

5.4 Cvičenia 5

1. Riešte diferenciálnu rovnicu:

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| a) $\frac{1}{x}y' = \frac{1}{x^2+1}$: | $[\frac{1}{2} \ln(x^2+1) + C]$ |
| b) $(\sqrt{xy} + \sqrt{x})y' - y = 0$; | $[2\sqrt{y} + \ln y - 2\sqrt{x} = C]$ |
| c) $2^{x+y} + 3^{x-2y}y' = 0$; | $[\frac{(\frac{2}{3})^x}{\ln \frac{2}{3}} - \frac{18^{-y}}{\ln 18} = C]$ |
| d) $yy' + x = 1$; | $[(x-1)^2 + y^2 = C^2]$ |
| e) $(y+xy)dx + (x-xy)dy = 0$; | $[x-y + \ln xy = C]$ |
| f) $x^2y' + y = 0$; | $[y = Ce^{1/x}]$ |
| g) $x + xy + yy'(1+x) = 0$; | $[x+y = \ln[C(x+1)(y+1)]]$ |
| h) $(1+x^2)y' + y\sqrt{1+x^2} = xy$; | $[y = \frac{C\sqrt{1+x^2}}{x + \sqrt{1+x^2}}]$ |
| i) $(1+e^x)yy' = e^x$; | $[y^2/2 - \ln(1+e^x) = C]$ |
| j) $y' = -2x/y$; | $[y^2 = -2x^2 + C]$ |

2. Nájdite všetky riešenia diferenciálnej rovnice, ktoré prechádzajú bodom A ;

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| a) $y^2 + x^2y' = 0, A = (-1, 1)$; | $[x+y=0]$ |
| b) $2(1+e^x)yy' = e^x, A = (0, 0)$; | $[2e^{y^2} = e^x + 1]$ |
| c) $(1+y^2)dx = xydy, A = (2, 1)$; | $[x^2 = 2 + 2y^2]$ |
| d) $y' = 2\sqrt{y} \ln x, A = (e, 1)$; | $[y = [x(\ln x - 1) + 1]^2, x \in (0, \infty)]$ |
| e) $y' = \frac{x}{\sin y}, A = (1, \pi/3)$; | $[y = \arccos(1 - \frac{1}{2}x^2), x \in (0, 2)]$ |
| f) $y' = \frac{\sin y}{x}, A = (1, \pi/2)$; | $[y = 2\operatorname{arctg} x, x \in \mathbf{R}^+]$ |
| g) $y' = \frac{1+y^2}{x}, A = (-1, 0)$; | $[y = \operatorname{tg} \ln(-x), x \in (-e^{\pi/2}, -e^{-\pi/2})]$ |
| h) $y' = e^x e^y, A = (1, -1)$; | $[y = -\ln(2e - e^x)]$ |
| i) $y' = -y \operatorname{tg} x, A = (0, -2)$; | $[y = -2 \cos x, x \in (-\pi/2, \pi/2)]$ |
| j) $y' = -\frac{y \ln y}{x}, A = (-1, e)$. | $[y = e^{-1/x}, x \in \mathbf{R}^-]$ |

3. Riešte diferenciálnu rovnicu:

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|---|--|
| a) $y' + 2xy = 4x$; | $[y = Ce^{-x^2} + 2, x \in \mathbf{R}]$ |
| b) $y' + y = e^x$; | $[y = e^x/2 + Ce^{-x}, x \in \mathbf{R}]$ |
| c) $y' - y \cot x = \sin x$; | $[y = (x+C) \sin x]$ |
| d) $(x^2+1)y' + 4xy = 3$; | $[y(x^2+1)^2 = x^3 + 3x + C]$ |
| e) $xy' + y = x \sin x$. | $[y = -\cos x + (\sin x + C)/x]$ |
| f) $xy' + y = \ln x + 1$; | $[y = \ln x + C/x]$ |
| g) $y' + x^2y = x^2$; | $[y = 1 + Ce^{-x^3/3}]$ |
| h) $y' + y/(1+x) + x^2 = 0$; | $[y = \frac{C - x^3/3 - x^4/4}{1+x}]$ |
| i) $y' + \frac{xy}{1+x^2} = \frac{1}{x(1+x^2)}$; | $[y = \frac{1}{\sqrt{1+x^2}}(C + \ln \frac{-1+\sqrt{1+x^2}}{x})]$ |
| j) $y' - 2y = 3e^{2x}$; | $[y = (3x+C)e^{2x}]$ |
| k) $y' + y = 2x^2 - 2x + 1$; | $[y = Ce^{-x} + 2x^2 - 6x + 7]$ |
| l) $y' + 4y = 5 \sin 3x$. | $[y = Ce^{-4x} + 4/5 \sin 3x - 3/5 \cos 3x]$ |

4. Nájdite riešenia daných diferenciálnych rovníc, ktoré prechádzajú bodom A :

a) $y' = \frac{2x}{1+x^2}y$, $A = (1, 2)$; $[y = 1 + x^2, x \in \mathbf{R}]$

b) $y' = 2xy + 1$, $A = (0, 0)$; $[y = e^{x^2} \int_0^x e^{-t^2} dt, x \in \mathbf{R}]$

c) $y' + 3y/x = 2/x^3$, $A = (1, 1)$; $[y = -1/x^3 + 2/x^2, x > 0]$

d) $y' + y = x + 1$, $A = (0, 1)$; $[y = x + e^{-x}]$

e) $y' - y \sin x = \sin x \cos x$, $A = (-\pi/2, 1)$; $[y = 1 - \cos x, x \in \mathbf{R}]$

f) $y' + y/x - 3x = 0$, $A = (1, 2)$; $[y = 1/x + x^2, x > 0]$

g) $y' \cos x - y \sin x = 2x$, $A = (0, 0)$; $[y = \frac{x^2}{\cos x}, x \in (-\pi/2, \pi/2)]$