

15.5 Laplaceova transformácia

Hľadanie obrazu ku predmetu $y(t)$ v premennej t a obrazu v premennej s :

```
> with(inttrans):
```

Príklad 1:

```
> y(t) := t^3 + sinh(3*t);
```

$$y(t) := t^3 + \sinh(3t)$$

```
> Y(s) := laplace(t^3 + sinh(3*t) = y(t), t, s);
```

$$Y(s) := \frac{6}{s^4} + \frac{3}{s^2 - 9} = \text{laplace}(y(t), t, s)$$

Príklad 2:

```
> y(t) := e^(-2*t)*sin(3*t)*cos(5*t);
```

$$y(t) := e^{(-2t)} \sin(3t) \cos(5t)$$

```
> Y(s) := laplace(e^(-2*t)*sin(3*t)*cos(5*t), t, s);
```

$$Y(s) := \frac{4}{s^2 + 4s \ln(e) + 4 \ln(e)^2 + 64} - \frac{1}{s^2 + 4s \ln(e) + 4 \ln(e)^2 + 4}$$

Inverzná Laplaceova transformácia:

```
> with(inttrans):
```

```
> Y(s) := 1/(s-a) + 1/(s^2+b) + 1;
```

$$Y(s) := \frac{1}{s-a} + \frac{1}{s^2+b} + 1$$

```
> y(t) := Invlaplace(1/(s-a) + 1/(s^2+b) + 1, s, t) = invlaplace(1/(s-a) + 1/(s^2+b) + 1, s, t);
```

$$y(t) := \text{Invlaplace}\left(\frac{1}{s-a} + \frac{1}{s^2+b} + 1, s, t\right) = e^{(at)} + \frac{\sin(\sqrt{b}t)}{\sqrt{b}} + \text{Dirac}(t)$$

```
> restart:with(inttrans):
```

```
> Y(s) := 1/(((s-a)^2+1)*((s-a)^2+4));
```

$$Y(s) := \frac{1}{((s-a)^2+1)((s-a)^2+4)}$$

```
> y(t) := Invlaplace(1/(((s-a)^2+1)*((s-a)^2+4)), s, t) = invlaplace(1/(((s-a)^2+1)*((s-a)^2+4)), s, t);
```

$$y(t) := \text{Invlaplace}\left(\frac{1}{((s-a)^2+1)((s-a)^2+4)}, s, t\right) = \frac{1}{6} e^{(at)} (2 \sin(t) - \sin(2t))$$

```
>
```

Riešenie diferenciálnych rovníc pomocou Laplaceovej transformácie:

>

> **de1 := diff(y(t),t\$2) + 5*diff(y(t),t) + 6*y(t) = 0;**

$$de1 := \left(\frac{d^2}{dt^2} y(t) \right) + 5 \left(\frac{d}{dt} y(t) \right) + 6 y(t) = 0$$

> **dsolve({de1, y(0)=0, D(y)(0)=1}, y(t),method=laplace);**

$$y(t) = -e^{(-3t)} + e^{(-2t)}$$