIANCE of differences.
# sqrt(sd) is their sample standard deviation.
TS=(d/sqrt(sd))*sqrt(n)
KH=qt(1-alpha/2,n-1)
cat("Critical region: (-Inf,", -KH, ") union (", KH, ", Inf)\n")
# Decision using the critical region
if (TS < -KH || TS > KH) {
  cat("Statistic in the critical region: reject H0.\n")
} else {
  cat("Statistic outside the critical region: do not reject H0.\n")
}

# Check using the paired t-test (stats, included with R).
test_result=t.test(x,y,paired=TRUE,alternative="two.sided",
                   conf.level=0.99)
print(test_result)
p=test_result$p.value
if (p > alpha) {
  cat("p > alpha: do not reject H0.\n")
} else {
  cat("p <= alpha: reject H0.\n")
}
# Interpretation: at alpha = 0.01, reject H0.
# There is evidence of a systematic difference in mean results.
# The mean difference, polarographic minus titration, is 0.27625.
# The paired t-test tests equality of means, not practical equivalence.
```