15.3 Riešenie diferenciálnych rovníc(všeobecné, partikulárne, grafy riešení):

Príklad 1: Hľadá riešenie diferenciálnej rovnice

$$de1 := \left(\frac{d^2}{dx^2}\,y(x)\right) - 3\left(\frac{d}{dx}\,y(x)\right) + 2\,y(x) = 0$$

```
> de1 :=diff(y(x),x$2)-3*diff(y(x),x)+2*y(x)=0;
```

$$de1 := \left(\frac{d^2}{dx^2}\,y(x)\right) - 3\left(\frac{d}{dx}\,y(x)\right) + 2\,y(x) = 0$$

Všeobecné riešenie

```
> dsolve(de1,y(x));
```

$$y(x) = \_C1\,\mathbf{e}^{(2\,x)} + \_C2\,\mathbf{e}^{x}$$

Partikulárne riešenie postupne pre začiatočné podmienky in1, in2,in3

```
> in1 :=y(0)=1,D(y)(0)=0;
```

$$in1 := y(0) = 1,\ D(y)(0) = 0$$

```
> in2 :=y(0)=2,D(y)(0)=0;
```

$$in2 := y(0) = 2,\ D(y)(0) = 0$$

```
> in3 :=y(0)=3,D(y)(0)=0;
```

$$in3 := y(0) = 3,\ D(y)(0) = 0$$

```
> dsolve({de1,in1},{y(x)});
```

$$y(x) = -\mathbf{e}^{(2\,x)} + 2\,\mathbf{e}^{x}$$

```
> dsolve({de1,in2},{y(x)});
```

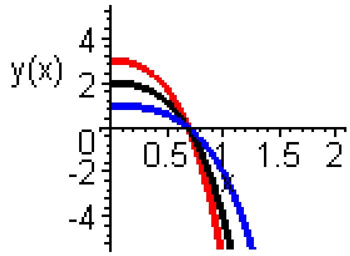
$$y(x) = -2\,\mathbf{e}^{(2\,x)} + 4\,\mathbf{e}^{x}$$

```
> dsolve({de1,in3},{y(x)});
```

$$y(x) = -3\,\mathbf{e}^{(2\,x)} + 6\,\mathbf{e}^{x}$$

Nakreslí grafy jednotlivých riešení

```
> with(DEtools):
DEplot({de1},\
{y(x)},x=0..2,[[in1],[in2],[in3]],linecolor=[blue,black,red],y
=-5..5,stepsize=.5);
```



Príklad 2: Hľadá riešenie diferenciálnej rovnice

$$de2 := \left(\frac{d^2}{dx^2}\,y(x)\right) + 3\left(\frac{d}{dx}\,y(x)\right) + 2\,y(x) = 0$$

```
> de2 :=diff(y(x),x$2)+3*diff(y(x),x)+2*y(x)=0;
```

$$de2 := \left(\frac{d^2}{dx^2}\,y(x)\right) + 3\left(\frac{d}{dx}\,y(x)\right) + 2\,y(x) = 0$$

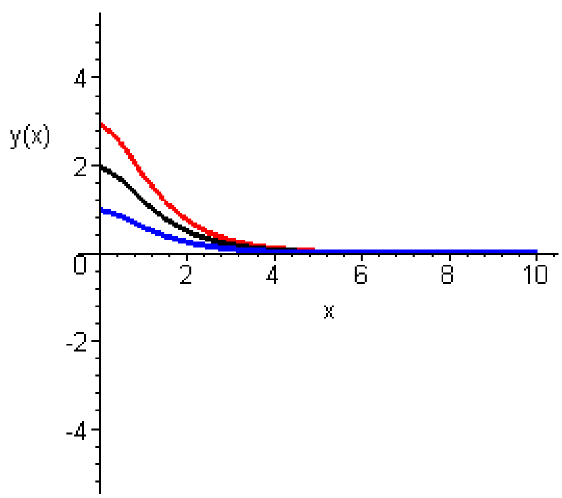
Všeobecné riešenie

```
> dsolve(de2,y(x));
```

$$y(x) = \_C1\,\mathbf{e}^{(-2\,x)} + \_C2\,\mathbf{e}^{(-x)}$$

Nakreslí grafy jednotlivých riešení pri zadaných začiatočných podmienkach

```
> DEplot({de2},\
{y(x)},x=0..10,[[in1],[in2],[in3]],linecolor=[blue,black,red],
y=-5..5,stepsize=.5);
```



Vidíme, že dané riešenia s rastúcim x sa približujú ku triviálnemu riešeniu.

Riešenie systému diferenciálnych rovníc r1, r2:

```
> r1 := diff(y(x),x)=z(x);
```

$$r1 := \frac{d}{dx}\,y(x) = z(x)$$

```
> r2 :=diff(z(x),x)=0.3*y(x)-0.02*z(x);
```

$$r2 := \frac{d}{dx} z(x) = 0.3\ y(x) - 0.02\ z(x)$$

Začiatočné podmienky

```
> in1 :=y(0)=1,z(0)=0;
```

$$in1 := y(0) = 1,\ z(0) = 0$$

```
> in2 :=y(0)=2,z(0)=0;
```

$$in2 := y(0) = 2,\ z(0) = 0$$

```
> in3 :=y(0)=3,z(0)=0;
```

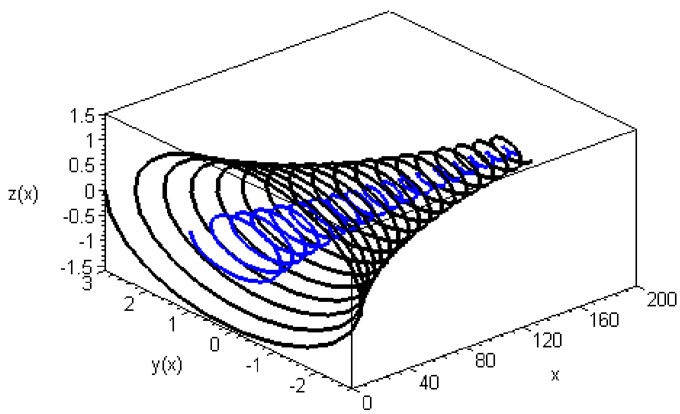
$$in3 := y(0) = 3,\ z(0) = 0$$

```
>
> r3 :=diff(z(x),x)=-.3*y(x)-.02*z(x);
```

$$r3 := \frac{d}{dx} z(x) = -0.3\ y(x) - 0.02\ z(x)$$

Nakreslí graf riešení odpovedajúcich začiatočným podmienkam in1, in2 pre sústavu r1, r3.

```
> with(DEtools):
>
DEplot3d({r1,r3},{y(x),z(x)},x=0..200,[[in1],[in3]],linecolor=
[blue,black],colour=black,stepsize=0.5);
```




```
> ode := diff(y(x),x)=z(x),diff(z(x),x)=-0.3*y(x)-0.02*z(x);
```

$$ode := \frac{d}{dx} y(x) = z(x),\ \frac{d}{dx} z(x) = -3\ y(x) - 2\ z(x)$$

```
> in4 := y(0)=1,z(0)=0;
```

$$in4 := y(0) = 1,\ z(0) = 0$$

```
> fun := {y(x),z(x)};
```

$$fun := \{ y(x),\ z(x) \}$$

Všeobecné riešenie sústavy : $ode := \frac{d}{dx} y(x) = z(x),\ \frac{d}{dx} z(x) = -3\ y(x) - 2\ z(x)$

```
> dsolve( {ode},fun);
```

$\{z(x) =$

$$e^{(-x)}\,(-\_C1\,\sin(\sqrt{2}\,x) + \_C1\,\cos(\sqrt{2}\,x)\sqrt{2} - \_C2\,\cos(\sqrt{2}\,x) - \_C2\,\sin(\sqrt{2}\,x)\sqrt{2}),$$

$$y(x) = e^{(-x)}\,(\_C1\,\sin(\sqrt{2}\,x) + \_C2\,\cos(\sqrt{2}\,x))\}$$

Partikulárne riešenie odpovedajúce začiatočnej podmienke in1:

```
> dsolve( {ode,in1},fun);
```

$$\{z(x) = -\frac{3}{20}\,e^{(-x)}\sqrt{2}\,\sin(\sqrt{2}\,x),\ y(x) = e^{(-x)}\left(\frac{1}{20}\sqrt{2}\,\sin(\sqrt{2}\,x) + \frac{1}{10}\cos(\sqrt{2}\,x)\right)\}$$

>

Fázový portrét sústavy $ode := \frac{d}{dx}\,y(x) = z(x),\ \frac{d}{dx}\,z(x) = -3\,y(x) - 2\,z(x)$:

```
> with(DEtools):
phaseportrait([D(y)(x)=z(x),D(z)(x)=-3*y(x)-2*z(x)], \
[y(x),z(x)],x=0..20,[[y(0)=10,z(0)=-1],[y(0)=20,z(0)=-
1]],stepsize=.05, \
scene=[y(x),z(x)],
linecolor=[gold,blue],method=classical[foreuler]);
```